

FLUORESCENT SCATTERING IN PLANETARY ATMOSPHERES

III. FORMATION OF LYMAN-BIRGE-HOPFIELD BANDS OF
N₂ IN THE MARTIAN ATMOSPHERE

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ABSTRACT

Formation of the Lyman-Birge-Hopfield bands of N₂ in a planetary atmosphere illuminated by the sun is investigated. This band system is expected to appear in the daytime ultraviolet spectrum of a planet observed from above. The band intensities are calculated for two model atmospheres of Mars: (1) a pure nitrogen atmosphere and (2) a nitrogen atmosphere with 1 per cent O₂ concentration. In the far ultraviolet the contribution of Rayleigh scattering to the continuum of the reflected spectrum is found to be much lower than the band intensities.

I. INTRODUCTION

The Lyman-Birge-Hopfield (LBH) bands of molecular nitrogen arise from forbidden transitions from the metastable state a¹ Π_g to the ground state X¹ Σ_g^+ . This band system falls in the far ultraviolet ($\lambda\lambda$ 1100–2400 Å), and its emission and absorption spectra have both been observed in the laboratory.

From the work of Thompson and Williams (1934), Herzberg (1946), and Oldenberg (1959) it appears that metastable N₂ molecules are critically sensitive to collisions with other molecules.

At temperatures of the order 100°–300° K, typical of the emitting regions of the upper atmosphere of the earth and probably Mars, nitrogen molecules are virtually all in the lowest vibrational level of the ground electronic state, X¹ Σ_g^+ . When excited by ultraviolet solar radiation, they make transitions to a¹ Π_g state and then back to the ground state emitting the LBH bands. Two factors, however, may seriously affect the emission of these bands.

a) At certain altitudes, where the collisional lifetime (i.e., the mean time between consecutive deactivating collisions) becomes less than the natural lifetime of the metastable levels, the excited molecules become deactivated before falling spontaneously to the ground state and emitting the LBH bands.

b) Molecular oxygen absorbs the incident ultraviolet radiation in the Schumann-Runge continuum. This continuum appears at 1759 Å and extends to 1300 Å (cf. Herzberg 1950).

In this paper scattering of solar radiation by two model atmospheres is investigated. Of these models, one is composed of pure nitrogen, and the second contains 99 per cent nitrogen and 1 per cent oxygen. The upper atmosphere of Mars probably has a composition between these limits. Even with O₂ absent in the lower atmosphere, a small amount will form in the region where CO₂ is partially dissociated (Chamberlain 1962).

The basic theory for this analysis is developed in Papers I and II of this series by Chamberlain and Sobouti (1962) and Sobouti (1962), respectively.

II. OUTLINE OF THE PROBLEM

Figure 1 represents a schematic energy-level diagram of the N₂ molecule. Vibrational levels and their corresponding rotational structure in the ground electronic state,

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$X^1\Sigma_g^+$, are denoted by v'' and J'' , respectively. In the excited state, $a^1\Pi_g$, they are indicated by v' and J' , respectively.

At temperatures of interest (100° – 300° K), only the ground vibrational level, $v'' = 0$, is populated. Upon the absorption of solar radiation, these molecules make transitions upward to the Π -state and then cascade back either to $v'' = 0$ (emitting the resonant band) or to any other vibrational level $v'' \neq 0$ (emitting a fluorescent band). Rotational selection rules permit only those transitions for which $J' - J'' = -1, 0, +1$, which are referred to as P, Q, R transitions, respectively. Any rotational or vibrational transition within a given electronic state is forbidden in homonuclear molecules.

From these considerations it emerges that the population of a J' level in a vibrational level v' is determined solely by the corresponding P, Q, R transitions from the ground level $v'' = 0$. In the resonant bands the intensities of any P, Q, R trio with the same upper level J' will be coupled to one another, since these transitions occur through both absorption and emission. Fluorescent bands, on the other hand, are exclusively emission

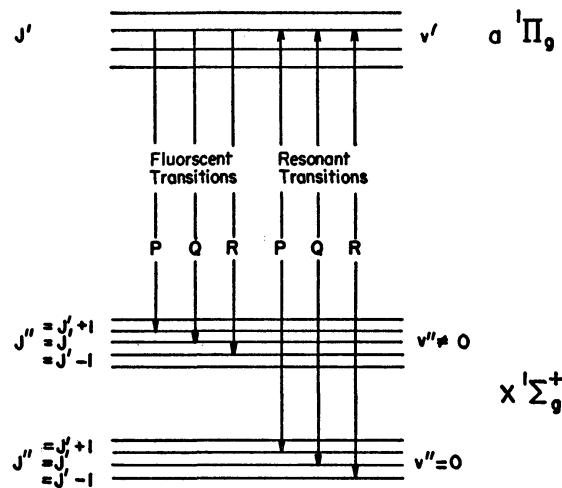


FIG. 1.—A schematic energy-level diagram illustrating rotational transitions in a band. Resonant transitions occur both in absorption and in emission. Fluorescent transitions, on the other hand, are exclusively emission features.

features. Hence the intensities of P, Q, R transitions in fluorescent bands will not be dependent on one another but will, however, depend on the behavior of the P, Q, R trio of the resonant bands (see Fig. 1).

It should be noted that in the present problem the *so-called* Λ -type doubling of the Π -state is neglected. Such doubling would split each rotational level of the Π -states into two sublevels, one with positive and one with negative statistics. Then in view of the selection rule $+ \leftrightarrow -$, the Q transitions become decoupled from P and R transitions (see Herzberg 1950, pp. 210 and 251–254). Nonetheless, it is shown at the end of section VI that the total intensity of a band remains very much the same with or without the Λ -type doubling.

III. ABSORPTION COEFFICIENTS

a) Line Absorption Coefficient

For a Doppler-broadened line the absorption coefficient per molecule is¹

$$\alpha(\nu)(v''J'', v'J') = \frac{\pi e^2}{mc} f(v''J'', v'J') \frac{c}{\pi^{1/2} U_{\nu_c}} \exp\left[-\frac{c^2}{U^2} \left(\frac{\nu - \nu_c}{\nu_c}\right)^2\right], \quad (1)$$

¹ For the formulae of this section and further references to them see Chamberlain (1961), pp. 1–26.

where ν = frequency, ν_e = frequency at the center of the line profile, e = electronic charge in e.s.u., m = electronic mass, c = velocity of light, and

$$U = \left(\frac{2kT}{M} \right)^{1/2} = \text{most probable molecular velocity}. \quad (2)$$

The f -value for a rotational line is

$$f(v''J'', v'J') = \frac{8\pi^2 m}{3h e^2} \nu_e R_e^2 q(v', v'') \frac{S(J', J'')}{2J'' + 1}, \quad (3)$$

in which the electronic, vibrational, and rotational contributions have been taken to be independent of one another. Here h is the Planck constant and R_e is the matrix element for the electronic transition $\Pi-\Sigma$. This matrix element is an unknown quantity, and, as we shall see later, at best a rough estimate of it may be inferred from the literature. However, we do not require a precise knowledge of this parameter, since it is only the relative values of the absorption coefficients that enter critically into the theory. It is true that, for a pure nitrogen atmosphere, the absolute f -values appear in the expression for the effective width of the rotational lines (cf. eq. [30]); while O_2 is present, they are needed to determine the albedos (cf. eq. [26]). Nevertheless, these effective widths and the albedos depend only weakly on the absolute f -values.

The quantity $q(v', v'')$ is the *Franck-Condon* factor. Nicholls (1962) has published extensive tables of this parameter for various band systems of different elements.

The strength of the transition is $S(J', J'')$, which is given by the Hönl-London formulae (Herzberg 1950, p. 208). For a $\Pi-\Sigma$ transition, these formulae take the following form:

$$S(P) = \frac{J'}{4}, \quad S(Q) = \frac{2J' + 1}{4}, \quad S(R) = \frac{J' + 1}{4}. \quad (4)$$

The line absorption coefficient per unit volume is defined by

$$\kappa^{(\nu)}(v''J'', v'J') = N(v''J'') \alpha^{(\nu)}(v''J'', v'J'), \quad (5)$$

where $N(v'', J'')$ is the number density of molecules in the (v'', J'') level. It is related to the total population of the vibrational level, $N(v'')$, by a Boltzmann distribution,

$$N(v''J'') = \frac{N(v'')}{Z} (2J'' + 1) \exp \left[-\frac{hc}{kT} B(v'') J'' (J'' + 1) \right], \quad (6)$$

where $(2J'' + 1)$ is the statistical weight (i.e., the degree of degeneracy) of the rotational level, $hcB(v'')J''(J'' + 1)$ is the energy of the rotational level (Herzberg 1950, pp. 107 and 553), and Z is the partition function,

$$Z = \sum_{J''} (2J'' + 1) \exp \left[-\frac{hc}{kT} B(v'') J'' (J'' + 1) \right]. \quad (7)$$

It should be noted that only the resonant bands (which arise from the ground vibrational level) have non-zero volume absorption coefficients.

As a simplification we shall neglect incoherent scattering within individual line profiles. That is, we shall consider an absorption at frequency $\delta\nu(0)$ from the line center to be followed by an emission at the corresponding $\delta\nu(v'')$ such that $\delta\nu(0)/\nu_e(0) = \delta\nu(v'')/\nu_e(v'')$. The ratios of volume absorption coefficients then become independent of frequency (cf. eqs. [1], [3], [4], [5], and [6]),

$$\begin{aligned} \kappa_P^{(\nu)} : \kappa_Q^{(\nu)} : \kappa_R^{(\nu)} &= J' \exp \left[-\frac{hc}{kT} B(v'') (3J' + 2) \right] : \\ &(2J' + 1) \exp \left[-\frac{hc}{kT} B(v'') J' \right] : (J' + 1) \exp \left[\frac{hc}{kT} B(v'') J' \right]. \end{aligned} \quad (8)$$

In deriving these ratios we have used the values $J'' = J' + 1$, $J'' = J'$, and $J'' = J' - 1$ for P , Q , and R transitions, respectively.

b) Continuous Absorption Coefficient

Continuous absorption in both resonant and fluorescent frequencies may arise from O_2 molecules, which absorb the ultraviolet radiation and become dissociated to atomic oxygen. Let β be the absorption cross-section of an O_2 molecule. The continuous absorption coefficient per unit volume will be

$$\sigma = N(O_2) \beta , \quad (9)$$

where $N(O_2)$ is the number density of O_2 molecules. The coefficients β and σ are practically constant across a line or even across a band. But they may vary appreciably from one band to another.

Watanabe (1958) has published the cross-section of O_2 for the wavelength region $\lambda\lambda 850-1750$ Å. Ditchburn and Young (1962) have measured it in the range $\lambda\lambda 1850-2500$ Å.

IV. ALBEDOS

The albedo of scattering for a transition $v'J' \rightarrow v''J''$ is the probability of such a transition among all possible transitions originating from $v'J'$ level. With induced emission neglected, it may be written in terms of Einstein's A -coefficients as

$$\varpi(v'J', v''J'') = \frac{A(v'J', v''J'')}{\sum_{v''} \sum_{J''} A(v'J', v''J'')} . \quad (10)$$

This scattering albedo does not include the effects of continuous absorption, which will be incorporated in equation (26) below. The A -coefficients are proportional to $S(v'J', v''J'')/\lambda^3$, where S is the *strength* of the transition. As in equation (3), this strength may be factorized into an electronic part, a vibrational part, and a rotational part (Chamberlain 1961, pp. 1-26):

$$S(v'J', v''J'') = R_e^2 q(v', v'') S(J', J'') . \quad (11)$$

Hence the albedo for a rotational line becomes

$$\varpi(v'J', v''J'') = \frac{q(v', v'')/\lambda^3(v', v'')}{\sum_{v''} [q(v', v'')/\lambda^3(v', v'')]} \frac{S(J', J'')}{\sum_{J''} S(J', J'')} . \quad (12)$$

In this equation $\lambda(v', v'')$ is an approximate wavelength typical of the band as a whole. An extensive collection of wavelengths for the LBH system and other bands of N_2 and N_2^+ is published by Wallace (1962). Let us define the albedo of an entire band as

$$\varpi(v', v'') = \frac{q(v', v'')/\lambda^3(v', v'')}{\sum_{v''} [q(v', v'')/\lambda^3(v', v'')]} , \quad (13)$$

which can be readily evaluated from the tables of *Franck-Condon* factors published by Nicholls (1962). The rotational strength $S(J', J'')$ is given by equations (4), and the summation in equation (12) has only the three terms, P , Q , and R . The expression for line albedo now becomes

$$\varpi(v'J', v''J'') = \varpi(v', v'') \frac{2S(J', J'')}{(2J' + 1)} , \quad (14)$$

or

$$\varpi(P) = \varpi(v', v'') \frac{J'}{2(2J'+1)}, \quad (14a)$$

$$\varpi(Q) = \varpi(v', v'') \frac{1}{2}, \quad (14b)$$

and

$$\varpi(R) = \varpi(v', v'') \frac{J'+1}{2(2J'+1)}. \quad (14c)$$

V. OPTICAL DEPTH OF THE MODEL ATMOSPHERES

We mentioned before that collisions may deactivate the excited N₂ molecules, and prevent the scattering process in dense regions of the atmosphere. As the lower boundary of the scattering zone, we shall adopt that altitude at which the collisional lifetime, T_c , becomes equal to the natural or radiative lifetime, T_r , of the excited molecules. Below this height, virtually all radiation is absorbed, and the scattering atmosphere may be regarded as having a lower boundary. Any radiation crossing this boundary from above may be regarded as lost.

Lichten (1957) measured the natural lifetime of the a¹II_g state of N₂ as $T_r = (1.7 \pm 0.3) \times 10^{-4}$ sec. The lifetime for an electronic state, however, is an ill-defined quantity. Because of the wavelength dependence of the Einstein A's, different vibrational levels constituting an electronic state have different lifetimes. Therefore, a definition for the lifetime of the electronic state will require a specification of the distribution of population over the vibrational levels of that electronic state. Obviously, such a parameter that depends on the environment of the molecule (say the parameters governing the distribution) will not be so useful a concept as the lifetime for a single level that depends solely on the constants of that level.

Nonetheless, we might interpret Lichten's measurement as follows. Absorption and emission in different bands of II-Σ transitions are each proportional to the Franck-Condon factors of the bands. Those bands having higher $q(v', v'')$ values will play a stronger role in determining the measured lifetime of the electronic state. As an approximation, we take Lichten's value of 1.7×10^{-4} sec to be the lifetime of the 3-0 band, which, among the bands arising from the ground vibrational level, has the highest value of $q(v', v'')$.

This lifetime may serve to estimate the matrix element, R_e , of equation (3) from the relation

$$\frac{1}{T_r(v', v'')} = A(v', v'') = \frac{64\pi^4}{3h} R_e^2 \frac{q(v', v'')}{\lambda^3(v', v'')}. \quad (15)$$

The lifetime of any other band will then be

$$T_r(v', v'') = T_r(3, 0) \frac{\lambda^3(v', v'')}{\lambda^3(3, 0)} \frac{q(3, 0)}{q(v', v'')}. \quad (16)$$

Oldenberg (1959), interpreting the results of an experiment by Thompson and Williams (1934), estimated the collisional lifetime of the metastable state, $T_c = 5 \times 10^{-5}$ sec. The pressure and mean-free path in the experiment were about 3×10^{-3} mm Hg and 2 cm, respectively,² which correspond to a temperature of approximately 100° K. From these data, by means of elementary formulae of gas-kinetic theory, one may derive the collisional lifetime for arbitrary density and temperature. The lifetime is

$$T_c = 2 \times 10^{11} N^{-1} T^{-1/2} \text{ sec}, \quad (17)$$

² The mean-free path, 2 cm, is quoted by Oldenberg.

where N is the number density of gas. Now putting $T_c = T_r$, we obtain the critical density at which the collisions dominate,

$$N_{cr} = 4 \times 10^{12} T^{1/2} \text{ cm}^{-3}. \quad (18)$$

In the upper atmosphere of Mars, according to the model developed by Chamberlain (1962), the critical density of about 4.5×10^{13} particles per cm^3 is reached at a temperature of approximately 130° K and a height of about 140 km. The integrated number of N_2 molecules above the corresponding critical height,

$$\mathfrak{N}(N_2) = \int_{z_{cr}}^{\infty} N(N_2) dz, \quad (19)$$

is about 5×10^{19} molecules per cm^2 column.

In the upper atmosphere of Mars, Chamberlain suggests, a trace of O_2 is formed by recombination of O atoms, which in turn are produced by photodissociation of CO_2 . [Dunham (1949) estimates spectroscopically that the total O_2 content of the Martian atmosphere is less than 0.15 that for the terrestrial atmosphere.] The O_2 in the upper atmosphere is mainly confined to altitudes below about 125 km. Above this height, O_2 is rapidly photodissociated. In view of these considerations, Model I is a pure nitrogen atmosphere and Model II has a maximum of 1 per cent molecular oxygen. One may expect the scattering region of the Martian atmosphere to have a composition intermediate between these two limits.

The total optical depth of an atmosphere is

$$\tau^{(\nu)}(v'J', v''J'') = \mathfrak{N}(v'', J'') \alpha^{(\nu)}(v''J'', v'J') + \mathfrak{N}(\text{O}_2) \beta. \quad (20)$$

The first term on the right side is the optical depth in the line due to N_2 molecules, and the second term is the optical depth in the continuum due to O_2 molecules.

Table 1 lists the optical depth at the center of the resonant P, Q, R lines of the LBH system. These values pertain to both models at a mean temperature of 100° K . Table 2 gives the optical depth in the continuum at the corresponding frequencies of the bands. The latter table pertains to Model II only.

The optical depth in the continuum is nearly constant across a band, but the depth in the lines varies appreciably from one line to another (see Table 1). The lines with larger optical depths will generate most of the scattered light. For simplicity, we may neglect the lines with optical depths less than unity and treat the lines with depths larger than unity with the radiative-transfer techniques appropriate to lines with large central opacities.

VI. INTENSITIES OF RESONANT BANDS

To find the intensity of a resonant band $v'-0$, we shall derive the intensity of a P, Q, R trio for a given J' (see Fig. 1) and then sum up over all values of J' . The intensities of P, Q , and R transitions, scattered in a direction $\theta = \cos^{-1} \mu$ (the angle between the direction of emergence and the normal to the plane of the atmosphere), are given by equations (9) and (81) of Paper II:

$$I_i(\mu) = \frac{\mu_0}{4} \varpi_i \mathfrak{H}_i(\mu) \sum_j F_j \frac{k_j \mathfrak{H}_j(\mu_0)}{K_j \mu + K_i \mu_0}, \quad i, j = P, Q, R, \quad (21)$$

where πF_j is the solar flux in frequency j incident on the atmosphere in a direction $\theta_0 = \cos^{-1} \mu_0$. In equation (21) and other subsequent equations the parameters k_j, s_j , and $K_j = k_j + s_j$ are proportional to the line absorption coefficients $\kappa_j^{(\nu)}$, the continuous absorption coefficient σ_j , and the total absorption coefficient $\kappa_j^{(\nu)} + \sigma_j$, respectively (see eqs.

TABLE 1

OPTICAL DEPTH AT THE CENTER OF THE RESONANCE LINES OF THE LBH BANDS

v'	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\tau_p(v' J', v'' = 0)$															
0	0.7	1.2	1.5	1.6	1.5	1.3	1.1	0.8	0.6	0.4	0.3	0.2	0.1	0.1	0.1
1	1.8	3.1	3.9	4.2	4.0	3.6	2.9	2.2	1.6	1.1	0.6	0.4	0.3	0.1	0.1
2	2.6	4.6	5.8	6.2	6.0	5.3	4.3	3.3	2.4	1.6	1.1	0.7	0.4	0.2	0.2
3	2.8	4.9	6.2	6.6	6.4	5.6	4.6	3.6	2.6	1.8	1.1	0.7	0.4	0.2	0.2
4	2.5	4.3	5.4	5.8	5.6	4.9	4.0	3.1	2.2	1.5	1.0	0.6	0.4	0.2	0.2
5	1.9	3.3	4.1	4.4	4.2	3.7	3.1	2.3	1.7	1.2	0.8	0.5	0.3	0.2	0.2
6	1.3	2.2	2.8	3.0	2.9	2.5	2.1	1.6	1.2	0.8	0.5	0.3	0.2	0.1	0.1
7	0.8	1.4	1.8	1.9	1.8	1.6	1.3	1.0	0.7	0.5	0.3	0.2	0.1	0.1	0.1
8	0.5	0.8	1.1	1.1	1.1	1.0	0.8	0.6	0.4	0.3	0.2	0.1	0.1	0.0	0.0
9	0.3	0.5	0.6	0.6	0.6	0.5	0.4	0.3	0.2	0.2	0.1	0.1	0.0	0.0	0.0
$\tau_q(v' J', v'' = 0)$															
0	2.2	3.3	4.1	4.4	4.3	3.9	3.3	2.6	2.0	1.4	1.0	0.6	0.4	0.2	0.2
1	5.8	8.9	10.9	11.8	11.6	10.5	8.9	7.1	5.3	3.8	2.6	1.6	1.0	0.6	0.6
2	8.6	13.1	16.1	17.4	17.1	15.5	13.1	10.5	7.9	5.6	3.8	2.4	1.5	0.9	0.9
3	9.2	14.1	17.3	18.6	18.3	16.6	14.1	11.2	8.4	6.0	4.1	2.6	1.6	0.9	0.9
4	8.1	12.3	15.1	16.3	16.0	14.5	12.3	9.8	7.4	5.2	3.5	2.3	1.4	0.8	0.8
5	6.1	9.3	11.4	12.3	12.1	11.0	9.3	7.4	5.6	4.0	2.7	1.7	1.0	0.6	0.6
6	4.2	6.4	7.8	8.4	8.2	7.5	6.3	5.1	3.8	2.7	1.8	1.2	0.7	0.4	0.4
7	2.6	4.0	4.9	5.3	5.2	4.7	4.0	3.2	2.4	1.7	1.2	0.7	0.5	0.3	0.3
8	1.6	2.4	2.9	3.2	3.1	2.8	2.4	1.9	1.4	1.0	0.7	0.4	0.3	0.2	0.2
9	0.9	1.4	1.7	1.8	1.8	1.6	1.4	1.1	0.8	0.6	0.4	0.2	0.2	0.1	0.1
$\tau_R(v' J', v'' = 0)$															
0	1.5	2.2	2.6	2.9	2.9	2.7	2.4	2.0	1.6	1.1	0.8	0.5	0.3	0.2	0.2
1	4.1	5.8	7.1	7.8	7.9	7.4	6.5	5.3	4.2	3.1	2.2	1.5	0.9	0.6	0.6
2	6.0	8.6	10.5	11.5	11.6	10.9	9.5	7.9	6.2	4.6	3.2	2.1	1.4	0.8	0.8
3	6.4	9.2	11.3	12.3	12.4	11.6	10.2	8.4	6.6	4.9	3.4	2.3	1.5	0.9	0.9
4	5.6	8.1	9.8	10.8	10.8	10.2	8.9	7.4	5.8	4.3	3.0	2.0	1.3	0.8	0.8
5	4.3	6.1	7.5	8.2	8.2	7.7	6.8	5.6	4.4	3.2	2.3	1.5	1.0	0.6	0.6
6	2.9	4.2	5.1	5.6	5.6	5.2	4.6	3.8	3.0	2.2	1.5	1.0	0.7	0.4	0.4
7	1.8	2.6	3.2	3.5	3.5	3.3	2.9	2.4	1.9	1.4	1.0	0.7	0.4	0.3	0.3
8	1.1	1.6	1.9	2.1	2.1	2.0	1.7	1.4	1.1	0.8	0.6	0.4	0.2	0.1	0.1
9	0.6	0.9	1.1	1.2	1.2	1.1	1.0	0.8	0.6	0.5	0.3	0.2	0.1	0.1	0.1

TABLE 2

CONTINUOUS OPTICAL DEPTH AT THE FREQUENCIES OF THE LBH BANDS, $\tau_o(v', v'')$

v'	v''	0	1	2	3	4	5	6	7	8	9	10	11	12
0	7.1	5.6	3.8	2.0	0.7	0.2								
1	7.2	6.6	5.1	3.2	1.5	0.6								
2	6.6	7.3	6.3	4.6	2.9	1.2	0.4							
3	3.5	6.7	7.2	5.8	4.1	2.3	0.9	0.3						
4	1.0	5.2	7.3	6.7	5.3	3.6	1.9	0.7	0.2					
5	0.3	1.1	6.5	7.3	6.5	4.9	3.1	1.4	0.4	0.1				
6	0.0	0.4	3.5	6.8	7.3	5.9	4.3	2.5	1.1	0.4				
7	0.7	0.2	1.0	4.9	7.2	7.2	5.4	3.8	2.1	0.8	0.3			
8	0.2	0.2	0.2	1.1	6.5	7.3	6.5	4.9	3.2	1.6	0.6	0.2		
9	9.5	0.1	0.1	1.0	3.5	6.8	7.3	5.9	4.3	2.8	1.2	0.4	0.1	

[68]–[70] of Paper II). We shall need only the relative values of these coefficients. The ratio of k 's is therefore given by equation (8):

$$\begin{aligned} k_P : k_Q : k_R &= J' \exp \left[-\frac{hc}{kT} B(0)(3J'+2) \right] : \\ (2J'+1) \exp \left[-\frac{hc}{kT} B(0)J' \right] : (J'+1) \exp \left[\frac{hc}{kT} B(0)J' \right]. \end{aligned} \quad (22)$$

In virtue of the assumption of coherent scattering, these ratios are independent of frequency within the line. The total absorption coefficient may be written as

$$K_i = k_i + s_i = k_i(1 + r_i), \quad i = P, Q, R, \quad (23)$$

where

$$r_i(\nu) = \frac{s_i}{k_i(\nu)} = \frac{\sigma_i}{\kappa_i(\nu)} = \frac{\mathfrak{N}(O_2)}{\mathfrak{N}_i(N_2)} \frac{\beta_i}{\alpha_i(\nu)}. \quad (24)$$

Here $\mathfrak{N}_i(N_2)$ is the integrated number of N_2 molecules in the corresponding lower level of the i th transition.

The \mathfrak{H} -functions in equation (21) are defined by equation (80) of Paper II. An approximate expression for these functions is developed by Stibbs (1963):

$$\mathfrak{H}_i(\mu) = \Sigma \varpi_l(\nu) \left[\sum_l \frac{\varpi_l(\nu)}{H[K_l \mu / K_i, \Sigma \varpi_l(\nu)]} \right]^{-1}, \quad i, l = P, Q, R, \quad (25)$$

where the *reduced albedo* $\varpi_i(\nu)$ is defined by

$$\varpi_i(\nu) = \frac{k_i(\nu)}{K_i(\nu)} \quad \varpi_i = \frac{\varpi_i}{1 + r_i(\nu)}. \quad (26)$$

The reduced albedo will be distinguished from the ordinary albedo by explicit notation of its frequency dependence. (For an explanation of the reduced albedo in eq. [25] see the comments following eqs. [79] and [80] of Paper II.) In equation (25), H is Chandrasekhar's H -function. For high values of the albedo, Stibbs's approximate solution of the \mathfrak{H} -functions appears to be more accurate than the approximate expression given by equation (25) of Paper II.

In practice, one measures the integrated intensity of lines or bands rather than the monochromatic intensity of equation (21). This integrated intensity is

$$\begin{aligned} \mathfrak{I}_i(\mu) &= \int I_i(\mu) d\nu \\ &= \frac{\mu_0}{4} \varpi_i \sum_j F_j \int \frac{k_j \mathfrak{H}_i(\mu) \mathfrak{H}_j(\mu_0)}{K_j \mu + K_i \mu_0} d\nu. \end{aligned} \quad (27)$$

For the two models the integration of this equation should be performed with different techniques of approximation.

For Model I, composed of pure nitrogen, there is no continuous absorption, and equation (27) takes the form

$$\mathfrak{I}_i(\mu) = \frac{\mu_0}{4} \varpi_i \sum_j F_j \int \frac{k_j \mathfrak{H}_i(\mu) \mathfrak{H}_j(\mu_0)}{k_j \mu + k_i \mu_0} d\nu. \quad (28)$$

The \mathfrak{H} -functions appropriate to this equation may be obtained from equation (25) by replacing all K_j 's by k_j 's. Under the assumption of coherent scattering, the ratio of k_i 's entering equation (25) and thence (28) is independent of frequency, so the whole inte-

grand in equation (28) becomes independent of frequency. The expression for $\mathfrak{J}_i(\mu)$ now becomes

$$\mathfrak{J}_i(\mu) = \frac{\mu_0}{4} \varpi_i \mathfrak{H}_i(\mu) \sum_j F_j \delta\nu_j \frac{k_j \mathfrak{H}_j(\mu_0)}{k_j \mu + k_i \mu_0}. \quad (29)$$

Here we have used an effective line width, $\delta\nu_j$, over which the absorption coefficient is significant. An approximate expression for it, as suggested in Paper I, may be taken as the interval between the two frequencies in the profile where $\tau_j(v) = 1$. From this criterion and the relation

$$\tau_j(v) = \tau_j^{(c)} \exp [-c^2(v - v_c)^2/v_c^2 U^2],$$

one gets

$$\delta\nu_j = 2 \frac{U}{c} v_c [\ln \tau_j^{(c)}]^{1/2}. \quad (30)$$

For Model II an evaluation of equation (27) does not require this sharp cutoff of the profile if continuous absorption effectively cuts it off first. From equations (23) and (1), one has

$$K_i = k_i [1 + r_i(v)] = k_i [1 + b_i \exp(x^2)], \quad (31)$$

where

$$b_i = \frac{\mathfrak{N}(O_2)}{\mathfrak{N}_i(N_2)} \frac{\beta_i}{\alpha_i(v_c)}, \quad x = c(v - v_c)/Uv_c,$$

and $\alpha_i(v_c)$ is the absorption coefficient per molecule at the center of the profile. By substituting for K_i in equation (27) and changing the variable of integration from v to x give

$$\mathfrak{J}_i(\mu) = \frac{Uv_c}{c} \frac{\mu_0}{4} \varpi_i \sum_j F_j \int_{-\infty}^{\infty} \frac{k_j \mathfrak{H}_i(\mu) \mathfrak{H}_j(\mu_0)}{k_j(e^{-x^2} + b_j)\mu + k_i(e^{-x^2} + b_i)\mu_0} e^{-x^2} dx. \quad (32)$$

Apart from the factor $\exp(-x^2)$, the rest of the integrand is a smooth and continuous function of x . The integral is in a suitable form to be evaluated by Laplace's method,³ which gives

$$\mathfrak{J}_i(\mu) = \pi^{1/2} \frac{Uv_c}{c} \frac{\mu_0}{4} \varpi_i \mathfrak{H}_i^{(c)}(\mu) \sum_j F_j \frac{k_j^{(c)} \mathfrak{H}_j^{(c)}(\mu_0)}{K_j^{(c)} \mu + K_i^{(c)} \mu_0}. \quad (33)$$

A superscript (c) in this equation indicates that the quantity in question should be evaluated at the center of the line, where $x = 0$ and $\exp(-x^2)$ is maximum.

It should be noted that the validity of solution (33) lies on the implicit assumption that the integrand of equation (32), apart from the exponential factor, be a slowly varying function of x (i.e., frequency). The \mathfrak{H} -functions ordinarily satisfy this condition and indeed do vary slowly with the albedo and the absorption coefficients, which are variable across the profile. However, the condition that the denominator in equation (32) also be a slow function of x clearly requires a large b_i , the ratio of continuous absorption to the line absorption at the center of the profile. Thus equation (33) was used for the resonant bands with $v'' = 0, 1, 2, 3, 4$, and 9 , at which the continuous optical depth is greater than, or equal to, unity (see Table 2). For the resonant bands with $v'' = 5, 6, 7$,

³ In general, one may write

$$\int_a^\beta f(x) e^{tg(x)} dx \approx f(\xi) e^{tg(\xi)} \left[\frac{2\pi}{-tg''(\xi)} \right]^{1/2},$$

where ξ is that value of x at which $g(x)$ is maximum and $g''(\xi)$ is the second derivative of $g(x)$ evaluated at ξ . The formula holds for $a < \xi < \beta$. Laplace's method is a special case of the more general method of steepest descent, which evaluates complex integrals.

and 8, however, at which the continuous optical depth is less than unity, equation (32) was integrated numerically. The details of this technique are developed in equations (36)–(40) of Paper I.

The scattered total intensity in the resonant band becomes

$$\mathfrak{I}(\mu) = \sum_{J'} \sum_{P, Q, R} \mathfrak{I}_i(\mu). \quad (34)$$

One observes that $\omega_P + \omega_R = \omega_Q$ and for large continuous absorption, $K_P \approx K_Q \approx K_R$. In this case one can show from equation (73) of Paper II that $I_P + I_R \approx I_Q$, a relation that holds exactly when Q transitions are not coupled to P and R transitions. In fact, by a method similar to that of section IIb of Paper II one can prove that the above relation holds to a high degree of approximation for any value of continuous absorption. The effect of possible Λ -type doubling in the II-state of the N_2 molecule is to decouple the Q transitions from P and R transitions. Nevertheless the above discussion shows that the total intensity of all three transitions remains unaffected.

VII. INTENSITIES OF FLUORESCENT BANDS

As in resonant bands, we shall again find the intensity in a fluorescent P, Q, R trio for a given J' and then sum over all values of J' (see Fig. 1).

From equation (24) of Paper I and equations (31)–(33) and (37) of Paper II, the scattered intensities of any two lines i and j , with the same upper level, are related by the following equation:

$$\frac{K_i}{\omega_i} I_i(K_i \mu) = \frac{K_j}{\omega_j} I_j(K_j \mu). \quad (35)$$

Let j be a resonant line (either P, Q , or R) in $v'' = 0$ band and i be the desired fluorescent line in $v'' \neq 0$ band. For a fluorescent line, $k_i(v'') = 0$ and $K_i(v'') = s_i(v'')$. Hence equation (35) becomes

$$I_i(\mu, v'') = \frac{\omega_i(v'')}{\omega_i(0)} \frac{K_i}{s_i(v'')} I_i\left(\frac{K_i \mu}{s_i(v'')}, v'' = 0\right), \quad i = P, Q, R, \quad (36)$$

where the resonant intensity on the right side is given by equation (21), which takes the form

$$I_i\left(\frac{K_i \mu}{s_i(v'')}, 0\right) = \frac{\mu_0}{4} \omega_i(0) \frac{s_i(v'')}{K_i} \mathfrak{H}_i\left(\frac{K_i \mu}{s_i(v'')}\right) \sum_j F_j \frac{k_j \mathfrak{H}_j(\mu_0)}{K_j \mu + s_i(v'') \mu_0}. \quad (37)$$

Substituting this expression in equation (36) yields

$$I_i(\mu, v'') = \frac{\mu_0}{4} \omega_i(v'') \mathfrak{H}_i\left(\frac{K_i \mu}{s_i(v'')}\right) \sum_j F_j \frac{k_j \mathfrak{H}_j(\mu_0)}{K_j \mu + s_i(v'') \mu_0}, \quad i, j = P, Q, R. \quad (38)$$

For Model I, with no continuous absorption, equation (36) reduces to

$$I_i(\mu, v'') = \frac{1}{4} \frac{\mu_0}{\mu} \omega_i(v'') \mathfrak{H}_i(\infty) \sum_j F_j \mathfrak{H}_j(\mu_0). \quad (39)$$

An exact expression for $\mathfrak{H}_i(\infty)$ is derived in the appendix. Using this value, we find

$$I_i(\mu, v'') = \frac{1}{4} \frac{\mu_0}{\mu} \omega_i(v'') (1 - \Sigma \omega_l)^{-1/2} \sum_j F_j \mathfrak{H}_j(\mu_0). \quad (40)$$

The integrated intensity becomes

$$\mathfrak{J}_i(\mu, v'') = \frac{1}{4} \frac{\mu_0}{\mu} \varpi_i(v'') (1 - \Sigma \varpi_i)^{-1/2} \sum_j F_j \delta\nu_j \mathfrak{H}_j(\mu_0), \quad (41)$$

where $\delta\nu_j$ is again given by equation (30).

For Model II, if the corresponding continuous optical depths are less than unity, we integrate equation (38) numerically, otherwise integrate it by Laplace's method, which gives

$$\mathfrak{J}_i(\mu, v'') = \pi^{1/2} \frac{U\nu_c}{c} \frac{\mu_0}{4} \varpi_i(v'') \mathfrak{H}_i^{(c)} \left(\frac{K_i \mu}{s_i(v'')} \right) \sum_j F_j \frac{k_j^{(c)} \mathfrak{H}_j^{(c)}(\mu_0)}{K_j^{(c)} \mu + s_i(v'') \mu_0}. \quad (42)$$

For both models the total scattered intensity in the fluorescent band $v' - v''$ is given by summing all the lines, as in equation (34).

VIII. RAYLEIGH SCATTERING

The solar radiation may be scattered by the atmospheric molecules according to Rayleigh's law and form a continuum for the diffusely scattered LBH bands. This scattering will, however, take place in the presence of O₂, which continuously absorbs the sunlight. Let $\beta(O_2)$ and $\beta(Ray)$ be the cross-sections per molecule for the continuous absorption by O₂ and the Rayleigh scattering by N₂ molecules, respectively. The albedo of Rayleigh scattering then may be written as

$$\varpi(Ray) = \frac{N(N_2)\beta(Ray)}{N(N_2)\beta(Ray) + N(O_2)\beta(O_2)}. \quad (43)$$

An expression for $\beta(Ray)$ is

$$\beta(Ray) = \frac{128\pi^5}{3\lambda^4} a^2, \quad (44)$$

where the polarizability, a , of the scattering particle in terms of the index of refraction, n , and the number density of the particles, N , is

$$a = \frac{n - 1}{2\pi N}. \quad (45)$$

Allen (1955) has compiled the values of n for the air at standard temperature and pressure. From his data we have calculated a and then used it to obtain β at $\lambda\lambda 1200-2400$ Å. The values of $\beta(Ray)$, together, with the absorption cross-sections for O₂, O₃, and CO₂ are listed in Table 3. The cross-sections for O₂ and O₃ are from Watanabe (1958) and from Ditchburn and Young (1962). The cross-sections for CO₂ are from Inn, Watanabe, and Zelikoff (1953).

Shortward of 1759 Å, O₂ absorbs in Schumann-Runge continuum with cross-sections which are greater than the Rayleigh cross-sections by factors of the order of 10⁶. Hence not only the albedo of equation (43) will be correspondingly low (by factors of about 10⁴ for 1 per cent O₂ concentration), but also at that region of the atmosphere where O₂ has enormous optical depths the Rayleigh scattering will produce a negligibly small optical depth. Shortward of 1759 Å, the Rayleigh-scattered intensities are calculated from

$$I_{Ray}(\mu) = \frac{1}{4} \varpi F_\lambda \frac{\mu_0}{\mu + \mu_0} \left\{ 1 - \exp \left[-\tau \left(\frac{1}{\mu} + \frac{1}{\mu_0} \right) \right] \right\}, \quad (46)$$

which is the expression for the intensity of primary scattering (Chandrasekhar 1950, p. 217). In equation (46) τ is the sum of the optical depths of the atmosphere for Rayleigh

scattering and for the O_2 absorption and ω is the albedo given by equation (43). Because of the very small values of the optical depth with respect to Rayleigh scattering, the secondary and higher-order scatterings should be unimportant. We have also assumed that the Rayleigh scattering is isotropic, which may affect the final results by factors ranging from 1 to 2, depending on the scattering direction.

The Martian atmosphere contains about 2 per cent CO_2 (Grandjean and Goody 1955; Goody 1957), which, according to Chamberlain (1962), is confined to altitudes below 125 km. In the region $\lambda \lambda 1200-1750$ Å, absorption cross-sections for CO_2 are about 10^5 times greater than those for Rayleigh scattering. In view of these considerations, we have assumed that CO_2 absorbs virtually all the sunlight below 125 km and that Rayleigh scattering occurs only above this level. Above 125 km the atmosphere contains about 2.5×10^{20} molecules/cm² column. For both Models I and II, we have calculated the continuum intensity from equation (46) and have plotted it in Figure 3. The necessary values of the solar flux, πF_λ , have been taken from Friedman (1961) and Hinteregger (1961). The results of Figure 3 may be regarded as only slightly better than an order-of-magnitude estimate.

TABLE 3

ABSORPTION CROSS-SECTIONS PER MOLECULE FOR RAYLEIGH
SCATTERING, O_2 ABSORPTION, AND O_3 ABSORPTION*

λ (Å)	β (Ray) (cm ²)	$\beta(O_2)$ (cm ²)	$\beta(O_3)$ (cm ²)	$\beta(CO_2)$ (cm ²)
1200	4 2(-24)	~ 1 5(-18)	~1 (-17)	~7(-20)
1400	1 9(-24)	13 8(-18)	~7 (-18)	~6(-19)
1600	9 8(-25)	5 3(-18)	1 1(-18)	~2(-19)
1750	6 6(-25)	2 6(-19)	7 8(-18)	~2(-20)
1800	5 7(-25)	15 0(-24)	7 4(-19)	
2000	3 3(-25)	13 0(-24)	2 9(-19)	
2200	2 1(-25)	6 4(-24)	1 9(-18)	
2400	1 4(-25)	1 0(-24)	8 6(-18)	

* In this table a number $q \times 10^p$ is written $q(p)$

Longward of 1759 Å, O_2 absorbs in the Herzberg continuum with cross-sections only 6 to 30 times those of the Rayleigh scattering. Hence, for small O_2 concentrations, the latter may compete with, or even dominate, the O_2 absorption. Cross-sections for CO_2 are not known in this wavelength region. If they are much lower than the values listed in Table 3, then lower regions of the atmosphere may participate in Rayleigh scattering and produce a strong continuum. On the other hand, O_3 has large cross-sections in the near ultraviolet, and an ozone concentration of $1/10^6$ would be sufficient to suppress the Rayleigh scattering. The amount of ozone in the atmosphere of Mars, however, is likely to be negligible. The lack of an adequate knowledge of the cross-sections for CO_2 and of the concentrations of the minor constituents of the Martian atmosphere seems to rule out the possibility of a definite statement regarding the magnitude of the continuum in the near ultraviolet.

IX. RESULTS AND DISCUSSION

The values of the solar flux, πF_λ , used in calculating the band intensities are listed in Table 4. They are obtained from Friedman (1961) and Hinteregger (1961). The solar flux at Mars is only half the value at the earth. The wavelengths in this table are those of the resonant bands of the LBH system. It is assumed that the incident flux is constant across a band. The enhancement of the solar energy at the 9-0 band ($\lambda 1205$ Å) may or may not be real. This band is in the immediate neighborhood of a Si III line ($\lambda 1206$ Å).

that has a strong intensity in the solar spectrum (Hinteregger 1961). It is difficult to decide whether the apparent high flux at $\lambda 1205 \text{ \AA}$ is due to the extended wings of the Si III line or to a false impression resulting from poor wavelength resolution. There also seem to be certain emission features in the neighborhood of the 4-0 and 5-0 bands (Friedman 1961), so that the solar fluxes quoted for the positions of these bands should be accepted only with some reservation.

Table 4 also includes an effective width for the resonant bands, defined as the wavelength interval between the band head and that rotational line in the *P*-branch which has unit optical depth at its center. The band heads in the LBH system are degraded to the violet and the *P*-transitions are degraded to the red. The effective width, so defined, provides a typical value representing the extent of a band. The specific intensity (i.e., the intensity per unit wavelength interval) of a band may simply be obtained by dividing the entire intensity of the band by its effective width.

The band intensities for Models I and II are calculated by an IBM 7090 electronic computer and are listed in Tables 5 and 6. The uncertainty in the values of the incident solar flux is considerable. And the absolute intensities of Tables 5 and 6 may require important corrections when better data are available. On the other hand, the bands

TABLE 4
ADOPTED SOLAR FLUXES AT MARS ($10^{-3} \text{ erg/cm}^2 \text{ sec A}$)

	v'									
	0	1	2	3	4	5	6	7	8	9
$\lambda(\text{\AA})$	1450	1416	1384	1354	1325	1298	1273	1249	1227	1205
πF_λ	5 0	3 0	2 0	2 0	2 5	2 5	1 5	1 5	2 0	4 0
$\delta\lambda(\text{\AA})$	1 3	2 0	2 2	2 1	2 0	1 8	1 5	1 2	0 9	0 6

arising from one and the same v' level use the same value of solar flux. The relative intensities of such bands are not affected by the uncertainties in the solar flux.

One should also note that the formulae of this paper are developed primarily for large optical depths. They may not be very adequate for optical depths of the order of unity. For $\tau > 3$ the errors arising from the mathematical approximations should be trivial, appearing only in the third significant figure. The corresponding relative intensities may then be trusted up to the second figure. Those corresponding to $2 < \tau < 3$ may have only the first figure correct. The optical depths of Tables 1 and 2 should serve as guides in estimating the accuracy of the relative intensities.

The variations in the intensity of one resonant band and one fluorescent band across the disk of the planet are shown in Figure 2. The same bands are plotted for both Models I and II. Limb brightening is, of course, a characteristic feature of all bands in both models, although, for the resonance bands, the added path length also means extra attenuation of the emerging beam. The effect of continuous absorption in reducing the intensities of Model II is drastic. In the example of Figure 2, however, the continuous absorption is rather large. There are many other bands with considerably low continuous absorption across them (consult Table 2).

Figure 3 illustrates the band intensities of both models plotted versus wavelength. All these bands arise from the selective absorptions at only 10 resonant frequencies listed in Table 4. The contrast between the two diagrams of the two models, especially in the interval 1300-1700 \AA , due to O₂ absorption is again striking. The dotted curves in Figure 3 show the specific intensity (i.e., intensity per \AA) of the continuum produced by Rayleigh scattering, which is to be compared with the intensities for the LBH system.

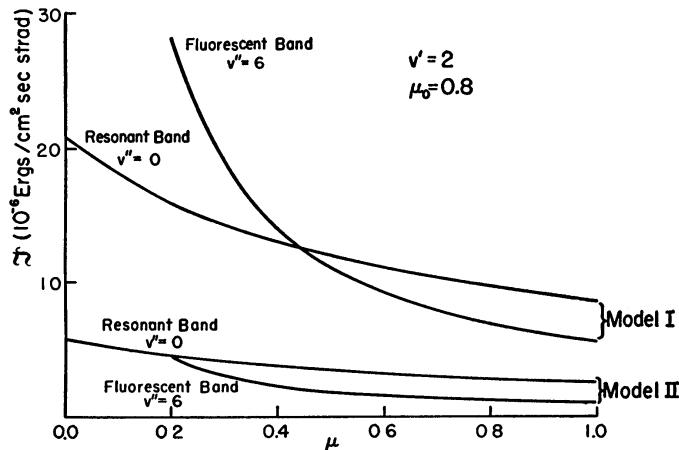


FIG. 2.—Variations of the diffusely reflected intensities of resonant and fluorescent bands across the disk of the planet. The data are from Tables 5 and 6.

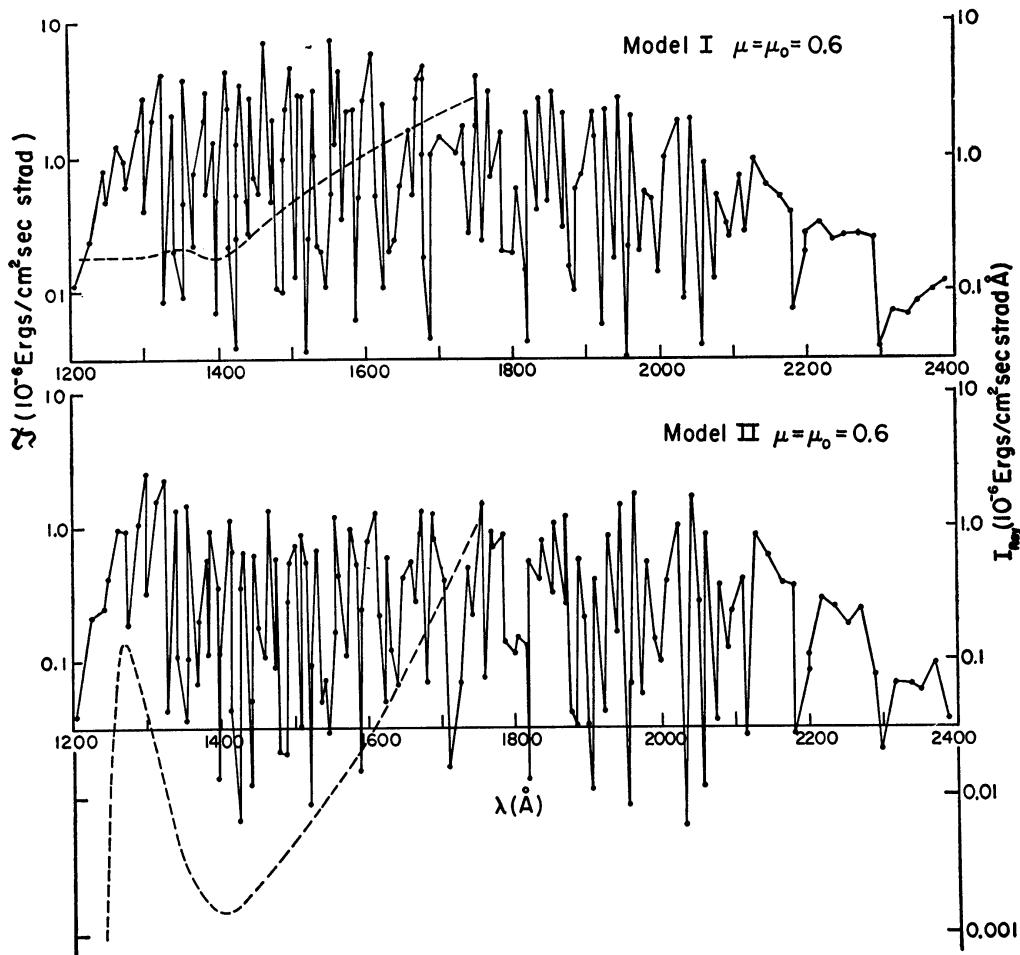


FIG. 3.—Diffusely reflected intensities of the LBH bands (both resonant and fluorescent) plotted versus wavelength. The dots are calculated intensities for entire bands and are taken from Tables 5 and 6. The connecting lines have been drawn in only to increase the legibility of the plots; the diagram should not be misconstrued as representing *specific intensity* (i.e., per unit wavelength interval). Estimates of the specific intensities may be derived from these total intensities and from the band widths of the resonance bands listed in Table 4. The dotted curves are the specific intensities for the continuum due to Rayleigh scattering.

The task of comparison is greatly simplified if one notes that the effective widths of the bands are of the order of 1-2 Å (consult Table 4), so that the same numbers representing the integrated intensities may also represent the specific intensities of the bands within factors ranging from 1 to 2. At least shortward of 1750 Å, the band intensities are considerably higher than the Rayleigh continuum and may be amenable to measurements.

I wish to express my indebtedness to Dr. J. W. Chamberlain for suggesting this problem and for his advice and patience throughout the course of the study.

APPENDIX

ON AN INTEGRAL PROPERTY OF \mathfrak{H} -FUNCTIONS

Chandrasekhar (1950, p. 106) establishes the following theorem for his H -functions:

$$\varpi \int_0^1 H(\mu) d\mu = 2 [1 - (1 - \varpi)^{1/2}]. \quad (\text{A1})$$

Employing his technique, we shall derive an analogous expression for

$$\sum_i \varpi_i \int_0^1 \mathfrak{H}_i(\mu) d\mu.$$

The defining equation for \mathfrak{H} -functions is (eq. [48] of Paper II)

$$\mathfrak{H}_i(\mu) = 1 + \frac{\mu}{2} \mathfrak{H}_i(\mu) \sum_l \varpi_l k_l \int_0^1 \frac{\mathfrak{H}_l(\mu')}{k_l \mu + k_i \mu'} d\mu'. \quad (\text{A2})$$

We multiply this equation by ϖ_i , integrate over μ , and sum over all values of i :

$$\begin{aligned} \sum_i \varpi_i \int_0^1 \mathfrak{H}_i(\mu) d\mu &= \sum_i \varpi_i \\ &+ \frac{1}{2} \sum_i \sum_l \varpi_i \varpi_l \int_0^1 \int_0^1 \frac{k_l \mu}{k_l \mu + k_i \mu'} \mathfrak{H}_i(\mu) \mathfrak{H}_l(\mu') d\mu d\mu'. \end{aligned} \quad (\text{A3})$$

The second term on the right side is independent of the pair of indices i, l , and the pair of variables μ, μ' . We interchange the indices i, l and the variables μ, μ' in equation (A3) and take the average of resulting equation with (A3):

$$\begin{aligned} \sum_i \varpi_i \int_0^1 \mathfrak{H}_i(\mu) d\mu &= \sum_i \varpi_i \\ &+ \frac{1}{4} \sum_i \sum_l \varpi_i \varpi_l \int_0^1 \int_0^1 \mathfrak{H}_i(\mu) \mathfrak{H}_l(\mu') d\mu d\mu', \end{aligned} \quad (\text{A4})$$

or

$$\frac{1}{4} \left[\sum_i \varpi_i \int_0^1 \mathfrak{H}_i(\mu) d\mu \right]^2 - \left[\sum_i \varpi_i \int_0^1 \mathfrak{H}_i(\mu) d\mu \right] + \sum_i \varpi_i = 0. \quad (\text{A5})$$

From this equation we have

$$\sum_i \varpi_i \int_0^1 \mathfrak{H}_i(\mu) d\mu = 2 [1 - (1 - \sum \varpi_i)^{1/2}]. \quad (\text{A6})$$

An expression for $\mathfrak{H}_i(\infty)$ may be obtained from equation (A6). From the defining equation (A2) we have

$$\mathfrak{H}_i(\infty) = 1 + \frac{1}{2} \mathfrak{H}_i(\infty) \sum_l \varpi_l \int_0^1 \mathfrak{H}_l(\mu') d\mu'. \quad (\text{A7})$$

Substituting for the summation term from equation (A6) and solving for $\mathfrak{H}_i(\infty)$ gives

$$\mathfrak{H}_i(\infty) = \mathfrak{H}_j(\infty) = (1 - \sum \varpi_l)^{-1/2}. \quad (\text{A8})$$

It appears that many other properties of Chandrasekhar's H -functions (and the X - and Y -functions encountered with optically finite atmospheres) can easily be generalized to the case of \mathfrak{H} -functions (and the \mathfrak{X} - and \mathfrak{Y} -functions, see Paper II)

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		INTENSITIES OF THE LBH BANDS (10^{-6} ERGS/CM 2 SEC STRAD) - MODEL I							Table 5		
		V'' = 0, μ = 0.0									
μ_0	V'	0	1	2	3	4	5	6	7	8	9
0.20	1.652	5.087	6.343	7.439	8.221	5.802	2.094	1.061	0.562	0.293	
0.40	1.659	5.151	6.477	7.626	8.415	5.911	2.121	1.070	0.566	0.294	
0.60	1.663	5.190	6.563	7.747	8.540	5.980	2.139	1.076	0.568	0.295	
0.80	1.666	5.218	6.624	7.833	8.630	6.029	2.151	1.080	0.569	0.295	
1.00	1.668	5.240	6.670	7.899	8.698	6.066	2.160	1.083	0.570	0.295	

μ_0	ν'	0	1	2	3	4	5	6	7	8	9
INTENSITIES OF THE LBH BANDS RESONANT BANDS $\nu'' = 0$, $\mu = 0.0$											
0.20	0.358	1.307	1.791	2.759	4.363	5.033	2.027	0.684	0.480	0.065	
0.40	0.358	1.314	1.811	2.805	4.449	5.122	2.053	0.689	0.483	0.065	
0.60	0.359	1.318	1.824	2.834	4.503	5.178	2.070	0.692	0.484	0.065	
0.80	0.359	1.322	1.833	2.854	4.541	5.218	2.081	0.694	0.485	0.065	
1.00	0.359	1.324	1.839	2.869	4.570	5.248	2.090	0.695	0.486	0.065	

Table 6

