NON-COMMUTATIVE INEQUALITIES AND UNCERTAINTY RELATIONS

ALI DADKHAH

Abstract. Let $\mathscr{H}$ and $\mathscr{K}$ be complex Hilbert spaces and let $\mathbb{B}(\mathscr{H})$ and $\mathbb{B}(\mathscr{K})$ be the algebras of all bounded linear operators on $\mathscr{H}$ and $\mathscr{K}$, respectively. A map $\Phi : \mathbb{B}(\mathscr{H}) \longrightarrow \mathbb{B}(\mathscr{K})$ is called n-positive if the map $\Phi_n : M_n(\mathbb{B}(\mathscr{H})) \longrightarrow M_n(\mathbb{B}(\mathscr{K}))$ defined by $\Phi_n([a_{ij}]_{n\times n}) = [\Phi(a_{ij})]_{n\times n}$ is positive, where $M_n(\mathscr{X})$ is the algebra of $n \times n$ matrices with entries in $\mathscr{X}$.
In the first part of the talk, we investigate some classes of positive mappings (not necessarily linear) in the setting of C^*-algebras. First, we obtain some inequalities and properties of 2-positive maps. We then give some results about the superadditivity, the starshapness, the homogeneity and the linearity of n-positive maps for $n \geq 3$.
The second part of our talk is about some inequalities in quantum information theory. The uncertainty principle in quantum mechanics is a fundamental relation with different forms, including Heisenberg's uncertainty relation and Schrödinger's uncertainty relation. The classical expectation value of an observable (self–adjoint element) A in a quantum state (density element) ρ is expressed by $\mathrm{Tr}(\rho A)$. Also, the classical variance for a quantum state ρ and an observable element A is defined by $V_\rho(A) := \mathrm{Tr}(\rho A^2) - (\mathrm{Tr}(\rho A))^2$. Heisenberg's uncertainty relation asserts that

$$V_\rho(A)V_\rho(B) \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2$$

for a quantum state ρ and two observables A and B. It gives a fundamental limit for the measurements of incompatible observables. A further strong result was given by Schrödinger as

$$V_\rho(A)V_\rho(B) - |\mathrm{Re}(\mathrm{Cov}_\rho(A,B))|^2 \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2,$$

where $[A,B] := AB - BA$ and the classical covariance is defined by $\mathrm{Cov}_\rho(A) := \mathrm{Tr}(\rho AB) - \mathrm{Tr}(\rho A)\mathrm{Tr}(\rho B)$. We present some inequalities related to the generalized variance and the generalized skew information. These inequalities provide non-commutative versions of the uncertainty relations.

Department of Pure Mathematics, Ferdowsi University of Mashhad
E-mail address: dadkhah61@yahoo.com

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