Modified Active-Neighborhood Formalism for Susceptible-Infected-Susceptible (SIS) Dynamics on Adaptive Multiplex Networks

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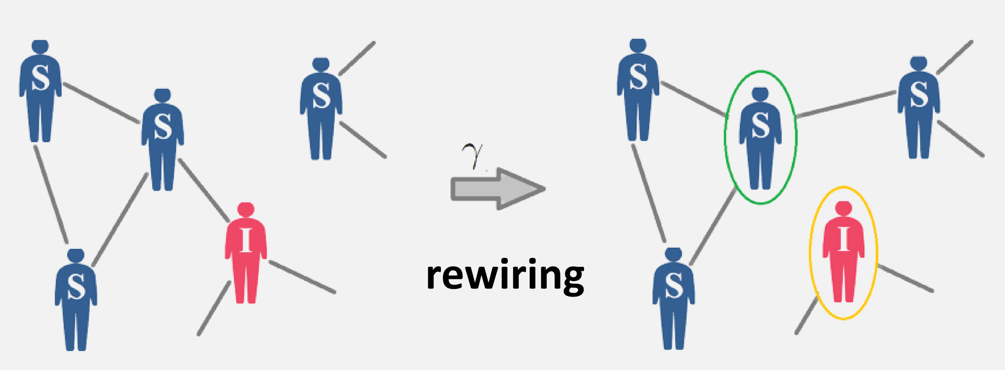
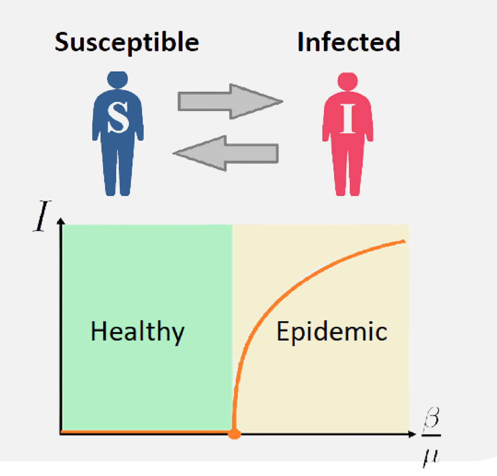
INTRODUCTION

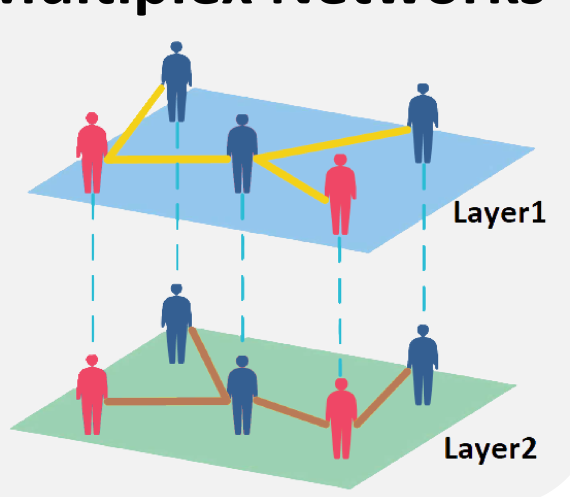
Adaptive Networks



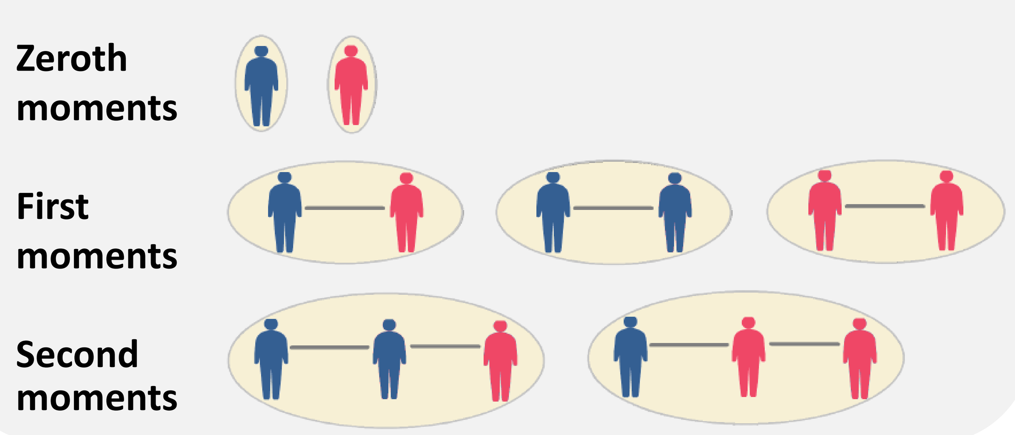
SIS Model



Multiplex Networks



Active Neighborhood



EQUATIONS OF THE MODEL

$$[SS] \equiv \sum_{k_1m_1k_2m_2} (k_1-m_1)s_{k_1m_1,k_2m_2}, \quad [II] \equiv \sum_{k_1m_1k_2m_2} m_1 i_{k_1m_1,k_2m_2} \quad [SI] \equiv \sum_{k_1m_1k_2m_2} m_1 s_{k_1m_1,k_2m_2} \equiv [IS],$$

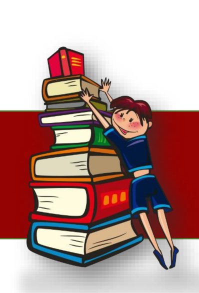
$$[ISI'] \equiv \sum_{k_1m_1k_2m_2} m_1 m_2 s_{k_1m_1,k_2m_2} \equiv [IS'I] \quad [SSI] \equiv \sum_{k_1m_1k_2m_2} m_1(k_1-m_1)s_{k_1m_1,k_2m_2}, \quad [SII] \equiv \sum_{k_1m_1k_2m_2} m_1(m_1-1)s_{k_1m_1,k_2m_2},$$

$$\begin{aligned}\frac{ds_{k_1m_1,k_2m_2}}{dt} &= \alpha i_{k_1m_1,k_2m_2} - (m_1\beta_1 + m_2\beta_2)s_{k_1m_1,k_2m_2} + (\alpha+\gamma_1)\left[(m_1+1)s_{k_1(m_1+1),k_2m_2} - m_1 s_{k_1m_1,k_2m_2}\right] + (\alpha+\gamma_2)\left[(m_2+1)s_{k_1m_1,k_2(m_2+1)} - m_2 s_{k_1m_1,k_2m_2}\right] \\ &+ \left(\beta_1\frac{[SSI]}{[SS]} + \beta_2\frac{[SSI']}{[SS]}\right)\left[(k_1-m_1+1)s_{k_1(m_1-1),k_2m_2} - (k_1-m_1)s_{k_1m_1,k_2m_2}\right] + \left(\beta_1\frac{[SS'I]}{[SS']} + \beta_2\frac{[SS'I']}{[SS']}\right)\left[(k_2-m_2+1)s_{k_1m_1,k_2(m_2-1)} - (k_2-m_2)s_{k_1m_1,k_2m_2}\right] \\ &+ \gamma_1\frac{[SI]}{S}\left[s_{(k_1-1)m_1,k_2m_2} - s_{k_1m_1,k_2m_2}\right] + \gamma_2\frac{[SI']}{S}\left[s_{k_1m_1,(k_2-1)m_2} - s_{k_1m_1,k_2m_2}\right]\end{aligned} \quad (1)$$

$$\begin{aligned}\frac{di_{k_1m_1,k_2m_2}}{dt} &= -\alpha i_{k_1m_1,k_2m_2} + (m_1\beta_1 + m_2\beta_2)s_{k_1m_1,k_2m_2} + \alpha\left[(m_1+1)i_{k_1(m_1+1),k_2m_2} + (m_2+1)i_{k_1m_1,k_2(m_2+1)} - (m_1+m_2)i_{k_1m_1,k_2m_2}\right] \\ &+ \gamma_1\left[(k_1-m_1+1)i_{(k_1+1)m_1,k_2m_2} - (k_1-m_1)i_{k_1m_1,k_2m_2}\right] + \gamma_2\left[(k_2-m_2+1)i_{k_1m_1,(k_2+1)m_2} - (k_2-m_2)i_{k_1m_1,k_2m_2}\right] \\ &+ \left[\beta_1\left(1+\frac{[SII]}{[SI]}\right) + \beta_2\frac{[ISI']}{[IS]}\right]\left[(k_1-m_1+1)i_{k_1(m_1-1),k_2m_2} - (k_1-m_1)i_{k_1m_1,k_2m_2}\right] + \left[\beta_2\left(1+\frac{[SI'I']}{[SI']}\right) + \beta_1\frac{[IS'I]}{[IS']}\right]\left[(k_2-m_2+1)i_{k_1m_1,k_2(m_2-1)} - (k_2-m_2)i_{k_1m_1,k_2m_2}\right]\end{aligned} \quad (2)$$

$$s_{k_1m_1,k_2m_2}(0) = (1-\rho_0)P_1(k_1)P_2(k_2)\binom{k_1}{m_1}\binom{k_2}{m_2}\rho_0^{m_1+m_2}(1-\rho_0)^{k_1+k_2-m_1-m_2} \quad (3)$$

$$i_{k_1m_1,k_2m_2}(0) = \rho_0 P_1(k_1)P_2(k_2)\binom{k_1}{m_1}\binom{k_2}{m_2}\rho_0^{m_1+m_2}(1-\rho_0)^{k_1+k_2-m_1-m_2} \quad (4)$$



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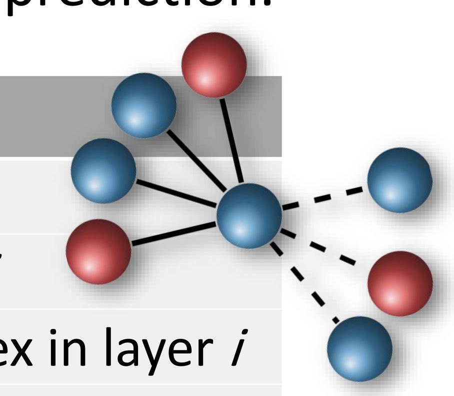
THE MODEL AND RESULTS

Active Neighborhood approach (ANA) as one of the most accurate theoretic approximation frameworks provides a remarkable analytic theory to predict dynamical features of epidemic spreading especially near critical points. On the other hand, many real networks are best described with adaptive multiplex networks due to the various nature of relationships connecting agents and mutual interactions between network structure and individual's state. We study a disease propagation following susceptible-infected-susceptible (SIS) dynamics on a two-layer adaptive multiplex network, consisting two types of edges; solid and dashed lines. We offer an extension of ANA and then study the epidemic threshold, phase transition, and two-parameter bifurcation diagrams of the model. We show that Monte-Carlo simulations for homogeneous adaptive multiplex networks are in a good agreement with ANA prediction.

Table 1 – Definitions of the Model Variables and Parameters

Par./Var.	Definition	Par./Var.	Definition
β_i	infection rate in layer i	k_i	total degree of a vertex in layer i
γ_i	rewiring rate in layer i	m_i	the number of infected neighbors of a vertex in layer i
α	recovery rate	$s_{k_1m_1,k_2m_2}$	fraction of susceptible vertices of degree k_i and m_i
ρ_I^*	stationary infected fraction	$i_{k_1m_1,k_2m_2}$	fraction of infected vertices of degree k_i and m_i



Compartments Evolution

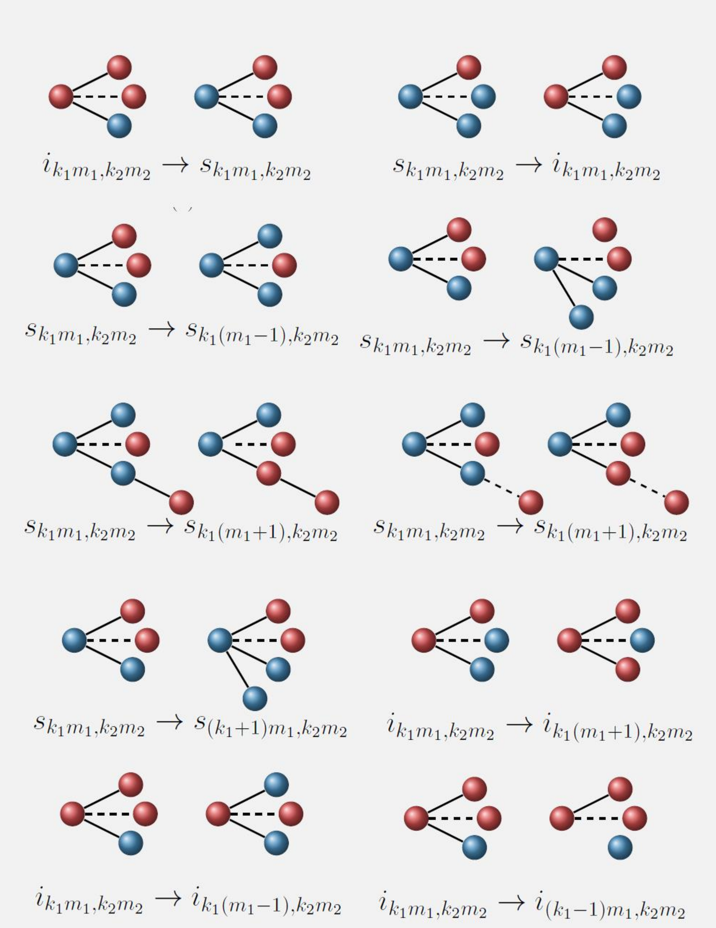


Fig.1 Schematic representation of different processes which result in equations (1) and (2). Blue and red points denote susceptible and infected vertices, respectively. Each edge might be of type 1 (solid line) or type 2 (dashed one).

Discontinuous Phase Transition

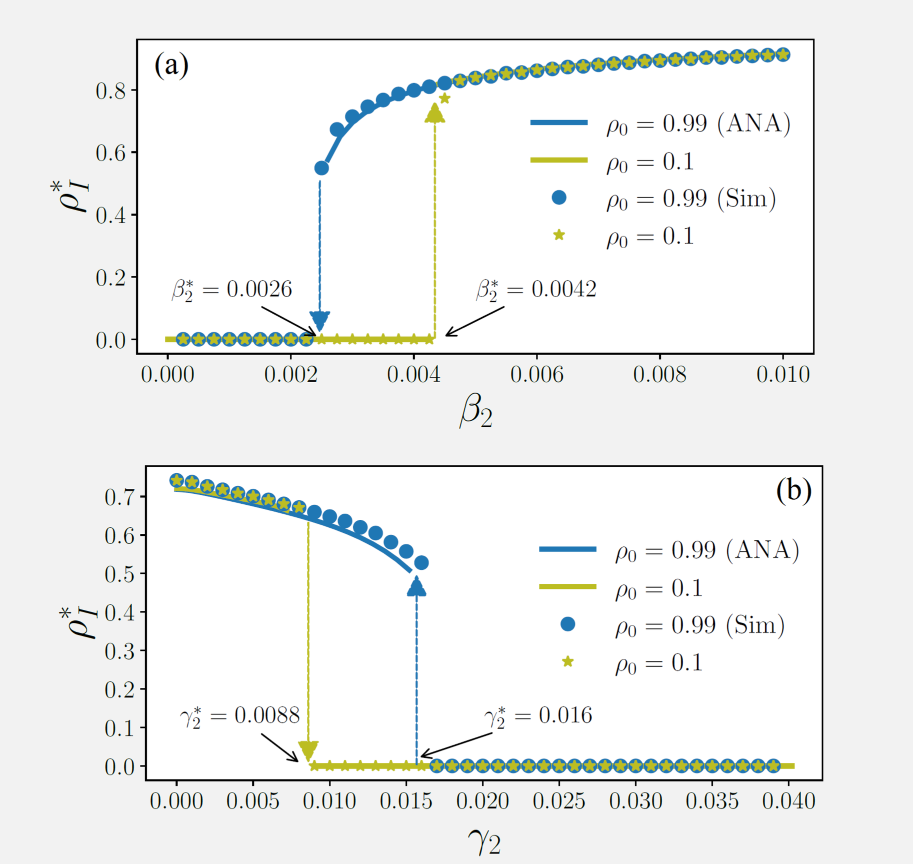


Fig.2 Stationary infected fraction ρ_I^* as a function of (a) infection with $\gamma_1 = \gamma_2 = 0.03$, (b) rewiring rate in layer 2 with $\beta_1 = \beta_2 = 0.02$. Symbols are related to the Monte-Carlo simulation, while solid lines represent ANA prediction. Dashed arrows identify boundary of the bi-stable region. The initial fraction of randomly infected vertices is put at 0.1 and 0.99 for green and blue results respectively.

Phase Diagram

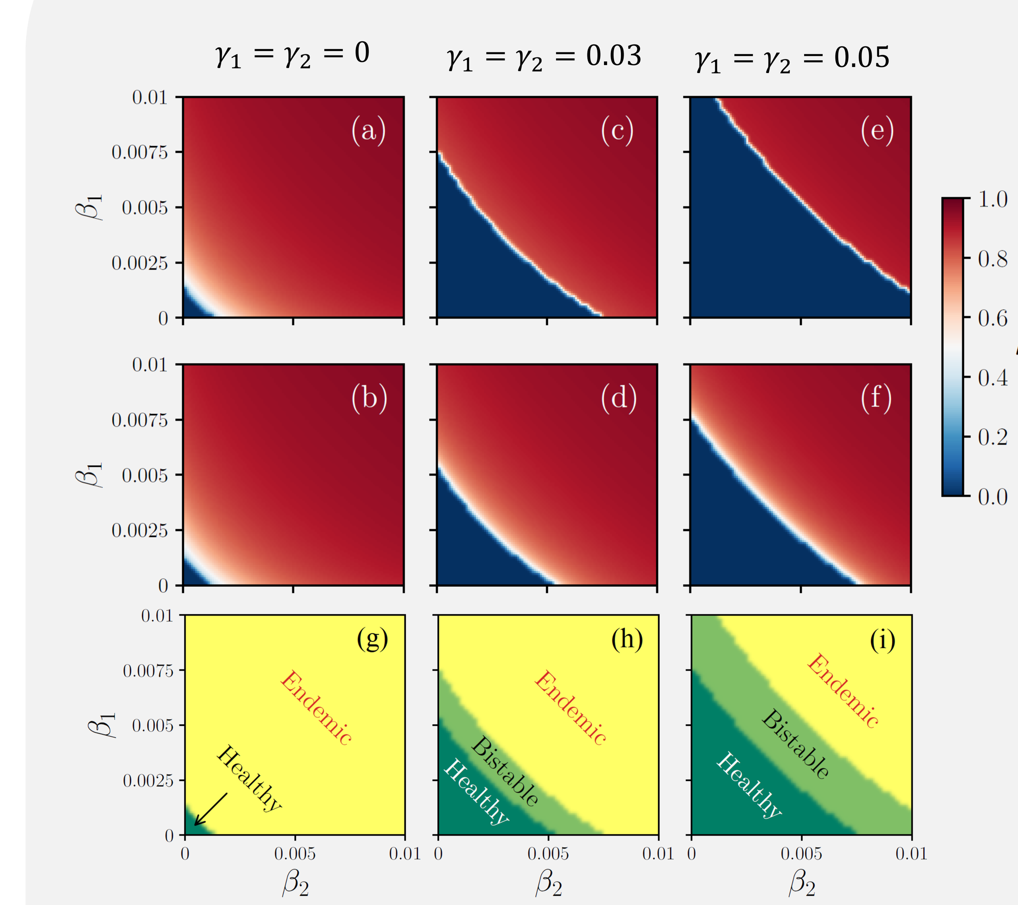


Fig.3 (a-f) Stationary infected fraction ρ_I^* as a function of infection rates in layers 1 and 2 with initially infected fraction of (first-row panel) 0.1 and (second-row panel) 0.99. (g-i) Phase diagram in the plane of $\beta_1 - \beta_2$. Rewiring rates are shown at the top of each column. Both layers have Poisson degree distributions with mean degree $< k_1 > = < k_2 > = 4$. Recovery rate is set $\alpha = 0.003$ for all diagrams.

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