Pancharatnam-Zak phase for two-dimensional systems: Gauge-independent topological invariant

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Introduction

Topological phases are a wide range of quantum phases that are classified using quantized topological invariants. In 2D systems, the Chern number is used to find the topological phases, that is independent of gauge [1]. But in some systems that have time reversal and inversion symmetry, the Chern number is zero. In these systems, 2D Zak phase is non-zero [2]. But the Zak phase is gauge dependent, therefore we defined 2D Pancharatnam-Zak phase to detect the topological phases of 2D systems [3]. This invariant, like 1D Pancharatnam-Zak phase introduced earlier [3], is independent of gauge.

Method

We first consider a two-dimensional system of electrons that are moving in a two-dimensional periodic potential $V(\vec{r})$ and are under the influence of a weak electric field $\vec{E}$:

$$\hat{H}_{\vec{\alpha}}(t)=\left(\left(\vec{p}-\hbar\vec{\alpha}(t)\right)^2/2m\right)+V(\vec{r}),$$

Where $\vec{p}$ and m are the momentum and mass of the electrons, respectively, and $\vec{\alpha}(t)=-e\vec{E}t/\hbar$.

The eigenstates of this Hamiltonian with the help of the Bloch's eigenstates can be wrote in the form $|\Psi_{n\vec{k}_m\vec{\alpha}}\rangle=e^{i\vec{k}_m\cdot\vec{r}}\left|u_n\left(\vec{k}_m+\vec{\alpha}\right)\right\rangle$. Here, setting $\vec{k}_m=0$ without loss of generality. Therefore, the geometric phase obtained during the evolution from the initial state at time t_i to the final state at time t_f is as follows:

$$\gamma_g=\sum_{n=1}^{occ.}\left[\operatorname{Arg}\left\langle u_n\left(\vec{\alpha}(t_i)\right)\middle|u_n\left(\vec{\alpha}(t_f)\right)\right\rangle + \left(i\int_{\vec{\alpha}(t_i)}^{\vec{\alpha}(t_f)} d\vec{\alpha}\cdot\left\langle u_n(\vec{\alpha})\middle|\nabla_{\vec{\alpha}}\middle|u_n(\vec{\alpha})\right\rangle\right)\right]$$

γ_g is 2D Pancharatnam-Zak phase, which is independent of gauge; If $\gamma_g=0$, the system is in the trivial phase, and if $\gamma_g=2\pi$, the system is in the non-trivial phase.

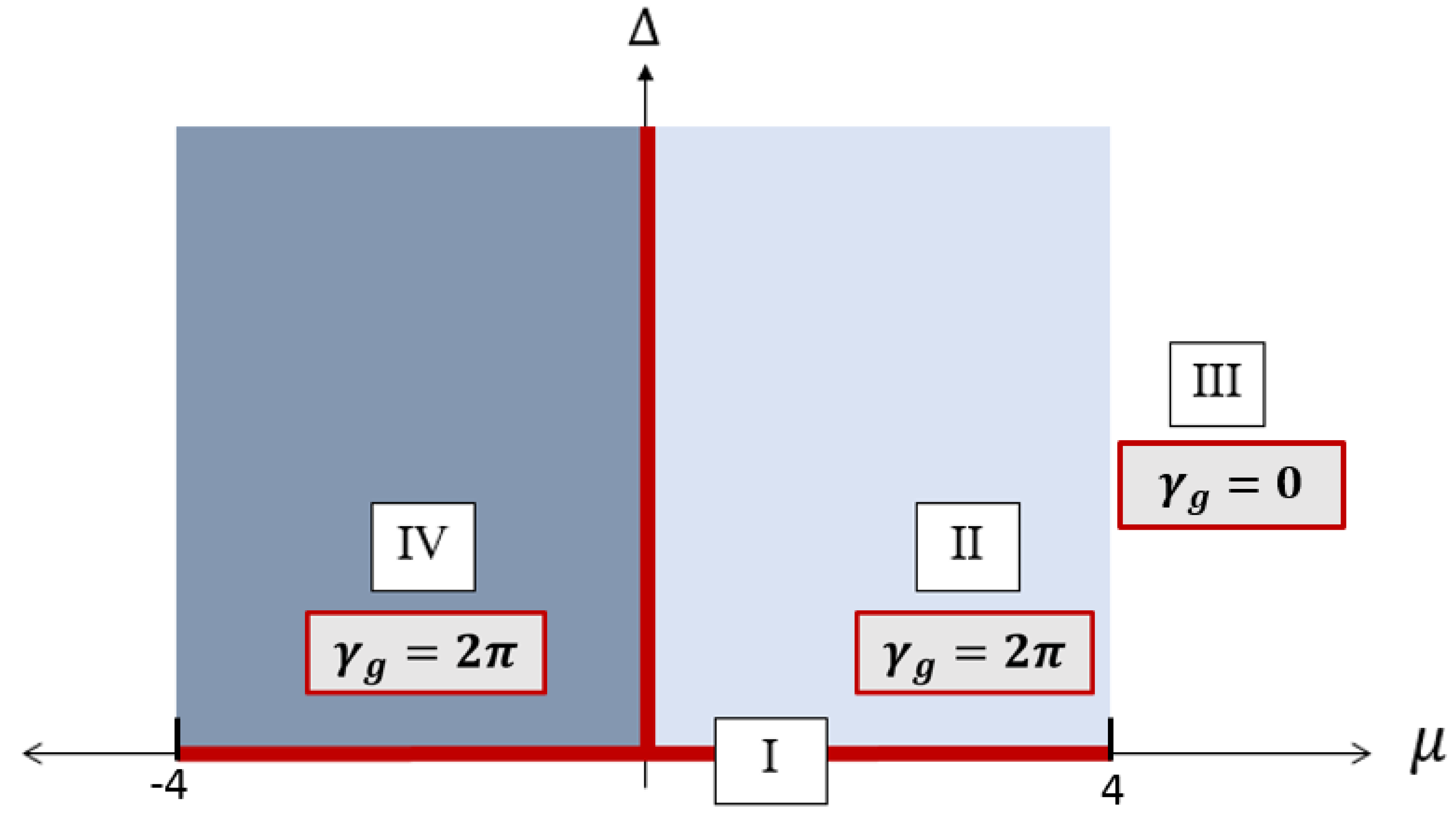

Figure 1: Phase diagram of 2D Kitaev model.
According to this phase diagram, this model has four distinct phases:
Phase I: System is metal.
Phase II: System has topological non-trivial phase.
Phase III: System is always gapped and is an insulator; and also the system has topological trivial phase.
Phase IV : System has topological non-trivial phase.

Results

If in 2D Pancharatnam-Zak phase, we use $|u_n(\vec{\alpha})\rangle e^{i\Lambda(\vec{\alpha})}$ instead of $|u_n(\vec{\alpha})\rangle$, γ_g remains invariant, so γ_g is independent of gauge.

The 2D Kitaev model is used to describe the two-dimensional p-wave superconductor. In this model J is the nearest neighbor hopping amplitude, and μ represents the chemical potential and Δ stands for the superconducting pairing amplitude:

$$\hat{H}_{Kitaev}=-J\sum_{\vec{r},\vec{a}}\hat{c}_{\vec{r}}^{\dagger}\hat{c}_{\vec{r}+\vec{a}}+h.c+\Delta\sum_{\vec{r},\vec{a}}\hat{c}_{\vec{r}}\hat{c}_{\vec{r}+\vec{a}}+h.c +\mu\sum_{\vec{r},\vec{a}}\left(\hat{c}_{\vec{r}}^{\dagger}\hat{c}_{\vec{r}}-(1/2)\right)$$

As an example, we have obtained the phase diagram of the 2D Kitaev model by examining the 2D Pancharatnam-Zak phase behavior in Figure 1.

According to this figure, the 2D Kitaev model has four distinct phases.

References

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