Chapter 5

Signal to Noise ratio

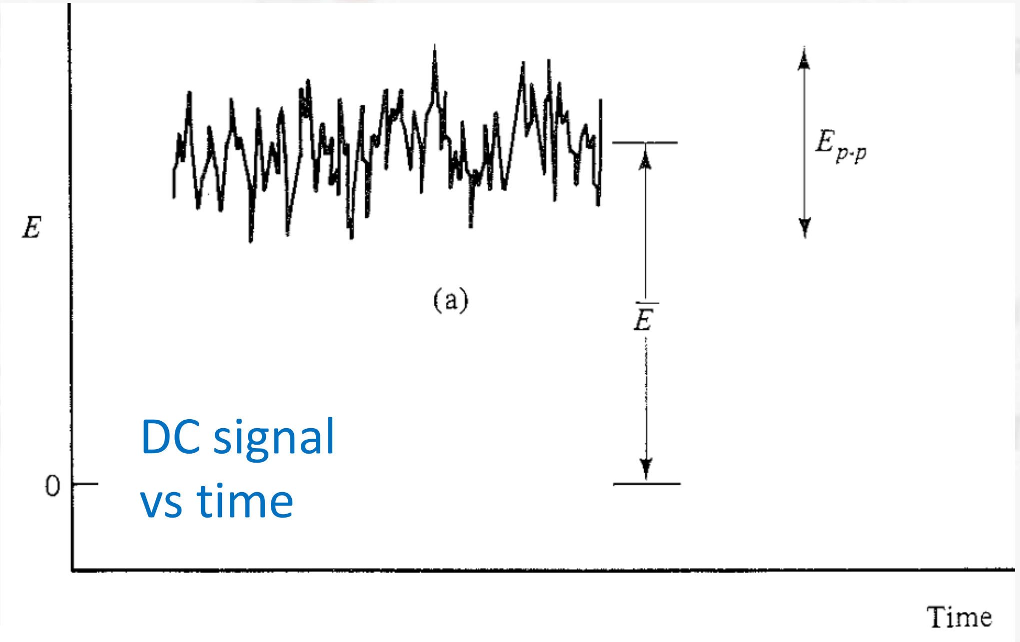
Noise

Unwanted fluctuations in signal (from source, transducer, ...)

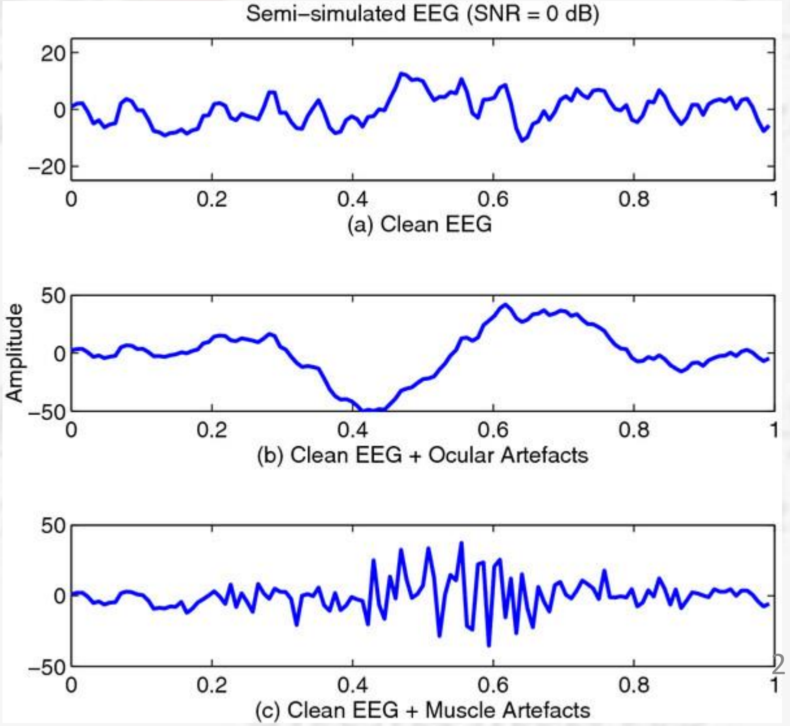
- Voice recording in a crowded room.
- photography in a low light room.

$$\frac{S}{N} = \frac{\overline{E}}{s_E}$$

$$\text{rms noise} = s_E = \left[\frac{\sum_{i=1}^{n}(E_i - \overline{E})^2}{n-1}\right]^{1/2}$$



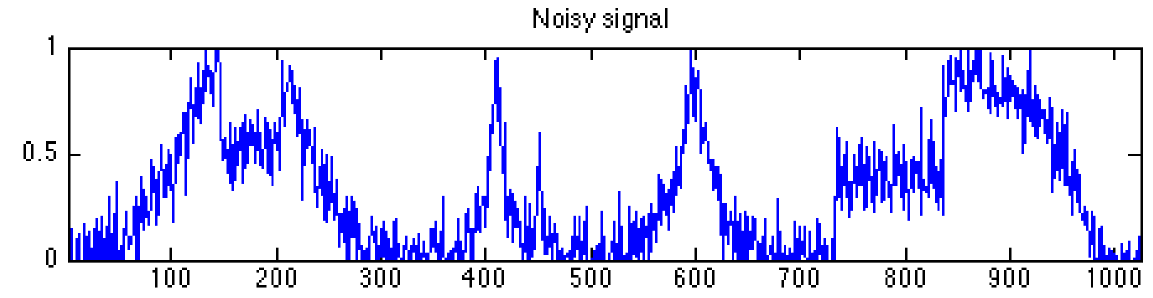
DC signal vs time



DC signal vs time

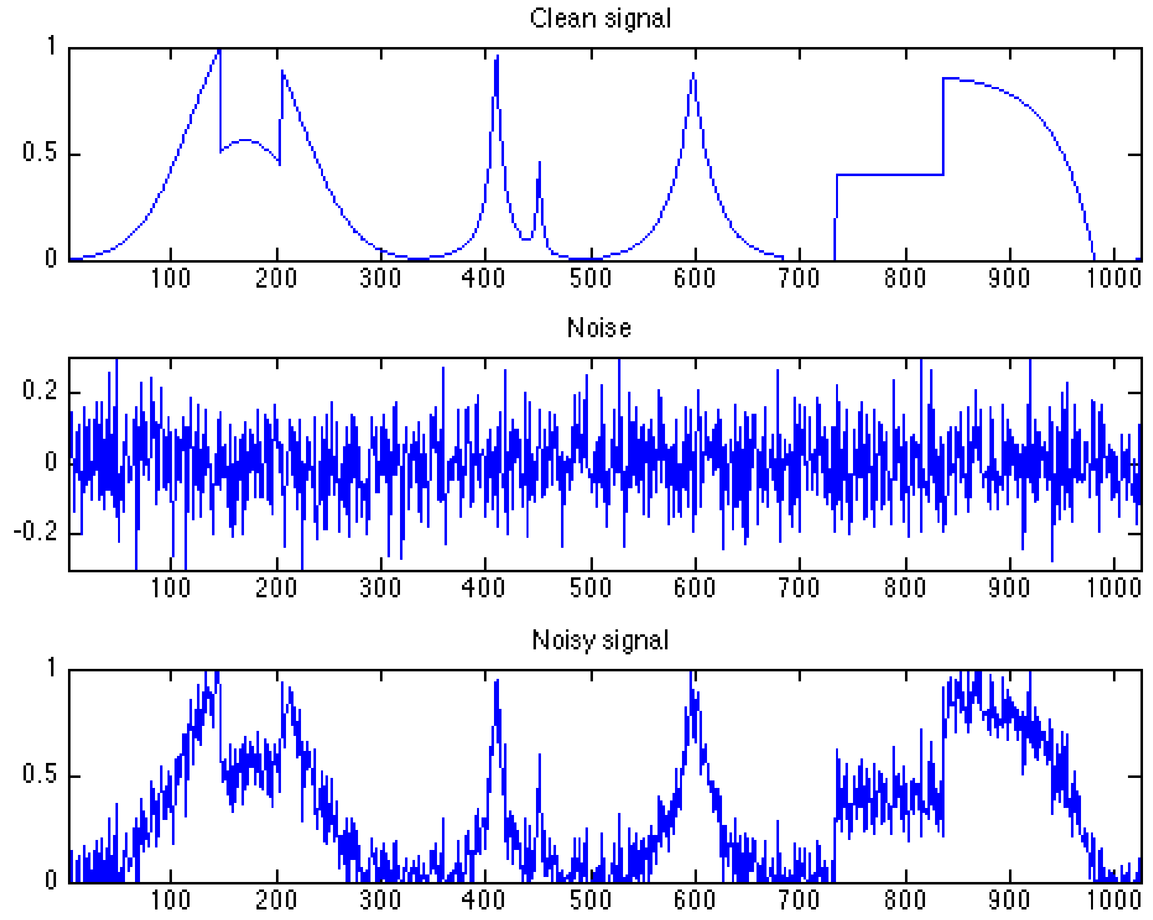
Not a constant DC signal

$$\text{rms noise} = s_E = \left[\frac{\sum_{i=1}^{n} (E_i - \overline{E})^2}{n - 1} \right]^{1/2}$$



Not a constant DC signal

$$\text{rms noise} = s_E = \left[\frac{\sum_{i=1}^{n}(E_i - \overline{E})^2}{n-1}\right]^{1/2}$$



Noise types

- **Random Noise** (Unpredictable)

  Summation of infinite number of changing *sine* waves.

  (With change in intensity and frequency)

  ** Fundamental: Particular nature of light and matter.

  Can never be eliminated.

  Example: Shot noise

  ** Excess (Non-fundamental): can be eliminated

  Example: Flicker noise

- **Non-Random Noise** : (always non-fundamental)

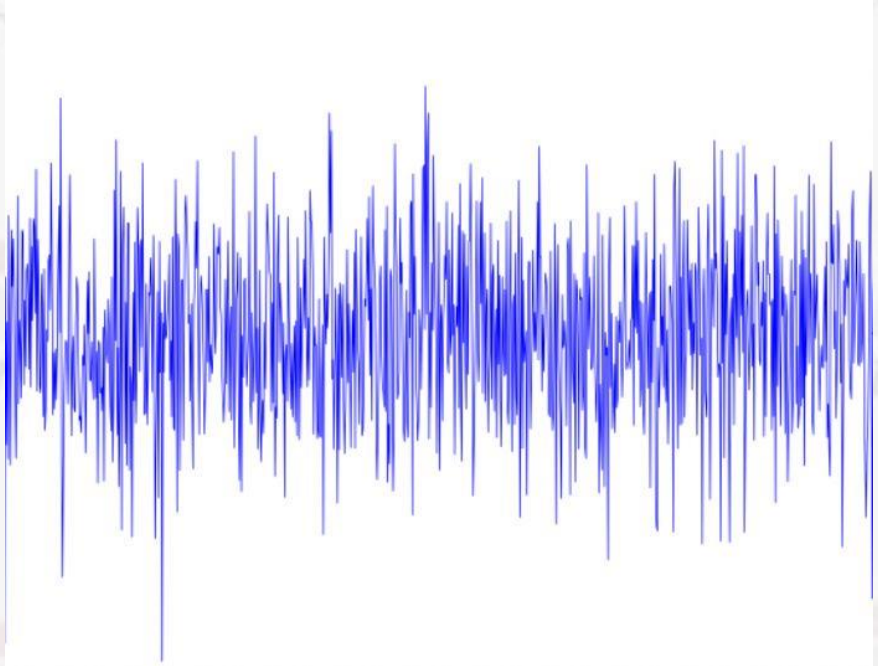
  Example: Impulse noise

  (when turning instrument on and off)

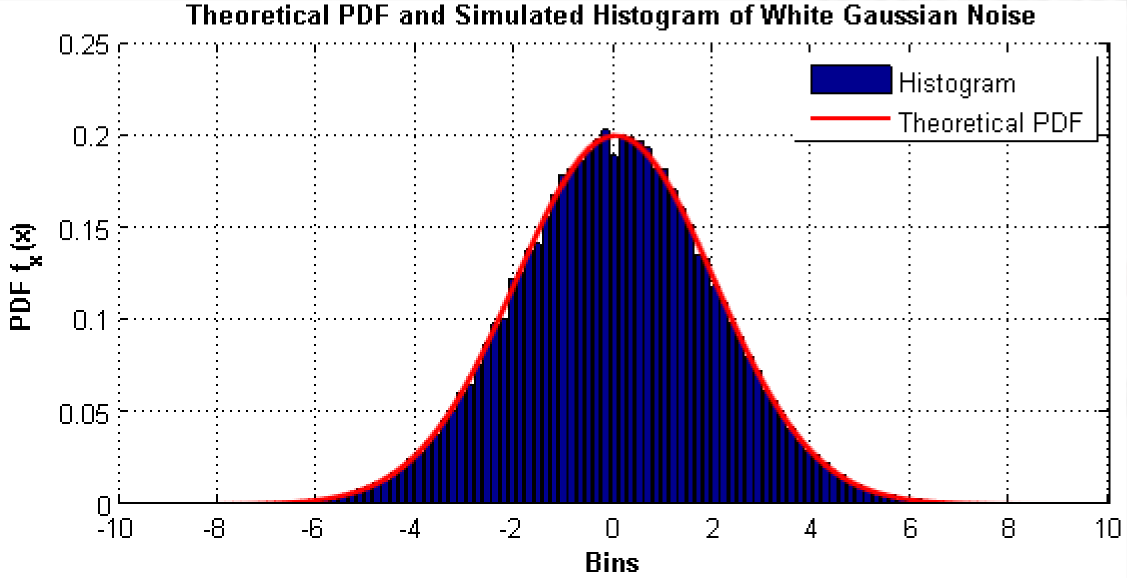
White noise

- Random
- Fundamental or excess

Gaussian Distribution

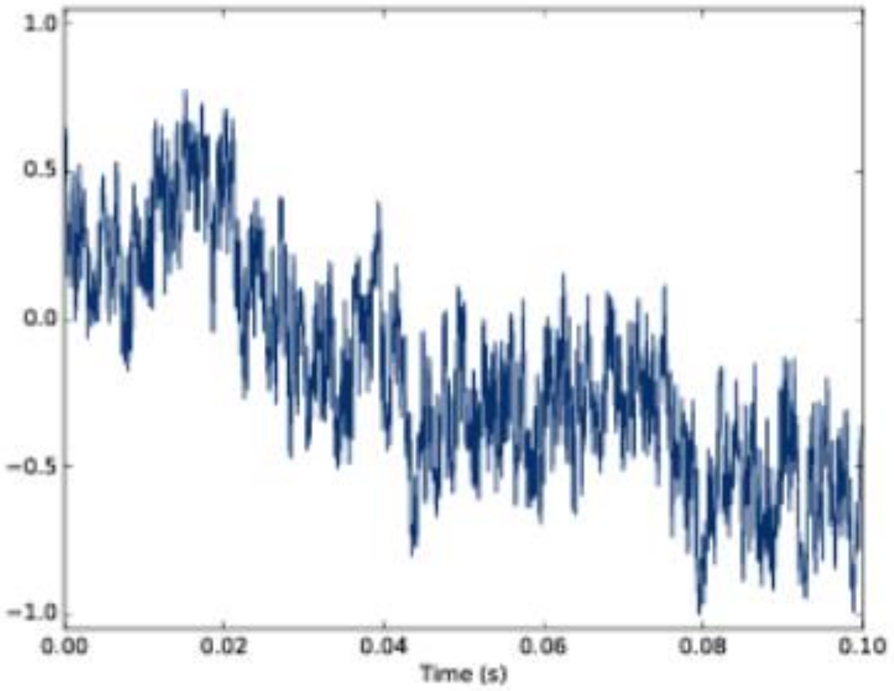


Low frequency sine waves: moderate
High frequency sine waves: moderate



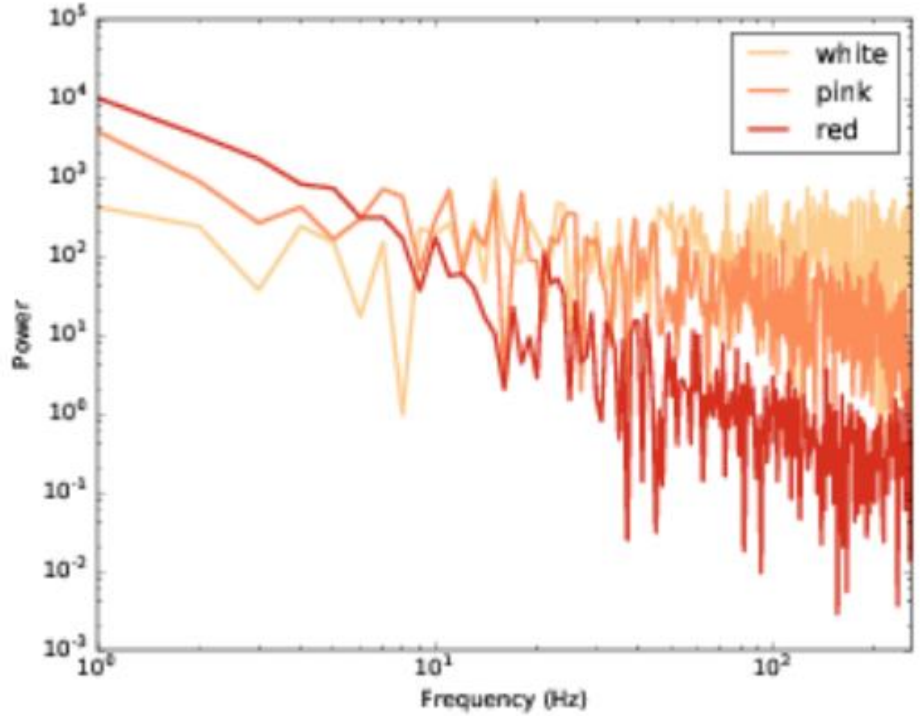
Pink noise (1/f noise)

Low frequency sine waves: stronger
High frequency sine waves: weak



Waveform of pink noise with β=1.

Power decreases as frequency increases

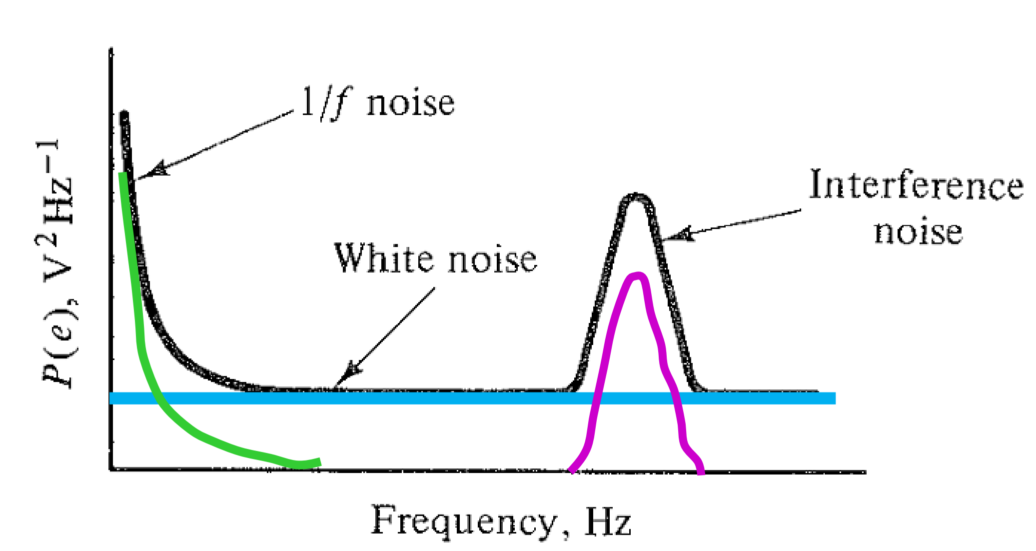


Spectrum of white, pink, and red noise on a log-log scale.

Example: drift

Noise power spectrum

White noise:
- Fundamental: Shot noise
- Nonfundam: Flicker noise
  independent to f



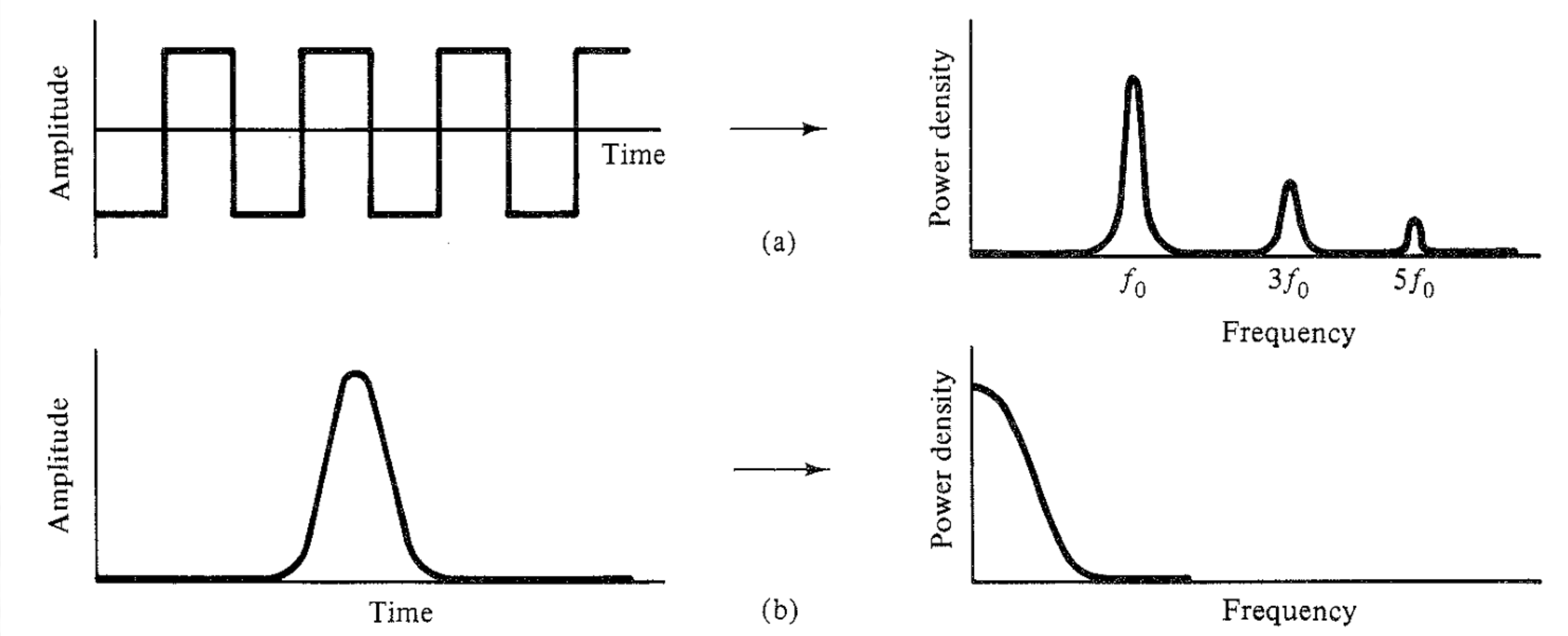
1/f noise (pink noise):
- power ∝ 1/f
- Random, nonfundamental

Interference noise: (environmental)
Nonfundam
descrete frequency

Flicker noise:
- Proportional (multiplicative) rms ∝ S

Shot noise:
- Proportional to $S^{1/2}$

Some signals and their power spectra



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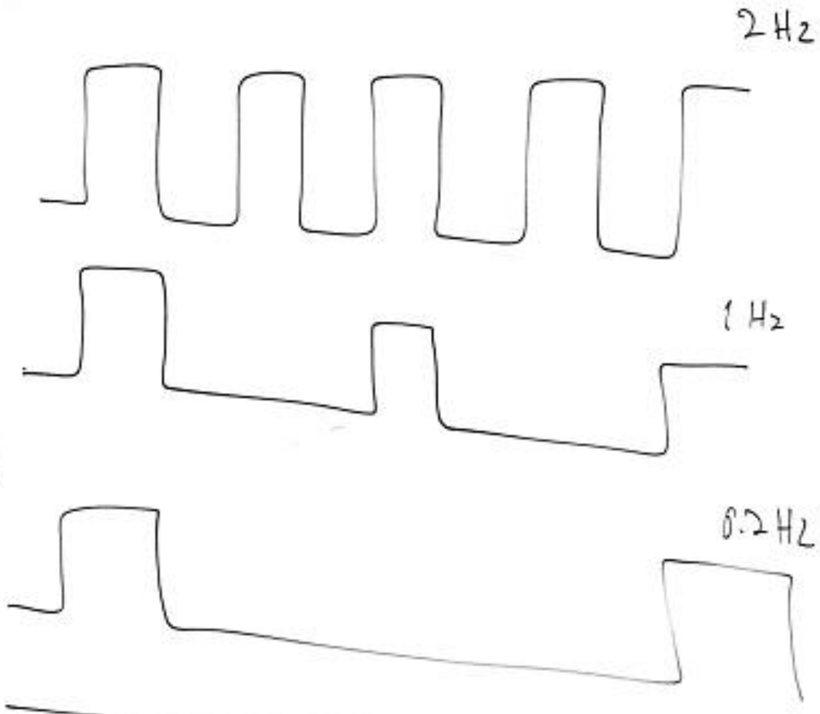
Signal to noise ratio

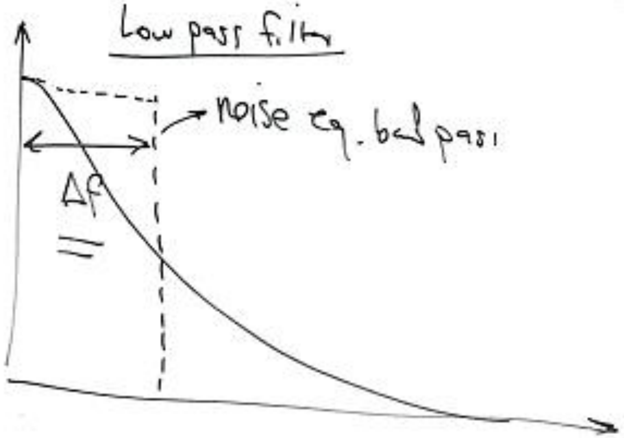
$\frac{S\uparrow}{N\downarrow} \rightarrow \uparrow$

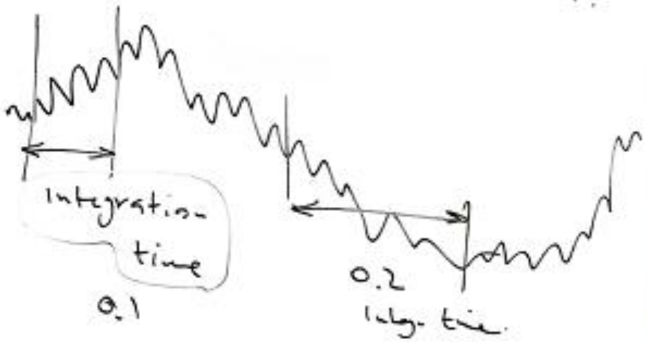
Flicker Noise $N_F \propto S$ $\frac{S\uparrow}{N_F\uparrow}$: کم بهبود $\frac{S}{N}$ با افزایش signal

Shot noise $N_S \propto \sqrt{S}$ $\frac{S\uparrow}{N_S} \propto \sqrt{S}$ بهبود $\frac{S}{N}$ با افزایش sign

Ex measuring S and noise in diff concentration of analyte (diff fluor. signals)



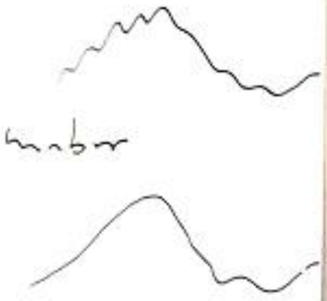




original spect : 1000,000 numbers

aver. of 10 : 100,000 number

av. of 10 1000



Frequency response of filters and integrators

Equivalent noise band passes (d), (e) and (f)

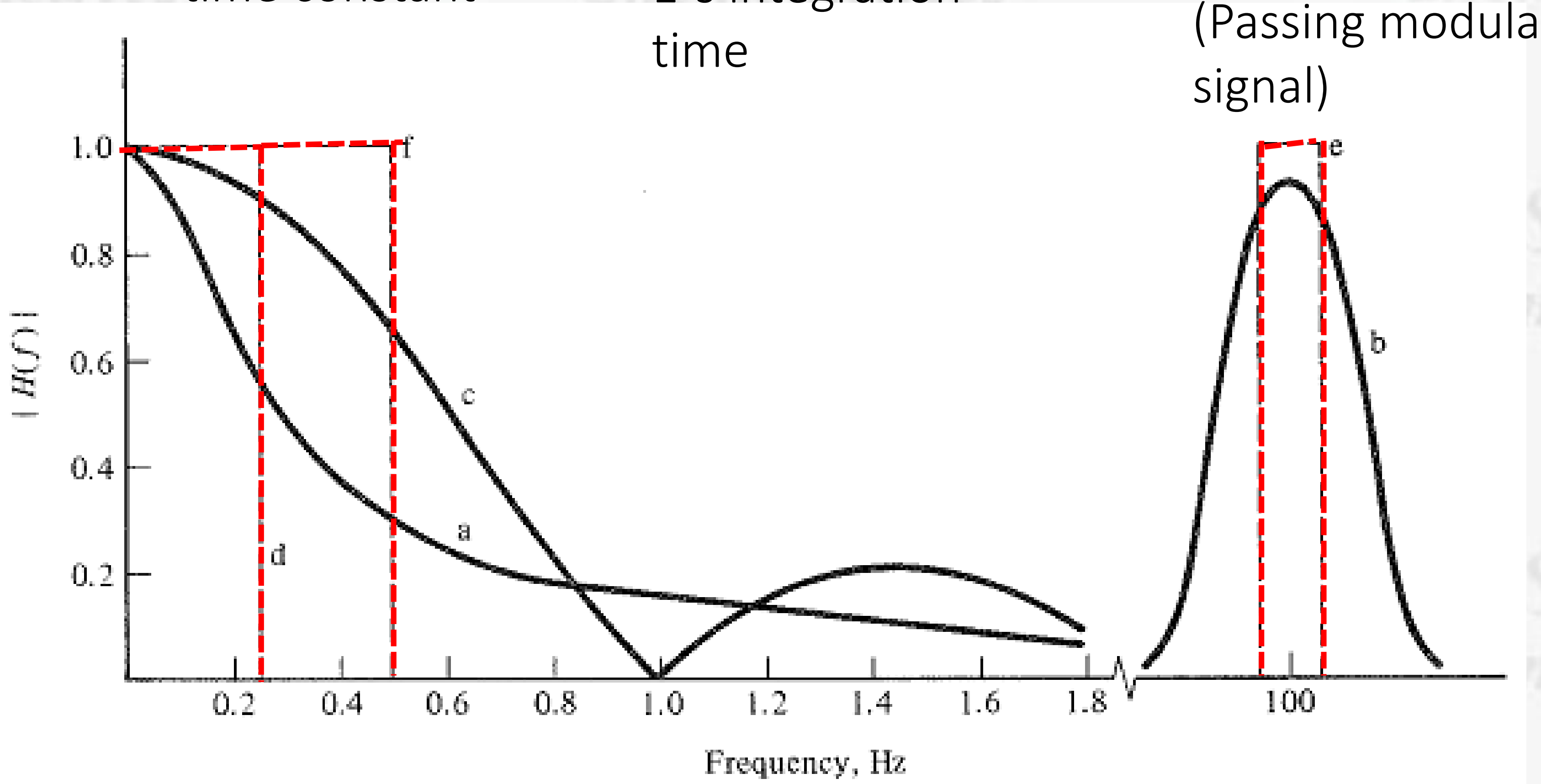
Area under each rectangle is equivalent to the area under corresponding curve (square of the amplitude transfer function)

$$|H(f)| = \left|\frac{E_o}{E_{in}}\right|$$

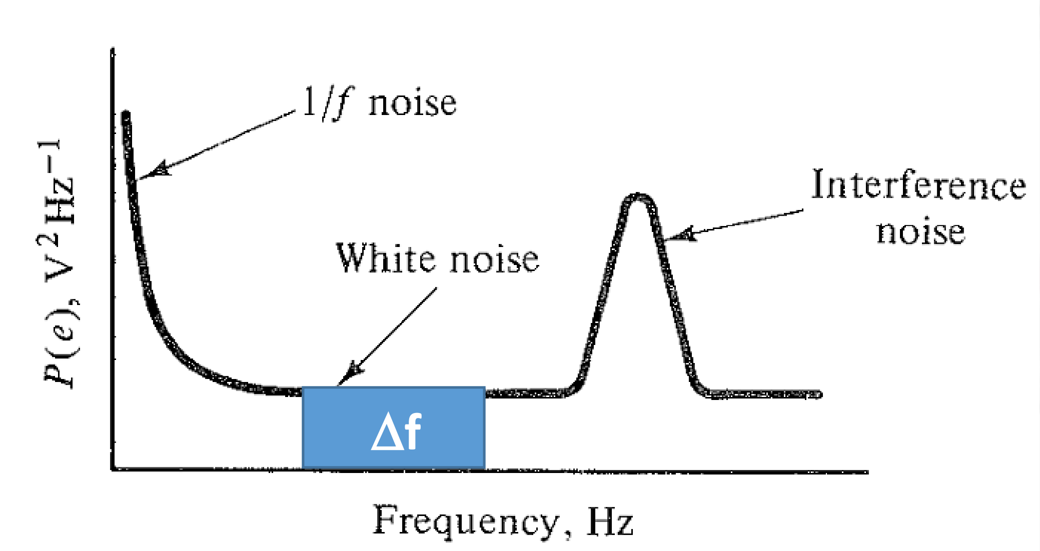
(a) Low pass filter with 1-s time constant

(c) Integrator with 1-s integration time

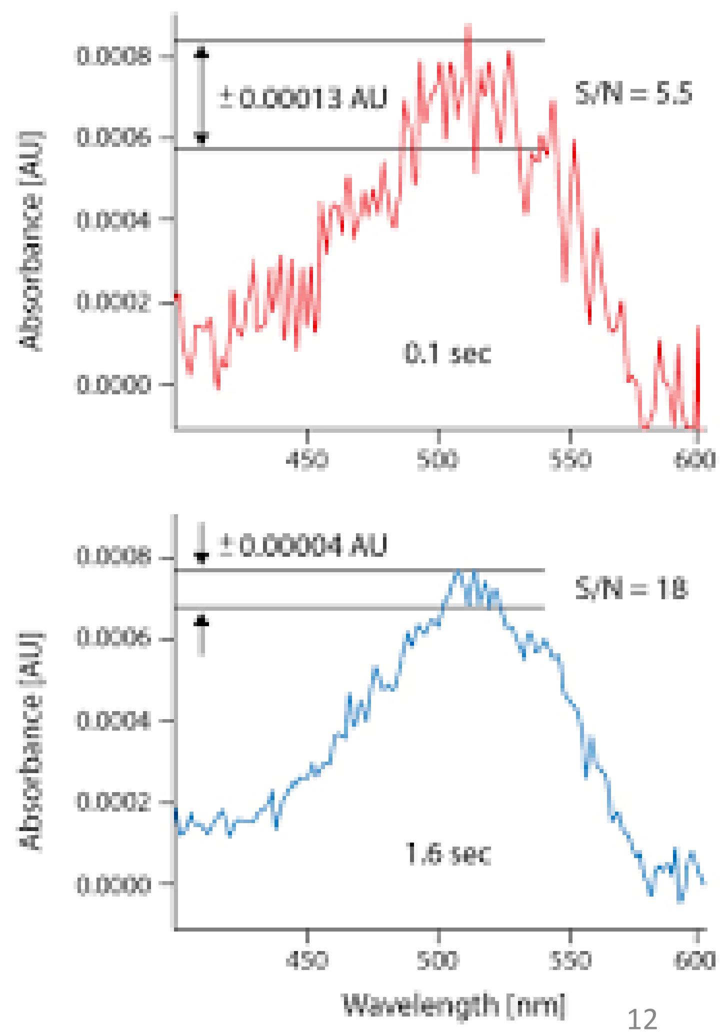
(b) Band pass centered at 100 Hz (Passing modulated signal)



Noise equivalent band pass (Δf)



$$\Delta f = \int_0^\infty |H(f)|^2 \, df$$



Δf= 1/(4 τ)

τ: integration time

= π/2 (f2 – f1)

band pass filter

TABLE 5-1

Bandpass characteristics of frequency-limiting circuits[a]

Circuit	$\lvert H(f)\rvert$	Lower cutoff frequency, f_1 (3 dB point)	Upper cutoff frequency, f_2 (3 dB point)	Signal frequency bandpass, $f_2 - f_1$ (Hz)	Noise equivalent bandpass, Δf (Hz)
Low-pass filter	$[1 + (2\pi f\tau)^2]^{-1/2}$	0	$(2\pi\tau)^{-1}$	$(2\pi\tau)^{-1}$	$(4\tau)^{-1}$
Integrator	$\frac{\sin \pi f t}{\pi f t}$	0	$\frac{0.433}{t}$	$\frac{0.433}{t}$	$(2t)^{-1}$
Bandpass filter	—	f_1	f_2	$\frac{f_m}{Q}$	$\frac{\pi f_m}{2Q}$

[a] τ, Time constant (*RC*) of low-pass filter (s); t, integration time of dc integrator (s); Q, quality factor for bandpass filter $= f_m/(f_2 - f_1)$; f_m, central frequency of bandpass filter (Hz).

$$s_E^2 = \int_0^\infty P(e)|H(f)|^2\, df$$

$$s_E^2 = P(e) \int_0^\infty |H(f)|^2\, df = P(e)\, \Delta f$$

Poisson distribution probability

$$P(n) = \frac{(rt)^n e^{-rt}}{n!}$$

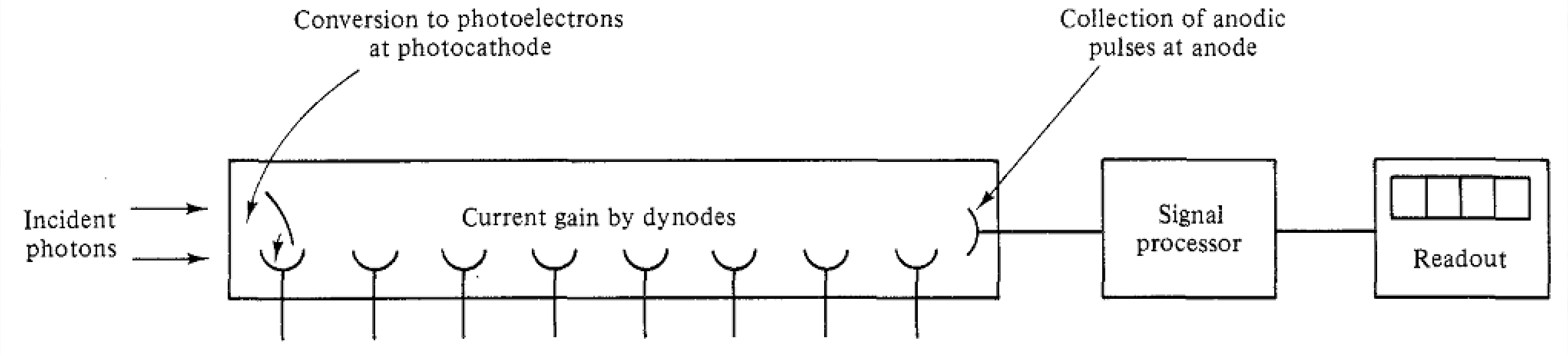
$n_{aver}=rt$ Quantum noise

$StDev(n_{aver)}=(rt)^{0.5}$

$n_{aver}/StDev(n_{aver)}=(rt)^{0.5}=n^{0.5}$

P(n): Probability of observing *n* events (anodic pulses) in time *t*.

r: Mean rate of event.



Photon counting signal processing:	$\Phi_{p,S}, \Phi_{p,B}$ (photon s^{-1})	$\xrightarrow{K(\lambda)}$	r_{cS}, r_{cB}, r_{cd} (electrons s^{-1})	$\longrightarrow$	r_{aS}, r_{aB}, r_{ad} (anodic pulses s^{-1})	$\xrightarrow{tf_d}$	n_S, n_B, n_d (counts)
Analog signal processing:	Φ_S, Φ_B (W)	$\xrightarrow{R(\lambda)}$	i_{cS}, i_{cB}, i_{cd} (A)	$\xrightarrow{m}$	i_{aS}, i_{aB}, i_{ad} (A)	$\xrightarrow{G}$	E_S, E_B, E_d (V)

$$n = \Phi_p t$$

$$(\sigma_n)_q = (\Phi_p t)^{1/2} = n^{1/2}$$

$$\frac{S}{N} = \frac{n}{(\sigma_n)_q} = n^{1/2}$$

TABLE 5-2

Signal and noise expressions for quantum or shot noise at transformation points[a]

Transformation point	Photon counting signal processing: Signal		Photon counting signal processing: Noise		Analog signal processing: Signal		Analog signal processing: Noise	
Photocathode[b]	$n_c = K(\lambda)\Phi_p = r_c t$ $= K(\lambda)n$	(1)	$(\sigma_n)_q = n_c^{1/2}$ $= [K(\lambda)n]^{1/2}$	(2)	$i_c = \frac{e}{t}n_c = er_c$ $= R(\lambda)\Phi$	(3)	$(\sigma_i)_q = \frac{e}{t}n^{1/2}$ $= \left(\frac{ei_c}{t}\right)^{1/2}$	(4)
Anode[c]	$n_a = n_c$	(5)	$(\sigma_n)_q = n_a^{1/2}$	(6)	$i_a = mi_c = mR(\lambda)\Phi$	(7)	$(\sigma_i)_s$ $= m(1+\alpha)^{1/2}\left(\frac{ei_c}{t}\right)^{1/2}$ $= \left[mei_a\frac{(1+\alpha)}{t}\right]^{1/2}$	(8)
Readout[d]	$n_r = f_d n_a$	(9)	$(\sigma_n)_q = (n_r)^{1/2}$ $= (f_d n_a)^{1/2}$	(10)	$E = Gi_a = mGR(\lambda)\Phi$	(11)	$(\sigma_E)_s = G(\sigma_i)_s$ $= \left[\frac{GmeE(1+\alpha)}{t}\right]^{1/2}$	(12)

$$(\sigma_E)_s = [2e\,\Delta f(1+\alpha)mGE]^{1/2}$$

$$(\sigma_E)_s = (mGKE)^{1/2}$$

TABLE 5-3

Signal and noise expressions for quantum and shot noise in analytical, background, and dark signals

Type of signal	Photon counting signal processing		Analog signal processing	
	Mean	Rms noise	Mean	Rms noise
Analytical	$n_S = f_d K(\lambda) r_S t$	$(\sigma_S)_s = n_S^{1/2}$	$E_S = mGR(\lambda)\Phi_S$	$(\sigma_S)_s = (mGKE_S)^{1/2}$
Background	$n_B = f_d K(\lambda) r_B t$	$(\sigma_B)_s = n_B^{1/2}$	$E_B = mGR(\lambda)\Phi_B$	$(\sigma_B)_s = (mGKE_B)^{1/2}$
Dark	$n_d = f_d r_{cd} t$	$(\sigma_d)_s = n_d^{1/2}$	$E_d = mGi_{cd}$	$(\sigma_d)_s = (mGKE_d)^{1/2}$