

Signal to noise ratio:

$$\frac{S}{N} = \frac{I_s}{\left\{ \underbrace{k(I_s + I_B + I_D)}_{\text{Shot n.}} + \underbrace{(f I_s)^2}_{\text{Analyte flick n.}} + \underbrace{(k I_B)^2}_{\text{Backgr. fl. n.}} + \underbrace{[(\sigma_d)_{ex/mG}]^2}_{\text{dark flick n.}} + \underbrace{(\sigma_{ar/mG})^2}_{\text{amplif. readout.}} \right\}^{1/2}}$$

Luminesc. ^{and} Emission
or

- * Blank noise limited [Analyte] ↓ ↓ (high quality instr.)
- ① Shot noise
 - ② nonfund. n.l.
 - ③ Backgr. S. n. lim.
 - ④ Detector noise limited.

* Analyte signal noise limited
 $I_s \leftarrow (\text{Analyte}) \uparrow$

$$T = \frac{E_{st} - E_{nt}}{E_{rt} - E_{ot}} = \frac{E_s}{E_r}$$

$$E_s = T E_r$$

$$\sigma_A \begin{cases} \propto \sqrt{T} & \text{Shot} \\ \propto \sqrt{t} & \text{flick.} \\ \propto \frac{1}{T} & \text{o.t. } \checkmark \end{cases} \quad \sigma_T \begin{cases} \propto \sqrt{T} & \text{Shot} \\ \propto T & \text{flick} \\ \propto \sqrt{t} & \text{o.t.} \end{cases}$$

$T_1 = 0.8$	$\frac{1}{T_1} = 1.25$	0.04
$T_2 = 0.4$	$\frac{1}{T_2} = 2.5$	↓
		0.08



Analytical signal limited S/N expression (Fluor or Emiss, high concn)

$$\frac{S}{N} = \frac{i_s}{\{K(i_s + i_B + i_d) + (\xi i_s)^2 + (i_B)^2 + [(\sigma_{ex}/mG)]^2 + (\sigma_{in}/mG)^2\}^{1/2}}$$

$$\frac{S}{N} = \frac{i_s}{\{K(i_s) + (\xi i_s)^2\}^{1/2}}$$

1- Analytical signal shot noise limited S/N expression:

$$\frac{S}{N} = \left(\frac{i_s}{K}\right)^{1/2}$$

2- Analytical signal flicker noise limited S/N expression: here S/N can be improved by reducing bandwidth or adjusting experimental condition. Increasing signal does not improve S/N in such condition:

$$\frac{S}{N} = \xi^{-1}$$

from
p149

Dependence of S/N on analytical signal:

$$\frac{S}{N} = \frac{i_s}{\{K(i_s + i_B + i_d) + \xi i_s^2 + \chi i_B^2 + [(\sigma_d)_{ex}/mG]^2 + (\sigma_{ar}/mG)^2\}^{1/2}}$$

Blank includes parts that are independent of i_s

$$\frac{S}{N} = \frac{i_s}{[K i_s + (\xi i_s)^2 + \sigma_{bk}^2]^{1/2}}$$

1- Shot noise limited: $\frac{S}{N} = \frac{i_s^{1/2}}{[K]^{1/2}}$

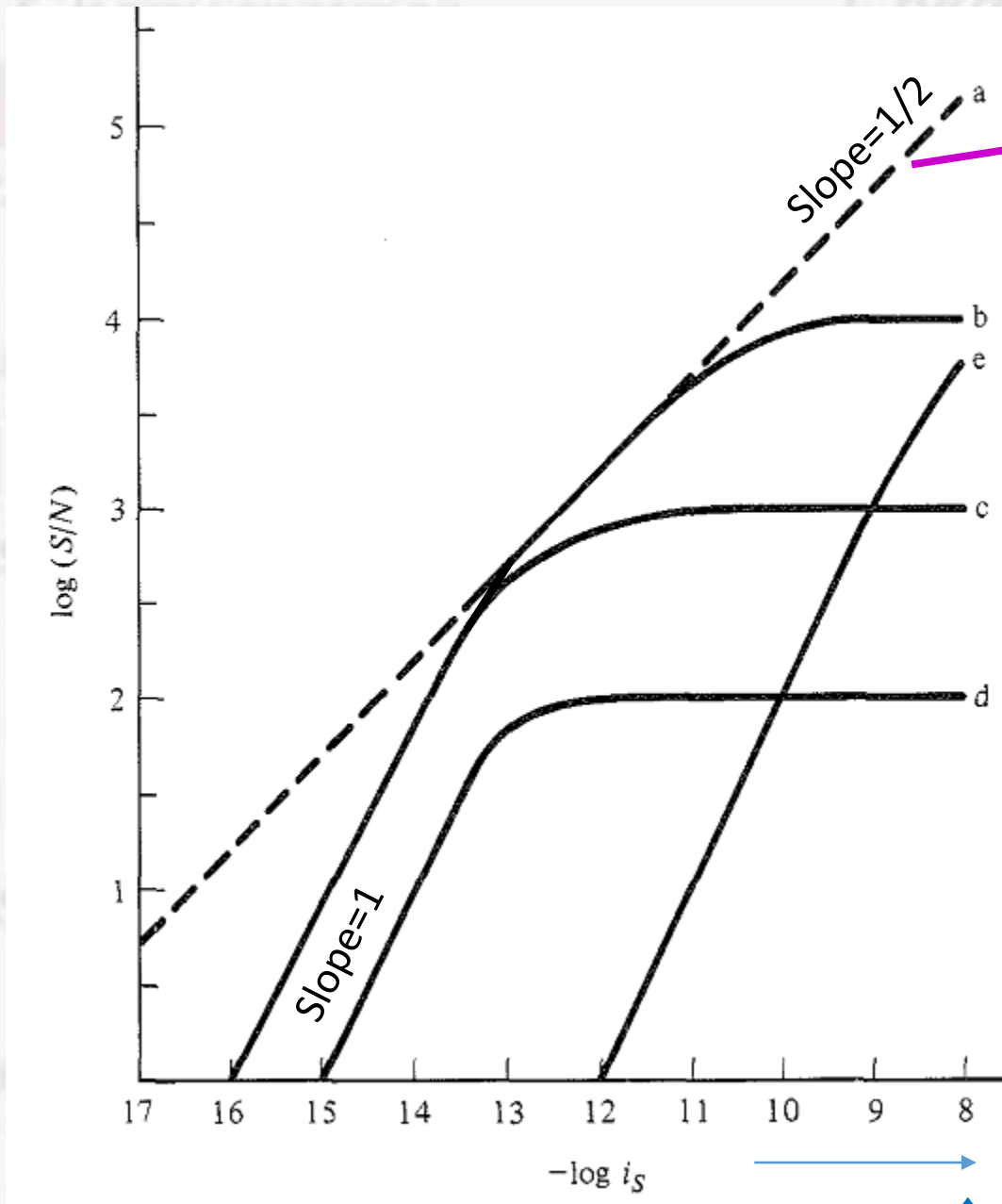
$$\log \frac{S}{N} = \log [K]^{1/2} + 1/2 \log i_s$$

2- Flicker noise limited: $\frac{S}{N} = \xi^{-1/2}$

$$\log \frac{S}{N} = -1/2 \log \xi + 0 \log i_s$$

3- Blank noise limited: $\frac{S}{N} = \frac{i_s}{\sigma_{bk}}$

$$\log \frac{S}{N} = -\log \sigma_{bk} + \log i_s$$



Limit line for S/N is the shot noise.

$\xi=10^{-2}, \sigma_{bk}=10^{-16}$

$\xi=10^{-2}, \sigma_{bk}=10^{-12}$

$\xi=10^{-3}, \sigma_{bk}=10^{-16}$

$\xi=10^{-2}, \sigma_{bk}=10^{-15}$

When σ_{bk} and/or ξ are/is high we do not observe shot noise. (low quality instrument)

[analyte] ↑

Signal to noise expression for absorption (vs T):

$$A = -\log T$$

or

$$A = -0.43 \ln T$$



$$\sigma_A = 0.43 \frac{\sigma_T}{T}$$

$$T = \frac{E_{st} - E_{ot}}{E_{rt} - E_{ot}}$$

by considering
only σ_{st}

$$\frac{d(\frac{U}{V})}{dx} = \frac{U'V - UV'}{V^2}$$

$$\sigma_T = \frac{\sigma_{st}(E_{rt} - E_{ot}) - (E_{st} - E_{ot})(0)}{(E_{rt} - E_{ot})^2} = \frac{\sigma_{st}}{E_r}$$

$$\sigma_A = 0.43 \frac{\sigma_{st}}{TE_r}$$

$$E_{st} = E_s + E_{ot} \quad \sigma_T = \frac{\sigma_{st}}{E_r} = \frac{(mGE_s + (\xi E_s)^2 + \sigma_{ot}^2)^{1/2}}{E_r}$$

$$\sigma_T = \frac{\sigma_{st}}{E_r} = \frac{(mGTE_r + (\xi TE_r)^2 + \sigma_{ot}^2)^{1/2}}{E_r}$$

- $\sigma_T \propto \sqrt{T}$ → Shot noise limitin
- $\sigma_T \propto T$ → Fliker noise
- $\sigma_T \propto \text{constant}$ → Dark noise limited

Signal to noise expression for absorption (vs A):

Noise equation for Transmittance is exactly similar to luminesc & emission.

$$\sigma_T = \frac{\sigma_{st}}{E_r} = \frac{(mGTE_r + (\xi TE_r)^2 + \sigma_{0t}^2)^{1/2}}{E_r}$$

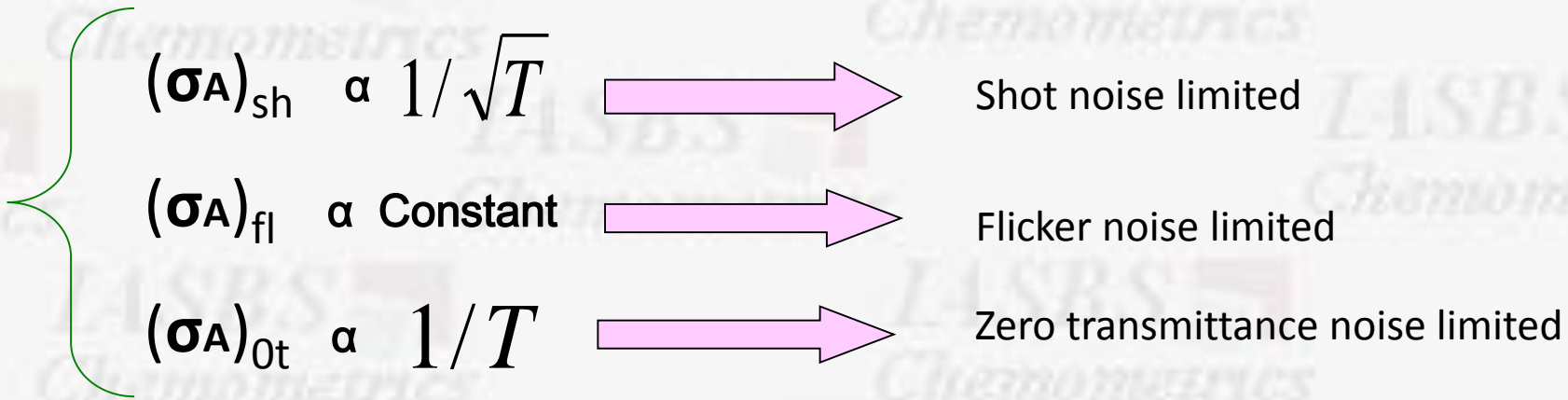
in

$$\sigma_A = 0.43 \frac{\sigma_T}{T}$$

→

$$\sigma_A = \frac{0.43(mGTE_r + (\xi TE_r)^2 + \sigma_{0t}^2)^{1/2}}{TE_r}$$

$$\sigma_A = 0.43 \left(\frac{mG}{TE_r} + \xi^2 + \frac{\sigma_{0t}^2}{TE_r^2} \right)^{1/2}$$



Example;

1) $\left\{ \begin{array}{l} \bar{T}_1=0.8 \quad \sigma_{A1}=0.02 \\ \bar{T}_2=0.4 \quad \sigma_{A2}=0.02 \end{array} \right.$

In this case flicker noise is dominant

2) $\left\{ \begin{array}{l} T_1=0.8 \quad \sigma_{A1}=0.04 \\ T_2=0.4 \quad \sigma_{A2}=0.08 \end{array} \right.$

Zero transmittance current is limiting

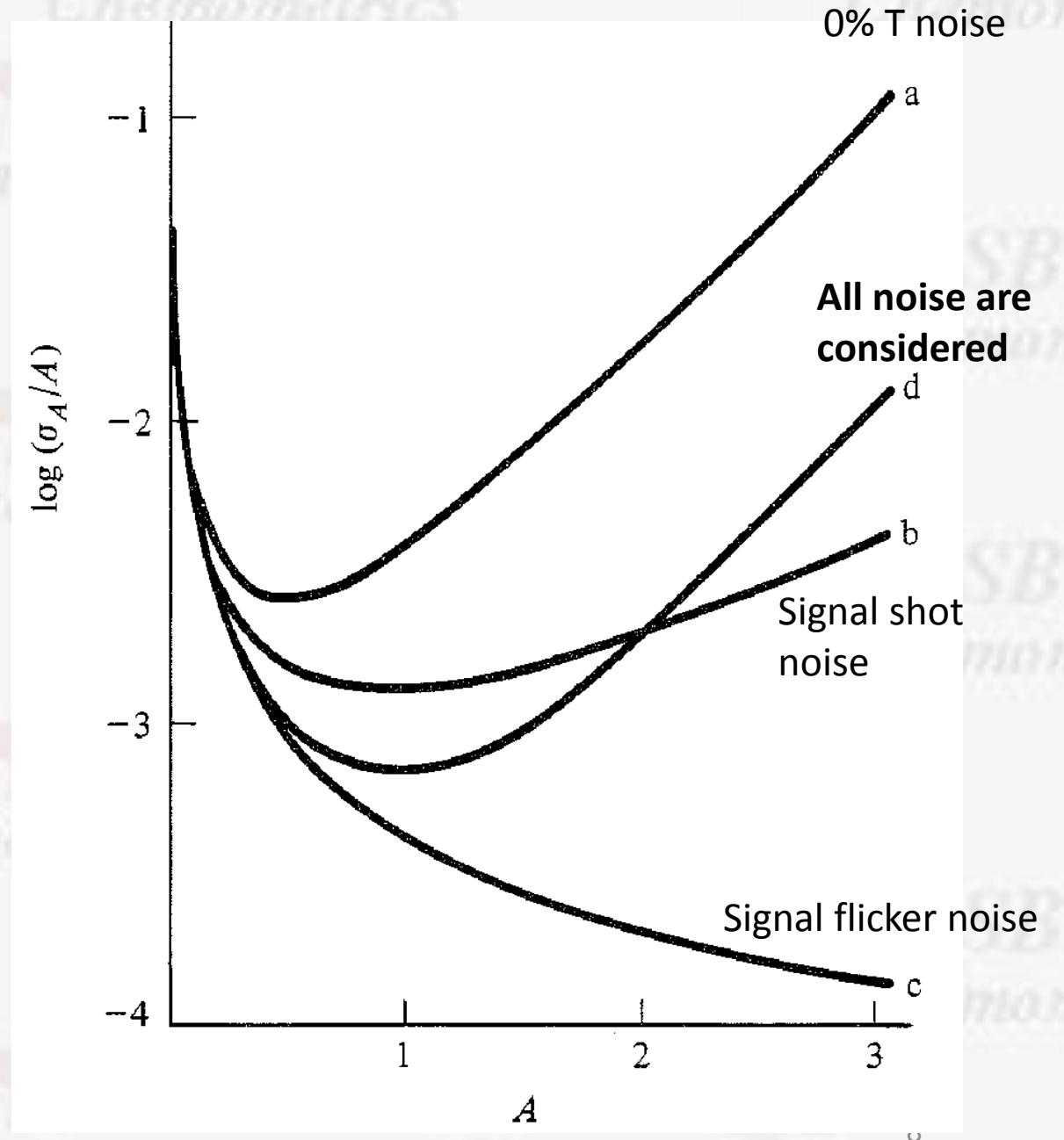
Noise to signal diagram;

$$\sigma_A = 0.43 \frac{\sigma_{st}}{TE_r} = 0.43 \frac{\sigma_{st}}{E_s}$$

$$\frac{\sigma_A}{A} = \frac{\sigma_A}{-lnT} = 0.43 \frac{\sigma_{st}}{-TE_r lnT}$$

$$\sigma_{st} = (mGE_s + (\xi E_s)^2 + \sigma_{0t}^2)^{1/2}$$

$$\frac{\sigma_A}{A} = 0.43 \left(\frac{mG}{TE_r A^2} + \frac{\xi^2}{A^2} + \frac{\sigma_{0t}^2}{(TE_r A)^2} \right)^{1/2}$$



Zero Transmittance (0%) Noise limited

When analyte concentration is very high and T is considerably low
-or when detector noise is high (IR detectors)

$$\frac{\sigma_A}{A} = 0.43 \left(\frac{\cancel{mG}}{\cancel{T}E_r A^2} + \frac{\cancel{\xi^2}}{A^2} + \frac{\sigma_{0t}^2}{(TE_r A)^2} \right)^{1/2}$$

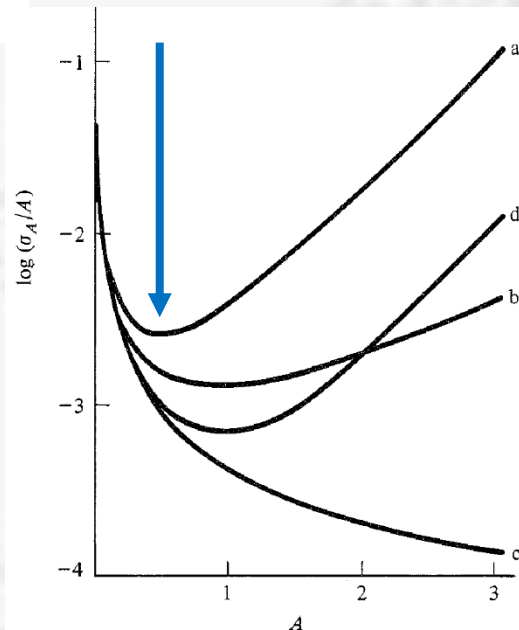
$$\sigma_{0t} = (\sigma_d^2 + \sigma_{ar}^2 + \sigma_{bE}^2)^{1/2}$$

$$\sigma_{0t} = [(\sigma_{bE})_s^2 + (\sigma_{bE})_f^2 + (\sigma_d)_s^2 + \sigma_{ar}^2 + (\sigma_d)_{ex}^2]^{1/2}$$

$$\frac{\sigma_A}{A} = \frac{\sigma_{0t}}{-E_r T \ln T} = \frac{\sigma_{0t}}{-E_{fs} T \ln T}$$

Differentiation = 0

→ min $A=0.434$ min $\sigma A/A$



- Intensive source
- Wider slit → $E_r \uparrow$ → $\sigma A/A \downarrow$
- (Sample absorb ↓) $T \uparrow$

Signal Shot noise limited

$$\frac{\sigma_A}{A} = 0.43 \left(\frac{mG}{TE_r A^2} + \frac{\xi^2}{A^2} + \frac{\sigma_{\delta t}^2}{(TE_r A)^2} \right)^{1/2}$$

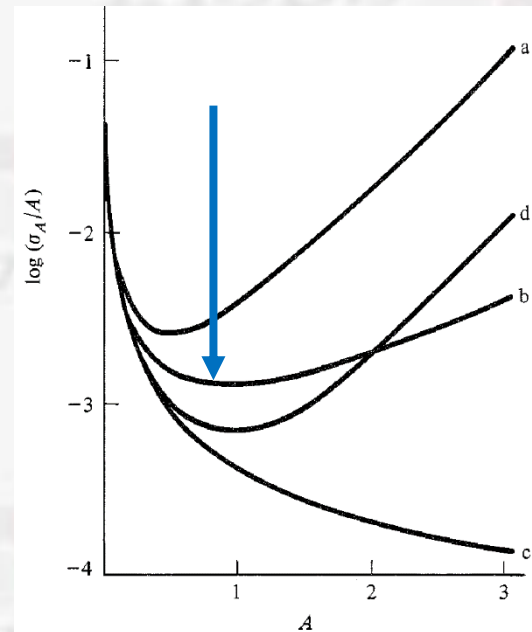
$$\frac{\sigma_A}{A} = \frac{[mGK/(TE_r)]^{1/2}}{-\ln T} = \frac{(K/Ti_r)^{1/2}}{-\ln T}$$

derivative = 0
 $\rightarrow A=0.87$ min σ_A/A

-integration time \uparrow
 $\rightarrow \Delta f \downarrow \rightarrow K \downarrow \rightarrow \sigma_A/A \downarrow$

-Intensive source
 -Wider slit $\rightarrow i_r \uparrow$

- $T \uparrow \rightarrow i_s \uparrow$



Signal-Flicker noise limited

$$\frac{\sigma_A}{A} = 0.43 \left(\frac{\cancel{mG}}{\cancel{TE_r}A^2} + \frac{\xi^2}{A^2} + \frac{\cancel{\sigma_{\delta t}^2}}{(\cancel{TE_r}A)^2} \right)^{1/2}$$

$$\frac{\sigma_A}{A} = \frac{\xi_s}{-\ln T} = \frac{\xi_s}{A}$$

$$\sigma_A = \xi_s$$

$$\xi_s = (\xi_1^2 + \xi_2^2)^{1/2}$$

Source flicker
factor

Container
flicker factor

$\xi \downarrow \Rightarrow S/N \uparrow$

-More Stable light source.

-Double beam

