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Sampling moiré technique and the dynamics of a spreading droplet on a solid surface

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Abstract

The rich physics of the fluid dynamical phenomena at the scale of capillary length needs novel techniques for experimental investigations. We have examined the recently developed method of sampling moiré for a problem in the area of fluid dynamics. We have experimentally quantified the dynamics of a droplet in the spreading regime, where the well known scaling results for the dynamics are expected from the theory of fluid dynamics. The simplicity of the method allows us to study and see a very good comparison between our experimental data and the theoretical results.

Keywords: sampling moiré, droplet spreading, droplet shape

(Some figures may appear in colour only in the online journal)

1. Introduction

Moiré fringe patterns are the base elements in constructing many experimental measurement techniques. Measuring in-plane and out-of-plane displacements of solid specimens are classical examples that have proved the strong ability of coherent moiré techniques [1–3]. Moiré interferometry is also the subject of many investigations [4, 5]. Recently, a simple and accurate phase measurement method, called the sampling moiré method, was developed for measuring thickness distributions of transparent plates from a single image [6]. The accuracy and systematic error behind this method are also very well studied and discussed [7–9]. In addition to the simplicity of the method, the non destructive nature of this technique allows one to study the samples without any physical damage. A class of problems in fluid dynamics, deals with the shape of interfaces of multi-phase fluid systems [10]. In this case a competition between different forces—surface tension, viscous and gravitational forces—eventually determines the dynamics of the interface. Especially, phenomena at the scale of capillary length, roughly speaking 1 mm in size, are much more interesting to study. In addition to direct imaging and studying the recorded images, classical interferometry is the traditional method for experimental studies in these systems [11–13].

In this paper we show that the sampling moiré method can provide a good method to investigate the dynamical properties of a spreading droplet of a transparent fluid over a rigid substrate. In this case the interface of fluid and air develops in time. Studying the dynamics of such a multi-phase flow interface is interesting either from a fundamental or technical point of view. Investigating the dynamics of a droplet over a rigid surface can reveal both the dynamical characteristics of the fluid, like viscosity and surface tension, and more importantly, the contact angle of a droplet. The contact angle encodes the surface properties of a fluid and also the underlying rigid substrate.

The structure of this paper is as follows: after a short review of the moiré method in section 2, we will present the details of our experiment and results in section 3. A theoretical argument for the spreading dynamics that is based on scaling facts is presented in section 4, and finally an outlook is given in section 5.

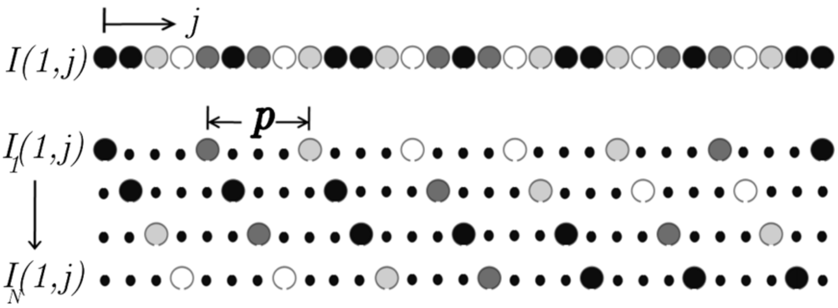
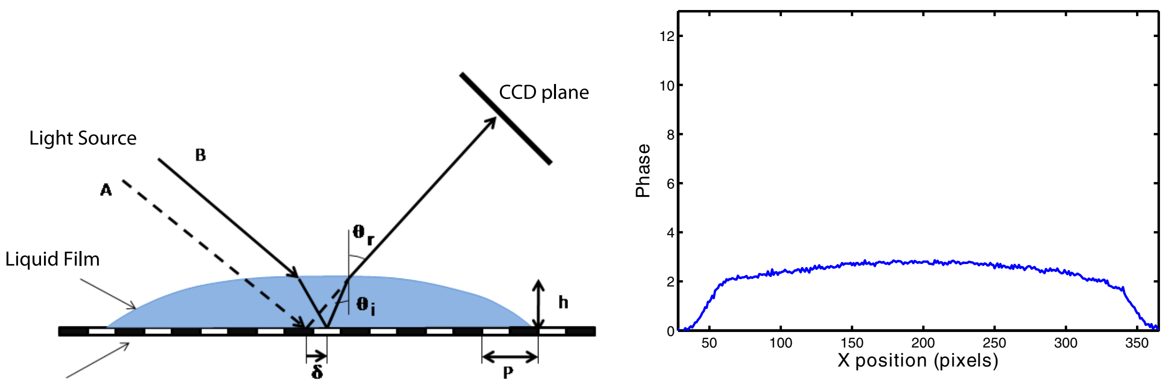


Figure 1. Procedure of constructing moiré fringe patterns. A first row of a single image taken from a Ronchi grating is sampled p times. The sampled images will be used in constructing the phase distribution of the image.



(*a*) (*b*)

Figure 2. Left: geometry of a droplet spreading over a flat glass. A Ronchi grating is located behind the glass. Right: a cross section of phase distribution constructed from moiré fringes.

2. Method of sampling moiré

A single image taken from a Ronchi grating that encoded the optical characteristics of a transparent sample is the main experimental element of the sampling moiré method [6]. Figure 1 presents briefly the details of this method. Let us denote the intensity distribution of an original image taken from a grating by $I(i, j)$. The wavelength of this grating that is measured in terms of pixel numbers is denoted by p. Here we outline the procedure just for the first row of the matrix: $I(1, j)$, this can be repeated for other rows. Different fringe patterns denoted by $I_n(i, j)$ can be constructed from the original image in the following way: for the nth fringe matrix, we choose all the $n + j$ elements ($j = 1, 2, \ldots, p$) from the original image, and interpolate all the interval elements by the neighboring data points. Here we have used a simplified case where the number of fringe patterns is exactly p, but in general those could differ. Now, having in hand p sampled fringe patterns, we can calculate the phase distribution by:

$$\varphi(i, j) = -\arctan \frac{\sum_{n=1}^{p} I_n(i, j) \sin\left(\frac{2\pi(n-1)}{p}\right)}{\sum_{n=1}^{p} I_n(i, j) \cos\left(\frac{2\pi(n-1)}{p}\right)}. \quad (1)$$

Performing the above procedure for a reference grating (not deformed) and a grating deformed by a sample, we can subtract the corresponding phases and obtain the phase shift distribution $\Delta\varphi(i, j)$. Real deformation of the grating is related to this phase shift by:

$$\delta(i, j) = -(p/2\pi)\Delta\varphi(i, j), \quad (2)$$

where the deformation is measured in units of pixel size. Very small displacements, up to 1/500 of the grating pitch can be achieved by the moiré method [9].

Using geometrical optics, we can obtain equations that relate the displacement field of the grating to the height profile of a transparent sample. Figure 2 shows the set-up of our experiment. A droplet of a transparent fluid, spreading over a rigid substrate shifts the image of a grating that is located behind the droplet. As shown in this figure, the angles that the reflected ray makes with the surface of the droplet are denoted by θ_i and θ_r. We see that:

$$h(i, j)(\tan\theta_r - \tan\theta_i) = \delta(i, j), \quad (3)$$

where $h(i, j)$ is the height profile of the droplet. By simplifying the equations we will have:

$$h(i, j) = \frac{1}{\tan\theta_r(i, j)} \frac{\sqrt{n^2 - \sin^2\theta_r(i, j)}}{\sqrt{n^2 - \sin^2\theta_r(i, j)} - \cos\theta_r(i, j)} \delta(i, j), \quad (4)$$

where n is the refraction index of the fluid relative to the refraction index of air. Here we are assuming that the refraction index is uniform all through the medium. For a very nearly flat droplet, the case that is valid for a spreading droplet with a pancake shape, the phase shift is linearly proportional to the height profile as:

$$h(i, j) = \kappa\Delta\varphi(i, j), \quad (5)$$

where the calibration parameter κ is a time independent constant.

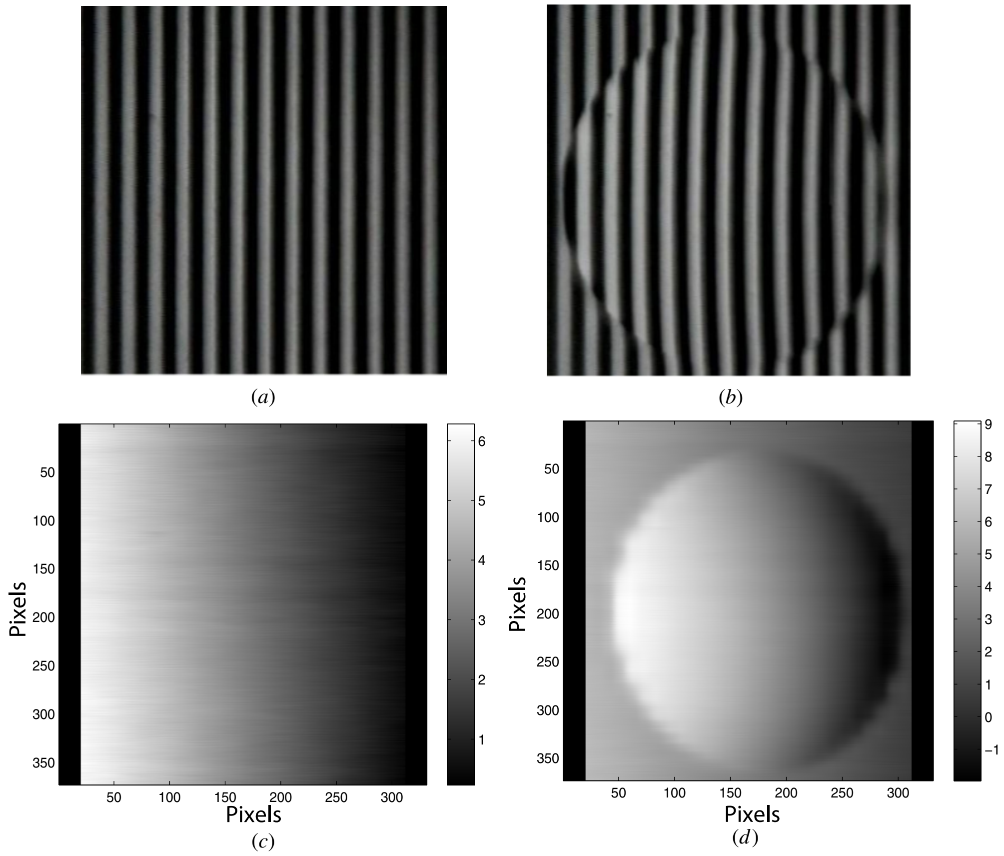


Figure 3. An example of phase analysis by sampling moiré method. (*a*) Reference image of a Ronchi grating. (*b*) Image of the grating after deformation. (*c*) Unwrapped phase distribution of grating before deformation. (*d*) Unwrapped phase distribution of grating after deformation by a droplet. Here down-sampling with a sampling pitch $p = 21$ pixels and linear intensity interpolation are used to obtain the results.

3. Spreading dynamics

In our experiment we have used silicone oil droplets located on a glass plate. A typical droplet of silicone oil with the size of 1 mm^3, spreads over the glass and reaches a complete wetting state. The final height of the oil layer has a very thin molecular scale. Our goal is to investigate the early time dynamics of this spreading experiment. To analyze the dynamics of this phenomena, we record a movie of a grating that is dynamically deformed by a droplet. In our experiment a grating with pitch length 500 μm, a CCD camera (TEO-517A, 640 × 480 pixels) and a lens (HELIOS, $f = 58$ mm) are used. Here the corresponding wavelength of the grating reads $p = 21$ pixels.

Before going to the dynamical case, we first present the details of moiré analysis for a single frame of this movie. Figures 3(*a*) and (*b*) show the intensity image of the Ronchi grating before and after deformation captured by the CCD camera. Figures 3(*c*) and (*d*) show the unwrapped phase distribution of the grating before and after the deformation obtained by down-sampling with a sampling pitch of 21 pixels. Here a linear intensity interpolation is used to obtain the fringe patterns. subtracting these two phase distributions (before and after deformation), we can obtain the phase shift distribution. A cross section of the phase shift distribution is plotted in figure 2(right). This cross sectional plot completely reflects the symmetry of the droplet. Using equations (2), (4), we can assign a height to the droplet captured in this frame. Repeating this scenario for all frames, we can achieve the dynamical properties of the droplets.

We have performed the spreading experiment with silicone oils with different viscosities and different sizes. The results of our moiré experiments are shown in figure 4. As one can see, for droplets with sizes smaller than 4 mm^3, a universal scaling relation quantifies the early time dynamics. In the logarithmic scale, all droplets with different viscosities show a linear dynamics with slope −0.2. In the next part, we will present a short theoretical discussion about this universal result.

4. Theoretical discussion

Understanding the spreading speed of a droplet is of fundamental interest in many practical problems. Paint drops and spin coating of lubricants on solid substrates are examples of the industrial interests. Measuring the time for approaching the final state with complete wetting is the key question. Here, we have chosen the sampling moiré method to study the dynamics of silicone oil drops with different viscosities spreading over a plate of glass. To have a theoretical description of the system, we shall think about the forces that drive the system. Gravitational and surface tension forces drive the dynamics of a droplet. For a fluid with density and surface tension denoted by ρ and σ, a very important length scale,

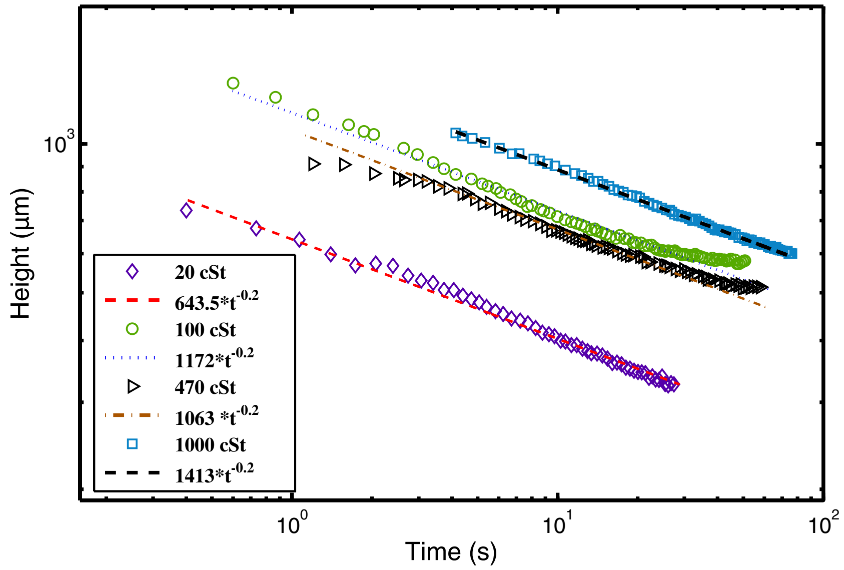


Figure 4. Dynamics of droplet height as a function of time is plotted for silicone oil with different viscosities. In this logarithmic scale, all experimental results are compared with straight lines with slope −0.2. The volume of all droplets is 4 mm^3.

called capillary length $\ell = \sqrt{\sigma/\rho g}$, measures the competition between these two forces. For all physical processes in a system with length scale less than ℓ, the surface tension dominates over the gravity and for large scale phenomena the gravity dominates. Including water, small drops of most usual fluids with a size less than 1 mm, lie in the regime of being surface tension-dominated. The central height of a drop, contact angle and the size of drop are coupled to each other. Theoretical arguments based on the solution of the Navier–Stokes equation, can reveal scaling relations for the dynamics [14, 15]. In a regime where the effects of gravity can be neglected, the Navier–Stokes equation reads [16]:

$$\eta \nabla^2 \mathbf{u} = -\nabla P,$$

where velocity and pressure of the fluid are denoted by **u** and P. This is essentially Newton's equation of motion for fluids. The left-hand side of this equation denotes the viscous force and the right-hand side stands for the surface tension force that can produce a gradient of pressure. The exact solution to this complicated equation is not possible, but a very intuitional scaling argument can be used to deduce the results. Let us denote the central height of the drop by $h(t)$ and its spreading diameter by $R(t)$ ($h \ll R$). For a spreading drop made of an incompressible fluid, the constant volume condition requires that: $R^2(t)h(t) = \text{constant}$. As the most important motion of the fluid takes place in the radial direction and parallel to the substrate, a typical tangential velocity of the fluid can be considered as $\dot{R}$. Obviously this velocity should vary from the bottom of the drop to its top, so a variation of this velocity in the perpendicular direction is expected. This helps us to have an approximation for the value of a viscous force as: $\eta\nabla^2\mathbf{u} \approx \eta\dot{R}/h^2$. To evaluate an approximation for the surface tension we can start from Laplace's equation for pressure drop across a curved interface with curvature r^{-1}, this reads as $P = \sigma/r$ [16]. For a droplet we have $1/r = \partial^2 h/\partial^2 R \approx h/R^2$, so the surface force reads: $\nabla P \approx \sigma h/R^3$. Putting all these facts into the Navier–Stokes equations, we will obtain a very simple dynamical equation:

$$\dot{h} \approx \frac{\sigma}{\eta} h^6,$$

this dynamical equation predicts a very simple and universal height profile as: $h(t) \approx t^{-0.2}$. This predicts that irrespective of the value of viscosity a constant exponent −0.2 describes the spreading dynamics for drops with sizes less than the capillary length. The results of our moiré experiments shown in figure 4 are in good agreement with the theoretical results.

5. Outlook

In summary, We have shown that the method of sampling moiré can be used to experimentally study the fluid dynamical phenomena at the scale of capillary length. As an example and to show the ability of this method, we have chosen spreading dynamics. The accuracy of our results are limited by the geometrical error associated with sampling moiré method. As has been discussed before [9], displacements up to 1/500 of the grating pitch can be detected by this method. In our experiment with a grating pitch of 500 μm, we can measure displacements of the order of 1 μm. For future works, we are studying the dynamics of contact angle. Measuring the contact angle of a reference fluid located on a substrate, provides an experimental gauge for the surface characteristics of the substrate. The dynamics of a pendant drop, a drop that is spreading on a roof, is also an interesting fluid dynamical problem that one may use this method to investigate.

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