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Analysis of the moiré pattern of moving periodic structures using reciprocal vector approach

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Abstract. Several mathematical approaches have been used to explore the moiré pattern. All of them have been considered for superimposing the static structures. In this paper, we have presented a theoretical approach to take into account the relative motion in superimposed gratings and its effect on the moiré fringe patterns. We have used a reciprocal vector approach and consequently obtained a comprehensive description of the dynamic behavior of the moiré patterns in a cinematic viewpoint. Formulations of the rotation and parallel moiré patterns of superposition of static and dynamic periodic structures have been derived in an unified form. Besides, some applications of moiré technique that have been already carried out in dynamical phenomena will be briefly reviewed.

1. Introduction

The moiré phenomenon has been known for a long time; it was already used by the Chinese in ancient times for creating an effect of dynamic patterns in silk cloth. However, modern scientific research into the moiré technique and its application started only in the second half of the 19th century. The word moiré seems to be used for the first time in scientific literature by Mulot [1].

The moiré technique has been applied widely in different fields of science and engineering, such as metrology and optical testing. It is used to study numerous static physical phenomena such as refractive index gradient [2, 3]. In addition, it has a severe potential to study dynamical phenomena such as atmospheric turbulence [4-7], wave-front sensing [8, 9], nonlinear refractive index measurements [10, 11], vibrations [12], displacements and stress [13, 14], velocity measurement [15], acceleration sensing [16], etc. Application of the moiré techniques to the displacement measurements and light deflection improves precision remarkably.

The moiré pattern can be created, for example, when two similar grids (or gratings) are overlaid at a small angle, or when they have slightly different mesh sizes. In many applications one of the superposed gratings is the image of a physical grating [3-11, 17]. When the image forming lights

propagate in a perturbed medium, the image grating is distorted and the distortion is magnified by the moiré pattern.
Several mathematical approaches can be used to explore the moiré phenomenon. The classical geometric approach [18, 19] is based on a geometric study of the properties of the superposed grids, their periods and angles. By considering relations between triangles, parallelograms, or other geometric entities generated between the superposed layers, this method leads to formulae that can predict, under certain limitations, the geometric properties of the moiré patterns. Another widely used classical approach is the parametric equations method [20]; this is a purely algebraic approach, based on the equations of each family of lines in the superposition, which also yields the same basic formulae. The best adapted approach for investigating this phenomenon in superposing of periodic structures is the spectral approach, which is based on the Fourier theory. This approach has been largely developed by Isaac Amidror [21]. The same enables one to analyze properties not only in the original grids and in their superposition but also in their spectral representations. First, one considers the use of Fourier series decompositions, purely in the image domain, for representing the original repetitive structures, their superposition's and their moirés. Second, the use of the Fourier theory for the interpretation of the moiré pattern in spectral terms as an aliasing phenomenon is introduced.
Calculating the moiré pattern using Ewald's sphere of reflection, has also been suggested [22-25]. The information about the fringe structure is obtained by adding spatial frequencies vectorially. The period and orientation of the structures can be represented by a vector in the spatial frequency plane. The length of the vector is the frequency and its angle is the orientation of the grating lines. In this approach the influence of the moiré fringes profile of the grid period is not considered [23].
It must be noted that a rigorous approach to the moiré pattern is presented by R. Gevers [26]. He succeeded in performing a treatment of the moiré effect taking into account the dynamical effects wholly by using the dynamical theory of electron diffraction.
Experimental moiré techniques have been utilized to investigate a variety of dynamic problems, including production of sinusoidal phase grating in photo-refractive crystals [27, 28], study of rapidly changing stress fields and stress pulse propagation [29, 30], propagating cracks [31], studies of atmospheric turbulence[6-8], and so on. Theoretical consideration of the behavior of the moiré fringes in the dynamical phenomena is not comprehensively demonstrated. In this paper we attempt to extend the analysis of the moiré patterns of moving gratings. Formulation of the moiré pattern of superposition of moving periodic structures is derived from the reciprocal vector approach. This approach proves to be very useful in the investigation of superposed periodic layers and their moiré effects. The individual structures are characterized by spatial frequency, orientation, profile (variation of transmittance over one period), and velocity parameters. As we will see, all the specific moiré formulae are derived using the presented approach.

2. Formulation

In the moiré phenomenon, when one of the superposed linear patterns is translated on top of the other in the direction perpendicular to its lines' direction, the moiré fringes move in a direction perpendicular to the fringes much faster than the moving grating. Thus, the moiré pattern has a velocity much larger than the velocity of the individual patterns. In the following we consider this kind of superposition of the periodic structures.
For simplicity of the calculations, transmittances of the superposed structures are assumed sinusoidal functions. Gratings periods, d_1 and d_2, and their reciprocal vectors $\mathbf{k_1}$ and $\mathbf{k_2}$ are related together as (see Fig. 1)

$$\mathbf{k_1} = \frac{2\pi}{d_1}\hat{\mathbf{k}}_\mathbf{1}, \qquad \mathbf{k}_2 = \frac{2\pi}{d_2}\hat{\mathbf{k}}_\mathbf{2} \tag{1}$$

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where, $\hat{\mathbf{k}}_1$ and $\hat{\mathbf{k}}_2$ are the unit vectors directed along the perpendicular to the gratings' lines, respectively. Transmittance functions of the gratings can be written as

$$T_1(\mathbf{r}) = a_1[1 + V_1 \cos(\mathbf{k_1}.\mathbf{r} + \varphi_1)], \quad (2)$$

$$T_2(\mathbf{r}) = a_2[1 + V_2 \cos(\mathbf{k_2}.\mathbf{r} + \varphi_2)], \quad (3)$$

where, $\mathbf{r}$ is the position vector in the plane of the gratings, V_i, a_i, and φ_i are the visibility, transmission coefficient, and initial phase of the transmittance function of the individual gratings (i = 1; 2). V_i and a_i satisfy the following constraints:

$$0 < a_i(1 + V_i) \le 1, \qquad 0 \le V_i \le 1, \qquad i = 1,2.$$

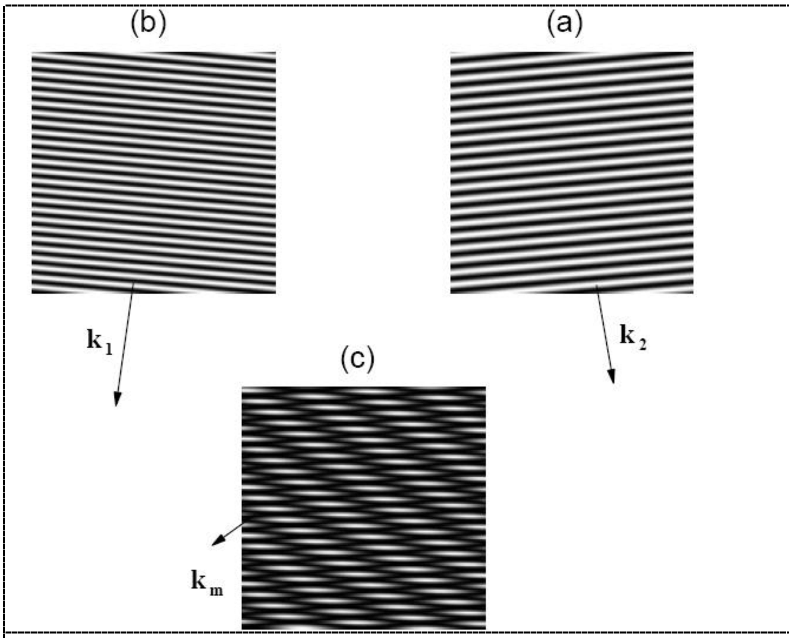

Figure 1. (a) and (b), typical sinusoidal gratings with reciprocal vectors $\mathbf{k_1}$ and $\mathbf{k_2}$ (c) corresponding multiplication moiré pattern with reciprocal vector $\mathbf{k_m}$

Now, consider the gratings to move in-plane by the velocities $\mathbf{v_1}$ and $\mathbf{v_2}$, respectively, in directions perpendicular to their lines' directions, i.e. $\mathbf{v_1} = v_1\hat{\mathbf{k}}_1$ and $\mathbf{v_2} = v_2\hat{\mathbf{k}}_2$. In this case the transmittance functions of the gratings can be written as:

$$T_1(\mathbf{r}, t) = a_1[1 + V_1 \cos(\mathbf{k_1}.\mathbf{r} - \omega_1 t + \varphi_1)], \quad (4)$$

$$T_2(\mathbf{r}, t) = a_2[1 + V_2 \cos(\mathbf{k_2}.\mathbf{r} - \omega_2 t + \varphi_2)], \quad (5)$$

so

$$\omega_1 = \mathbf{v_1}.\mathbf{k_1} = 2\pi \frac{v_1}{d_1}(\hat{\mathbf{k}}_1.\hat{\mathbf{k}}_1) = \frac{2\pi}{\tau_1},$$

$$\omega_2 = \mathbf{v_2}.\mathbf{k_2} = 2\pi \frac{v_2}{d_2}(\hat{\mathbf{k}}_2.\hat{\mathbf{k}}_2) = \frac{2\pi}{\tau_2},$$

where, τ_1 and τ_2, respectively, are the time required to transfer the gratings equal to their spatial periods. Now, to get the velocities of the phase translations we equate the differentials of Eqs. (4) and (5) equal to zero:

$$d(\mathbf{k_1}.\mathbf{r} - \omega_1 t + \varphi_1) = 0.$$

For constant $\mathbf{k_1}$, ω_1, and φ_1 it is equivalent to

$$\mathbf{k_1}.d\mathbf{r} - \omega_1 dt_1 = 0,$$

or

$$k_1 dr_1 - \omega_1 dt_1 = 0, \tag{6}$$

where r_1 is the component of $\mathbf{r}$ in the direction of $\mathbf{k_1}$, and $k_1 = |\mathbf{k_1}|$. Thus, the magnitude of the first grating velocity v_1, is

$$\frac{dr_1}{dt} = \frac{\omega_1}{k_1} = v_1. \tag{7}$$

By the same token, for the second grating we obtain:

$$v_2 = \frac{\omega_2}{k_2}. \tag{8}$$

The moving moiré pattern is produced by the multiplication of the transmittance functions of the moving gratings:

$$\begin{aligned} T = T_1 T_2 &= a_1 a_2 \left[1 + V_1 \cos(\mathbf{k_1}.\mathbf{r} - \omega_1 t + \varphi_1) + V_2 \cos(\mathbf{k_2}.\mathbf{r} - \omega_2 t + \varphi_2)\right] \\ &+ \frac{a_1 a_2 V_1 V_2}{2} \cos\left[(\mathbf{k_1} + \mathbf{k_2}).\mathbf{r} - (\omega_1 + \omega_2)t + (\varphi_1 + \varphi_2)\right] \\ &+ \frac{a_1 a_2 V_1 V_2}{2} \cos\left[(\mathbf{k_1} - \mathbf{k_2}).\mathbf{r} - (\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2)\right]. \end{aligned} \tag{9}$$

Because, the moiré pattern is the pattern with the lowest spatial frequency, the transmittance function of the moiré pattern can be deduced as

$$T_m = a_1 a_2 \left\{ 1 + \frac{V_1 V_2}{2} \cos\left[(\mathbf{k_1} - \mathbf{k_2}).\mathbf{r} - (\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2) \right] \right\}. \tag{10}$$

Thus, visibility V_m, transmission coefficient a_m, and initial phase φ_m of the moiré fringes are

$$V_m = \frac{V_1 V_2}{2}, \quad a_m = a_1 a_2, \quad \varphi_m = \varphi_1 - \varphi_2. \tag{11}$$

It should be mentioned that, here the formulation for the visibility of the fringes is introduced considering full coherence of the illumination or immediate superimposing of the gratings. However, in many applications such as imaging through the atmosphere a certain degree of coherence should be considered in the formulation [32].
The transmittance function of the moiré pattern can be expressed as

$$T_m = a_m \left[1 + V_m \cos(\mathbf{k_m}.\mathbf{r} - \omega_m t + \varphi_m) \right], \tag{12}$$

where

$$\mathbf{k_m} = \mathbf{k_1} - \mathbf{k_2}, \tag{13}$$

and

$$\omega_\mathrm{m} = \omega_1 - \omega_2. \tag{14}$$

By squaring both sides of the Eq. (13) we obtain the following expression

$$k_\mathrm{m}^2 = k_1^2 + k_2^2 - 2k_1 k_2 \cos\theta, \tag{15}$$

where θ is the angle between $\mathbf{k_1}$ and $\mathbf{k_2}$ (see Fig. 2). Using $k_1 = \frac{2\pi}{d_1}$ and $k_2 = \frac{2\pi}{d_2}$, the period of the moiré pattern is obtained as:

$$d_m = \frac{d_1 d_2}{\left(d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta\right)^{1/2}}, \tag{16}$$

In addition, according to Fig. 2 we have

$$\mathbf{k_2}.\hat{\mathbf{k}}_\mathbf{0} = -\mathbf{k_m}.\hat{\mathbf{k}}_\mathbf{0},$$

or

$$k_2 \sin\theta = k_m \sin\gamma, \tag{17}$$

where, γ is the angle between $\mathbf{k_1}$ and $\mathbf{k_m}$, and $\hat{\mathbf{k}}_\mathbf{0}$ is a unit vector directed along the perpendicular to $\mathbf{k_1}$ in the $\mathbf{k_1}$ and $\mathbf{k_2}$ plane, as illustrated in Fig. 2. Substitution of Eq. (15) into (17) leads to the moiré fringes' orientation formula

$$\sin\gamma = \frac{d_1 \sin\theta}{\left(d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta\right)^{1/2}}. \tag{18}$$

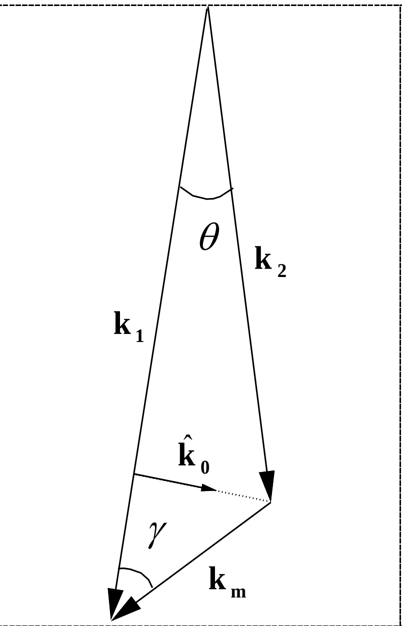

Figure 2. Representation of the gratings reciprocal vectors $\mathbf{k_1}$ and $\mathbf{k_2}$, and the corresponding reciprocal vector of the moiré pattern $\mathbf{k_m}$. $\hat{\mathbf{k}}_\mathbf{0}$ is a unit vector directed along the perpendicular to $\mathbf{k_1}$ in the $\mathbf{k_1}$ and $\mathbf{k_2}$ plane.

Now ,we set the differential of the argument in Eq. (10) equal to zero:

$$d\left[(\mathbf{k_1} - \mathbf{k_2}).\mathbf{r} - (\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2) = 0\right],$$

For constant $\mathbf{k_i}$, ω_i, and φ_i $(i = 1,2)$ it leads to

$$(\mathbf{k_1} - \mathbf{k_2}).d\mathbf{r} - (\omega_1 - \omega_2)dt = 0,$$

or

$$k_m dr_m - \omega_m dt = 0, \tag{19}$$

where r_m is the scalar component of $\mathbf{r}$ in the direction of $\mathbf{k_m}$. Thus, the magnitude of the velocity of the moiré fringes v_m, is

$$v_m = \frac{\omega_m}{k_m}. \tag{20}$$

Now, if we express the velocity of the moiré fringes in terms of the gratings `velocities and constants we get

$$v_{\mathrm{m}} = \frac{k_1 v_1 - k_2 v_2}{\left(k_1^2 + k_2^2 - 2k_1 k_2 \cos\theta\right)^{1/2}} = \frac{d_2 v_1 - d_1 v_2}{\left(d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta\right)^{1/2}} = \frac{d_2 v_1 - d_1 v_2}{d_1 d_2} d_m. \tag{21}$$

Special cases:

a) The period and orientation of the moiré fringes in the parallel moiré pattern are derived by considering $d_1 \neq d_2$, $\theta = 0$ in Eqs. (16) and (18):

$$d_m = \frac{d_1 d_2}{|d_1 - d_2|}, \tag{22}$$

and

$$\gamma = 0. \tag{23}$$

Thus, in this case the moiré fringes are parallel to the lines of the individual gratings.

b) The period and orientation of the moiré fringes in the rotation moiré pattern are derived by considering $d_1 = d_2 = d$ and $\theta \neq 0$ in Eqs. (16) and (18):

$$d_m = \frac{d}{2\sin(\theta/2)}, \tag{24}$$

and

$$\sin\gamma = \cos(\theta/2),$$

or

$$\gamma = (\frac{\pi}{2} - \frac{\theta}{2}). \tag{25}$$

That means, the moiré fringes are perpendicular to the bisector of the gratings' lines.

c) For the case $d_1 \neq d_2$, $\theta = 0$, $v_1 \neq 0$, $v_2 = 0$ the velocity of the moiré fringes becomes:

$$v_m = \frac{v_1}{d_1} d_m = v_1 \frac{d_2}{|d_1 - d_2|}. \tag{26}$$

This means moiré fringes move in the same direction as the grating, but much faster.

d) For the case $d_1 = d_2 = d$, $\theta \neq 0$, $v_1 \neq 0$, $v_2 = 0$, the velocity of the moiré fringes becomes:

$$v_m = \frac{v_1}{d_1} d_m = \frac{v_1}{2\sin(\theta/2)}. \quad (27)$$

In this case moiré fringes move in the direction almost perpendicular to the grating motion.

It should be noted that the presented approach is valid for non-relativistic velocities and for relativistic velocities some modification must be considered.

3. Conclusion

In many experiments, the moiré technique has been utilized to investigate a variety of dynamical problems. At the formation of the moiré patterns sometimes one of the superposed gratings is the image of a physical grating. When the image forming lights propagate in a dynamic medium, the image grating appears as a dynamic grating and dynamical behavior of the medium is magnified by the moiré pattern.

Theoretical consideration of the behavior of the moiré fringes in the dynamical phenomena is not completely demonstrated. In this paper a general theory for the moiré patterns of the moving periodic structures is derived from the reciprocal vectors approach. Formulation of the rotation and parallel moiré patterns in the superposition of static and dynamic periodic structures are unified. The individual structures are characterized by spatial frequency, orientation, profile (variation of transmittance over one period), and velocity parameters. All the specific moiré formulae are derived using the presented approach.

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