Application of the moiré deflectometry on divergent laser beam to the measurement of the angle of arrival fluctuations and the refractive index structure constant in the turbulent atmosphere

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Received February 7, 2008; accepted March 23, 2008;
posted April 2, 2008 (Doc. ID 92570); published April 29, 2008

When a slightly divergent laser beam passes through a turbulent ground level atmosphere and strikes a linear grating, fluctuating self-images are formed at Talbot distances. By superimposing a similar grating on one of the self-images, even for the case of parallel gratings' lines, fluctuating moiré fringes are formed owing to the beam divergence. Recording the successive moiré patterns by a CCD camera and feeding them to a computer, after filtering the higher spatial frequencies, produces highly magnified fluctuations of the laser beam. Using moiré fringe fluctuations we have calculated the fluctuations of the angle of arrival and the atmospheric refractive index structure constant. The implementation of the technique is straightforward, a telescope is not required, fluctuations can be magnified more than ten times, and the precision of the technique is similar to that reported in our previous work.

OCIS codes: 010.1330, 120.4120, 070.6760, 070.6110.

Three physical effects are observed when a light beam propagates through a turbulent atmosphere: optical scintillation, beam wandering, and fluctuations in the angle-of-arrival (AA). These effects are used for measuring turbulence characteristic parameters. Fluctuations of light propagation direction, referred to as the fluctuations of AA, are measured by various methods. In astronomical applications the AA fluctuations measurement is a basic step. Differential image motion monitor [1] and generalized seeing monitor systems [2] are based on AA fluctuations. The edge image waviness effect [3] is also based on AA fluctuations. In some conventional methods the fluctuations of AA are derived from the displacements of one or two image points on the image of a distant object in a telescope. In other techniques the displacements of the image of an edge are exploited.

Recently, we have developed a method, based on the moiré technique, for measuring AA fluctuations of the light propagating in the turbulent ground-level atmosphere on a telescope [4]. We have also used this technique to measure the refractive-index structure constant and the modulation transfer function of the ground-level atmosphere [5,6]. The mentioned works are based on incoherent imaging of a grating in turbulent atmosphere by a telescope. However, our present work is based on coherent imaging without using a telescope.

Moiré deflectometry has already been applied to studies of a hot turbulent jet generated by a heat gun [7] and turbulent mixing of liquids [8]. In the first work the method was based on the measurement of the local contrast degradation of moiré fringes owing to turbulence, and in the second work the analysis was based on the measurement of the minimum intensity of the moiré fringes. Now, in this work we present a novel high-precision measurement of AA fluctuations using the moiré technique and Talbot effect on a slightly divergent laser beam propagating through turbulent atmosphere. As a laser beam passes through a turbulent surface layer of the atmosphere and perpendicularly strikes a grating, fluctuating self-images are formed at Talbot distances. By superimposing another grating similar to the previous one on one of the self-images the moiré pattern is formed even for the case of parallel gratings' lines owing to beam divergence. We filter the unwanted spatial frequencies and record the successive moiré patterns, which contain the largely magnified AA fluctuations, by a CCD camera and store them in a computer. From the measurement of the relative displacements of moiré fringes we deduced the AA variance and the atmospheric refractive index structure constant C_n^2.

In the theoretical considerations we assume a spherical wavefront originating from a point source illuminates two gratings $G1$ and $G2$ of equal periods d, separated by Z_k along the optical axis with their lines and planes parallel together (Fig. 1). The distance between $G1$ and the source is L. The parameter

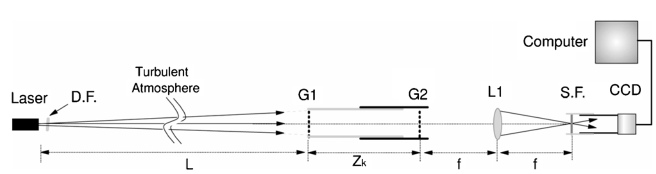

Fig. 1. Schematic of the experimental setup. $D.F.$, $G1$, $G2$, $L1$, and $S.F.$ represent the neutral density filter, first grating, second grating, Fourier transforming lens, and the spatial filter, respectively.

Z_k that denotes the kth self-image or Talbot's distance is given by [9]

$$2k\frac{d^2}{\lambda} = \frac{LZ_k}{L+Z_k}, \tag{1}$$

where λ is the light wavelength. According to Eq. (1) the period of the self-image grating depends on Z_k. Using geometrical optics, denoting the self-image period by $d+\delta d$, and considering Fig. 1 one can write

$$\frac{L}{d} = \frac{L+Z_k}{d+\delta d} \tag{2}$$

from which

$$Z_k = \frac{L}{d}\delta d. \tag{3}$$

In the plane of $G2$, multiplicative moiré fringes (parallel to the gratings' lines) appear with period

$$d_m = \frac{d^2}{\delta d}. \tag{4}$$

Owing to the atmospheric turbulence the self-image fluctuates on the second grating. According to the moiré formulation, for displacement l of the self-image in the direction normal to the grating lines, moiré fringes are shifted by s given by

$$s = \frac{d_m}{d} l. \tag{5}$$

However, the component of the AA fluctuations, α, in the latter direction is

$$\alpha = \frac{l}{Z_k}. \tag{6}$$

Substituting from Eqs. (3)–(5) in Eq. (6) we get

$$\alpha = \frac{s}{L}. \tag{7}$$

Thus, it is obvious that the precision of the AA fluctuations measurement is improved by increasing the propagation length L and decreasing the minimum measurable moiré fringe shift s. In practice, we have used

$$\alpha = \frac{s}{d_m}\frac{d}{Z_k} \tag{8}$$

for calculation, which is obtained by considering Eqs. (3) and (4) in Eq. (7). According to Eq. (8), by increasing the gratings' distance or decreasing the period of the gratings, the measurement precision is improved. In this Letter, we have used $d=1/15$ mm and $Z_k=0.39$ m; by considering $s/d_m=1/60$, the minimum measurable AA fluctuations are 2.8×10^{-6} rad or 0.58 arc sec.

A schematic of the experimental setup is shown in Fig. 1. The second harmonic of a cw diode-pumped Nd-YAG laser beam passes through a turbulent surface layer of the atmosphere and strikes the first grating $G1$. The second grating $G2$ is installed at 23rd Talbot distance. Lens $L1$ forms the Fourier transform of the moiré pattern on its second focal plane. A suitable spatial filter, $S.F.$, mounted in the focal plane of $L1$, removes the unwanted frequencies. The CCD camera installed after the spatial filter records successive moiré patterns and stores them in a computer. The gratings $G1$ and $G2$ of the period 1/15 mm and dimensions 50 mm × 50 mm are installed at the ends of a cylindrical tube of changeable length to choose the desired Talbot's distance. The holders of the gratings can be rotated around the optical axis for adjustment of the angle between the gratings. The source and the experimental setup are installed at the height of 45 cm, over an asphalted area, at the distance 27 m from each other. After alignment of the setup, by a neutral density filter the beam intensity is reduced to a level below the saturation level of the CCD.

The moiré fringes were recorded by sampling rate 25 frames/s. Several sets of experimental data corresponding to low, medium, and high turbulence conditions were recorded and digitized with an image grabber. In this Letter, we refer to two typical series of the measurements performed on September 20, 2007, at cloud free and moderately windy conditions. Each set of data was collected in 120 s and contained 3000 frames. The typical recorded moiré patterns is shown in Fig. 2(a). To get clearer fringes the image of the grating lines is removed by fast Fourier transform using MATLAB software [Fig. 2(b)]. After clearing all the moiré patterns, the traces of moiré fringe minima were specified. The traces of the minima could be determined to within one pixel accuracy. For more details see our previous work [4].

Once all 3000 data frames of the data set were processed, the moiré fringe displacement statistics (mean fringe trace, fringe displacement standard deviation, and displacement correlation function) were calculated. Using Eq. (8), the component α of the AA fluctuations was obtained. The plots in Figs. 3(a) and 3(c) show the horizontal components of AA fluctuations versus time, at a point on the first grating, for two sets of data obtained at two different turbulence conditions. The corresponding mean values of the AA fluctuations of wavefronts over the area of the first grating are plotted in Figs. 3(b) and 3(d), respectively. The moiré magnification and the setup configuration are the same for both plots. In the described experiments, the digitized frames consisted of 288 × 352

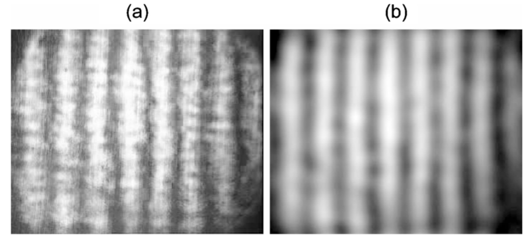

Fig. 2. (a) Typical moiré pattern and (b) the corresponding low-frequency illumination distribution.

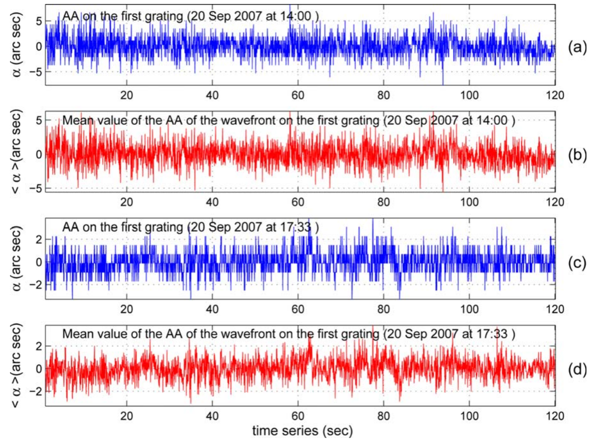

Fig. 3. (Color online) (a) and (c) Typical horizontal components of AA fluctuations at a point on the first grating versus time for two sets of data obtained at two different turbulence conditions. (b) and (d) Mean values of the AA fluctuations of the wavefront on the first grating surface corresponding to (a) and (c), respectively.

pixels, d_m was covered by 45 pixels, and the measurement precision of AA fluctuations was about 0.78 arc sec.

For horizontal propagation of light in weak turbulence, the C_n^2 has the form [3]

$$C_n^2 = \frac{\sigma_\alpha^2 D^{1/3}}{1.14 L f\left(\frac{L_0}{D}\right)}, \tag{9}$$

where, in our case, D is the diameter of the laser beam on the first grating, σ_α^2 is the variance of the mean value of the AA over the area of the first grating, L is the propagation length, and the function $f(L_0/D)$ that describes the effect of the turbulence outer scale, L_0, on the AA variance has been quantified [10]. The outer scale for propagation paths near the ground is estimated as $L_0 = 0.4h$ [3], where h is the altitude. Thus, in our case, $h = 45$ cm and $D = 40$ mm, the predicted value of $f(L_0/D)$ is 0.91. It should be mentioned that in literature, D in Eq. (9) is the diameter of the receiving aperture of the imaging system whereat the AA fluctuations averaged on it. To consider this averaging effect we take the average of the AA fluctuations over the area illuminated by the laser beam on the first grating. Substituting the given values in Eq. (9) the refractive index structure constant is calculated. The values of C_n^2 obtained for the data sets recorded at 14:00 and 17:33 (September 20, 2007) are $6.89 \times 10^{-13}\,\mathrm{m}^{-2/3}$ and $2.20 \times 10^{-13}\,\mathrm{m}^{-2/3}$, respectively, which are compatible with the typical values obtained by other techniques.

Integration of the Talbot effect in the measurement of the AA fluctuations, magnifies the fluctuations and simplifies the setup alignment and the optics compared to our previous method. Additionally, this technique allows us to use retroreflector in order to further reduce the propagation length in special cases. It also seems that moiré fringes with parallel gratings' lines improve the precision of the measurement compared to the rotational moiré fringe case, and this allows further reduction of the propagation length. The presented technique is very flexible and can be applied to a wide range of turbulent conditions, by choosing gratings of adequate pitch, size, and the distance between them. By increasing the gratings' pitch and decreasing the distance between them, the AA fluctuations can be measured in a highly turbulent atmosphere, practically, without limit. However, minimum measurable fluctuations are limited by the size and the spatial frequency of the gratings and also by the specifications of the detecting system. Also, the device can be used for the measurement of the correlations in directions parallel and perpendicular to the propagation direction in a turbulent atmosphere and in wavefront sensing.

We thank A. Darudi for useful discussions.

References

1. M. Sarazin and F. Roddier, Astron. Astrophys. **227**, 294 (1990).
2. A. Ziad, R. Conan, A. Tokovinin, F. Martin, and J. Borgnino, Appl. Opt. **39**, 5415 (2000).
3. M. S. Belen'kii, J. M. Stewart, and P. Gillespie, Appl. Opt. **40**, 1321 (2001).
4. S. Rasouli and M. T. Tavassoly, Opt. Lett. **31**, 3276 (2006).
5. S. Rasouli and M. T. Tavassoly, Proc. SPIE **6364**, 63640G (2006).
6. S. Rasouli, K. Madanipour, and M. T. Tavassoly, Proc. SPIE **6364**, 63640K (2006).
7. J. Stricker, and B. Zakharin, Exp. Fluids **23**, 76 (1997).
8. I. Glatt and O. Kafri, Chem. Eng. Sci. **39**, 1637 (1984).
9. K. Patorski, *Handbook of the Moiré Fringe Technique* (Elsevier, 1993).
10. R. J. Sasiela and J. D. Shelton, J. Opt. Soc. Am. A **10**, 646 (1993).