Measurement of modulation transfer function of the atmosphere in the surface layer by moiré technique

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ABSTRACT

The turbulence of the atmosphere puts an upper limit on the quality of the image of a ground object obtained by long-exposure photography from low or high altitudes in the atmosphere or in the space. By using good optics and high resolution film or CCD and a stable platform, this limit could be approached but not exceeded. A useful quantity for indicating the magnitude of this limit is the integral of the modulation transfer function (MTF) associated with the turbulence. In this work, we introduce a new method for measuring the MTF of the atmosphere in the surface layer, based on moiré technique. In this technique, from a low frequency Ronchi grating, installed at a certain distance from a digital camera equipped with a tele lens, successive images are recorded and then transferred to a PC. By rotating each image by $\theta/2$ and $-\theta/2$, say $\pm 3°$, and superimposing them, a large number moiré patterns are produced. The average transmittance function of the superimposed image gratings is measured in a moiré fringe interval. The latter function is measured by scanning the moiré pattern by a slit parallel to moiré fringes. It is shown theoretically that from the Fourier transform of the latter function the MTF of the atmosphere can be deduced, if the MTFs of the imaging system and the grating are given or their effects are negligible. The atmospheric MTFs have been measured at different turbulence conditions. Also, we have studied the behavior of the atmospheric MTF respect to exposure time.

Keywords: modulation transfer function, moiré deflectometry, atmospheric surface layer

1. INTRODUCTION

The changes in the properties of a light beam passing through an optical system or a transparent medium are characterized fully by the optical transfer functions (OTF) of the latters.[1,2] Although, it is formulated traditionally through the image-forming properties of lens systems,[1] the OTF concept is not limited to imaging systems, as evidenced by its application to the transmission of light through the atmosphere.[3–5] This situation results from the definition of the OTF in terms of the spatial frequency observed in the image plane of an image-forming system. Measurements of the atmospheric modulation transfer function (MTF) involve the use of an auxiliary lens that images the measured beam onto its image plane.[3–5] Multiplying the spatial frequency by the image distance permits the expression of the atmospheric MTF in terms of angular frequency, which is independent of the focal length of the imaging lens. However, this process does not eliminate the effects of this lens on the measured MTF, which must be taken into account.

Atmospheric turbulence with its associated random refractive-index inhomogeneities disturbs a light beam which propagates any significant distance through the atmosphere. The disturbance takes the form of distortion of the shape of the wavefront and variations of the intensity across the wavefront. If a collimated beam passes through the atmosphere and is then collected by a lens and brought to focus, the quality of the image formed is influenced by the atmospherically produced disturbances. That the distortion of the shape of the wavefront affects the image quality is obvious. It is not quite so obvious that the intensity variations across the wavefront affect the image quality. That this is so can be seen by considering the intensity variations as a form of random apodization of the lens. The apodization, of course, affects the image quality.

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As an initial measure of image quality, D. L. Fired[6] considered the MTF of an optical system composed of the atmosphere and a lens. He restricted his attention to thin, diffraction-limited lenses. In his study of the effect of wavefront distortion on the MTF, two distinct cases were to be considered. Some part of the distortion can be considered to be a random tilt of the wavefront. This tilt displaces the image but does not reduce its sharpness. If a very-short-exposure image is recorded, the image sharpness and the MTF are insensitive to the tilt, which can be a substantial part of the total distortion. If a long-exposure image is recorded, the image is spread during the exposure by the random variations of the tilt. Hence, the image sharpness and the MTF are affected by wavefront tilt as well as by the more complex shapes. The analytic distinction between the two cases, which he refered to as the long-exposure case and the short-exposure case, lies in the manner in which the average of the MTF is taken. In the short-exposure case, a random factor associated with the tilt is extracted from the MTF before taking the average. In the long-exposure case, no such factor is removed.

By the smear diameter of the image of a radial target, which is comprised of alternative black and white sectors, MTF of the atmosphere has been estimated.[7]

In this work, we introduce a new method, based on moiré deflectometry, for measuring the MTF of the atmosphere in the surface layer. In this technique from a low frequency Ronchi grating, installed at a certain distance from a digital camera equipped with a tele lens, successive images are recorded and stored in a PC. By rotating each image by $\theta/2$ and $-\theta/2$, say $\pm 3^\circ$, and superimposing them, a large number moiré patterns are produced. The average transmittance function of the superimposed image gratings is measured in a moiré fringe interval. The latter function is measured by scanning the moiré pattern by a slit parallel to moiré fringes. It is shown theoretically that from the Fourier transform of the latter function the MTF of the atmosphere can be deduced, provided that the MTFs of the imaging system and the target grating are given or their effects are negligible. The atmospheric MTFs has been measured at different turbulence conditions. Beside, the behavior of the atmospheric MTF respect to exposure time is studied.

Comparing with Fired's[6] manner, we can get the atmospheric MTF at short-exposure case, because in the moiré method space ensemble replace with the time ensemble of data at the long-exposure case.

2. ATMOSPHERIC OTF AND MTF

An electromagnetic wave propagating through the atmosphere will be amplitude and phase modulated by atmospheric density variations.[4] The average propagation quality of the atmosphere can be quantified by an OTF. The long-term OTF is particularly convenient since the light source and instrument related effects can be separated from the atmospheric contribution of a composite measurement. The specific one-dimensional OTF used in this paper is defined by the Fourier transform of the line spread function $S(x)$,[8]

$$OTF(\nu) = \frac{\int_{-\infty}^{\infty} S(x) e^{-2\pi i \nu x} dx}{\int_{-\infty}^{\infty} S(x) dx}, \tag{1}$$

where $S(x)$ is a one-dimensional profile of the irradiance distribution in the focal plane of an optical system and ν is a spatial frequency. $S(x)$ is a convolution of a linear target transmittance distribution, atmospheric stochastic effects, and the OTF of the optical instrument used to produce the image. If the optical system is assumed to be on axis, optically symmetric, and the atmosphere is assumed to be laterally stationary and isotropic, one line-spread function $S(x)$ will be representative of all rotations around the optical axis; under these conditions, the OTF and the MTF are equivalent:[8]

$$MTF(\nu) = |OTF(\nu)|. \tag{2}$$

A difference between the OTF and the MTF can arise when noise and time averages are encountered.[9]

In the above conditions, the observed MTF_o is simply the product of the target, atmosphere, and instrument components, that is

$$MTF_o(\nu) = MTF_t(\nu).MTF_a(\nu).MTF_i(\nu). \tag{3}$$

The MTF_t of the target is assumed to be that of an ideal Ronchi grating and is therefore unity. The MTF_i of the instrument can be measured in the laboratory; hence, an atmospheric MTF_a can be extracted from the observed data by removing the instrument contribution. With a diffraction limited optical system that is of the diameter larger than about 10-20 cm, the atmospheric contribution generally dominates the instrument effect and restricts the high spatial frequencies.

Several interrelated frequency scales occur in the mathematical description of optical and atmospheric modulation transfer functions.[3,6,8–11] Experimentally, the quantity ν expressed in cycles/length is observed. For atmospheric turbulence effects, it is convenient to remove the dependence on the optical system by multiplying ν by the effective focal length of the optical system F which defines an angular frequency scale $\nu' = F\nu$ expressed in cycles/radian.

3. PHYSICAL APPROACH

The random refractive-index inhomogeneities in the atmosphere, associated with turbulence, distort the characteristics of light traveling through the atmosphere and thereby limit the resolution with which an object can be viewed through the atmosphere.

Turbulence in the atmosphere between a point object and an optical imaging system causes the image of that point to be degraded in several ways. The image will fluctuate randomly in both intensity and position, and at any instant in time will generally have a larger and/or more irregular blur circle than that corresponding to the imaging system without turbulence. In similar ways, atmospheric turbulence will also degrade the image of an extended object. The exact amount and nature of the image degradation will vary with time for any point in the field of view and will vary with field angle for any fixed time. The process is random and complicated.

A low frequency Ronchi grating was programmed by a PC and printed on a paper with desired dimensions. As mentioned above at a distance from the grating, a digital camera snaps the image of the grating through turbulent atmosphere. By rotating the image by $\theta/2$, and $-\theta/2$, and superimposing the images, moiré pattern is formed. The image grating is no longer Ronchi grating and the OTF of the atmosphere and the imaging system have been imparted on it in the recording process. The OTF for each spatial angular frequency appears as a factor multiplied by the corresponding transmittance expansion coefficient in the Fourier expansion of the transmittance function of each image grating. Thus, for the case that the grating lines are perpendicular to x axis the transmittance coefficient can be expressed as follows[12]

$$i(x) = \sum_{n=-\infty}^{n=+\infty} a_n OTF(n\nu).exp(\frac{i2\pi nx\nu}{F}), \tag{4}$$

where, ν is the fundamental spatial angular frequency, a_n is the expansion coefficient of Ronchi ruling at spatial angular frequency $(n\nu)$, n is an integer number, and F is the focal length of imaging system. For a grating with a pitch D that installed at distance L from the imaging system the fundamental spatial angular frequency derived as:

$$\nu = \frac{L}{D} = \frac{F}{d},$$

where, d is the image grating pitch.

An image of the grating are rotated in way that their ruling directions make angles $\theta/2$ and $-\theta/2$ by y-axis. The transmittance of the superimposed gratings can be given by

$$I(x,y) = \sum_{n=-\infty}^{n=+\infty} \sum_{n=-\infty}^{n=+\infty} a_n a_m OTF(n\nu)OTF(m\nu).exp\{\frac{i2\pi\nu}{F}[(n+m)xCos(\frac{\theta}{2}) + (n-m)ySin(\frac{\theta}{2})]\}. \tag{5}$$

The resulted transmittance is a new periodical structure, called moiré pattern. The period of the moiré fringes is[13]

$$d_m = \frac{d}{2\sin(\theta/2)}. \tag{6}$$

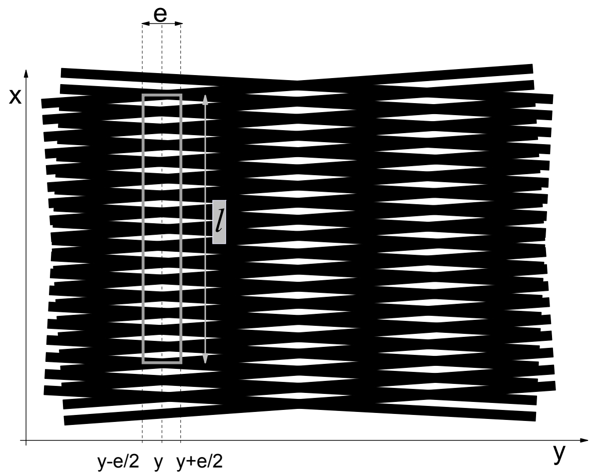

Figure 1. Superimposed gratings and the orientation of the slit in scanning the transmittance.

The moiré fringes appear parallel to x-axis. Now, the average transmittance function of the superimposed gratings in a moiré fringe interval is then measured by the method suggested in Ref. 14. According to this a slit of length l and width e is scanned by the superimposed gratings illuminated uniformly. The amount of average light passing through the slit, when the ordinate of its central line is y , Fig.1, can be given by:

$$\Phi(y) = I_0 \int_{-l/2}^{+l/2} dx \int_{y-e/2}^{y+e/2} I(x,y)dy, \tag{7}$$

where, I_0 is the illuminating irradiance, and $I(x,y)$, is given by Equation (5). Substituting from Equation (5) in Equation (7) we get

$$\Phi(y) = I_0 \sum_{n=-\infty}^{n=+\infty} \sum_{n=-\infty}^{n=+\infty} a_n a_m OTF(n\nu)OTF(m\nu)$$

$$\times \int_{-l/2}^{+l/2} exp\{\frac{i2\pi\nu}{F}[(n+m)xCos(\frac{\theta}{2})]\}dx \int_{y-e/2}^{y+e/2} exp\{\frac{i2\pi\nu}{F}[(n-m)ySin(\frac{\theta}{2})]\}dy, \tag{8}$$

For large enough l , the first integral in Equation (8) vanishes except for $n=-m$. For the latter case it becomes equal to l and Equation (8) takes the following form:

$$\Phi(y) = I_0 l \sum_{n=-\infty}^{n=+\infty} a_n a_{-n} OTF(n\nu)OTF(-n\nu) \int_{y-e/2}^{y+e/2} exp(\frac{i2\pi ny}{d_m})dy, \tag{9}$$

where, d_m is given by Equation (6). Integrating the integral in Equation (9) leads to

$$\Phi(y) = I_0 el \sum_{n=-\infty}^{n=+\infty} a_n a_{-n} OTF(n\nu)OTF(-n\nu)Sinc(\frac{\pi ne}{d_m})exp(\frac{i2\pi ny}{d_m}). \tag{10}$$

Now, according to definition, OTF is the Fourier transform of the point spread function (PSF) which is a real function.[15] Therefore, we have

$$OTF(-n\nu) = OTF^*(n\nu), \tag{11}$$

and since Ronchi ruling is an even function, we have

$$a_{-n} = a_n. \tag{12}$$

Using Eqs.(11) and (12) in Equation (10), we get:

$$\Phi(y) = I_0 el \sum_{n=-\infty}^{n=+\infty} a_n^2 MTF^2(n\nu) Sinc(\frac{\pi ne}{d_m}) exp(\frac{i2\pi ny}{d_m}), \tag{13}$$

where, according to Equation (2), $MTF(n\nu) = |OTF(n\nu)|$ is the modulation transfer function at frequency $(n\nu)$.

Since MTF^2 and Ronchi ruling are symmetrical functions we have: $MTF^2(\nu) = MTF^2(-\nu)$ and $a_n^2 = a_{-n}^2$. Besides considering that $MTF(0) = 1$ and for a Ronchi ruling $a_0 = \frac{1}{2}$, Equation (13) can be written in the following form:

$$\Phi(y) = I_0 el[\frac{1}{4} + 2\sum_{n=1}^{+\infty} a_n^2 MTF^2(n\nu) Sinc(\frac{\pi ne}{d_m}) Cos(\frac{2\pi ny}{d_m})]. \tag{14}$$

The maxima and minima of Equation (14) appear at $y = md_M$ and $y = (m+\frac{1}{2})d_m$, $m = 0, 1, 2, ...$ respectively. Therefore, we have:

$$2(\Phi_{max} + \Phi_{min}) = I_0 el. \tag{15}$$

Dividing Equation (14) by Equation (15) we get:

$$I_a(y) = \frac{1}{4} + 2\sum_{n=1}^{+\infty} a_n^2 MTF^2(n\nu) Sinc(\frac{\pi ne}{d_m}) Cos(\frac{2\pi ny}{d_m}), \tag{16}$$

where, $I_a(y)$ is the average transmittance on the slit at location y. Now, multiplying both sides of Equation (16) by $Cos(\frac{2\pi ny}{d_m})$ and integrating from $-\frac{d_m}{2}$ to $\frac{d_m}{2}$, we get

$$MTF(n\nu) = \frac{1}{a_n\sqrt{d_m Sinc(\frac{\pi ne}{d_m})}}[\int_{-d_m/2}^{d_m/2} I_a(y) Cos(\frac{2\pi ny}{d_m}) dy]^{1/2}. \tag{17}$$

In practice, $I_a(y)$ is specified by scanning a slit, of length l and width e, by the set of the superimposed grating images in an interval equal to the moiré fringe spacing. Using the resulted function in Equation (17), $MTF(n\nu)$ for different n can be evaluated.

3.1. Simulation Considerations

In order to test Equation (16) the following simulations have been carried out. For an ideal Ronchi grating MTF is equal to 1 at each frequency. Therefore, putting $MTF(n\nu) = 1$ in Equation (16) and calculating $I_a(y)$ for different y in a moire fringe interval we get the familiar triangular average transmittance shown in Fig.2. For real gratings, depending on $MTF(\nu)$, the average transmittance becomes more or less blunt at the corners. For example, for $MTF(\nu) = exp(-k\nu)$, $\nu = \frac{nL}{D}$, $k = 0.1, 0.07, 0.04mrad$, $D = 2.6cm$, $L = 89m$; Equation (16) leads to the curves shown in Fig. 2(a). Now, using this average transmittance in Equation (17) and calculating $MTF(\frac{nL}{D})$ for $D = 2.6cm$ and $n = 0, 1, 3, 5, 7, 9$. we get the MTFs at desired frequencies. These are shown in Figs. 2(b). The continuous curves are the introduced MTFs.

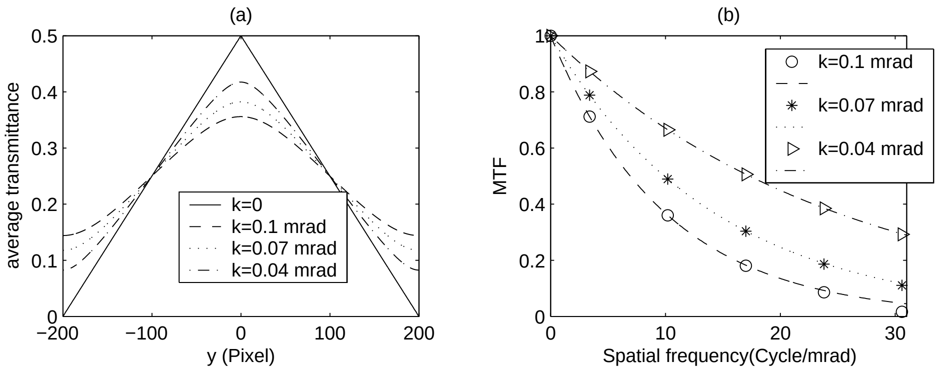

Figure 2. Simulations of the average transmittance function through superimposed similar ideal Ronchi gratings - triangle - and gratings with Exponential MTFs - curves (a). The corresponding MTF simulations (b).

4. EXPERIMENTAL CONSIDERATIONS

A Ronchi grating was programmed by a PC and printed with the period of 2.6cm on a paper of size $91cm \times 70cm$. The grating was installed vertically by a wooden frame fixing its lower part $5cm$ above the ground. At the distance of $89m$ from the grating a digital camera, model $NikonD100$, that equipped with a tele lens, model ZENIT-$N802950$, was installed at a height of 70 cm over an asphalted road. The images of the grating were recorded by resolution 3008×2000 pixels.

Several sets of measurements corresponding to different turbulence conditions were carried out. In this report, we refer to two series of measurements performed on 24 and 26 June 2006, at cloud free and moderately windy conditions. Each set of data was collected in 15 minute time interval from 7am to 6:30pm, with exposure times $\frac{1}{125}sec.$, $\frac{1}{180}sec.$, and $\frac{1}{250}sec.$. Using Matlab software the frames was rotated by angles equal to $3°$ and $-3°$ then multiplied together to produce the required moiré fringes. The sample grating image is shown in Fig. 3. A typical moiré pattern formed by the image gratings is illustrated in Fig. 4. In Fig. 5(a), (c), (e), (g), and (i) the experimental average transmittance functions are plotted - curves - in a moiré fringe interval, corresponding images recorded at 7am, 9am, 11am, 4pm, and 5.5pm, respectively. The triangles are the average transmittance of ideal superimposed Ronchi gratings. In Fig. 5(b), (d), (f), (h), and (j) the corresponding composed MTF are shown. For comparing the result at different turbulent conditions, the calculated average transmittance function in a moiré fringe interval at different day times and same exposure time are shown in Fig 6(a). The corresponding composed MTFs of the atmosphere, imaging system, and target are shown in 6(b). The calculated average transmittance functions in a moiré fringe interval at different exposure times and same turbulence condition are shown in Figs. 7(a) and 8(a). The corresponding images recorded at 7:00am and 5:30pm. The corresponding composed MTFs of the atmosphere, imaging system, and target are shown in Figs. 7(b) and 8(b).

The MTF of the imaging system can be measured in the indoor laboratory. The measured MTF are composed of degradations by the optical lens, the CCD camera, target grating, and also the frame grabber. The overall atmospheric MTF can be calculated by dividing the total measured MTF by that of the imaging system, assuming that the degrading effects of the system and the atmosphere are independent of one another. The atmospheric MTF was normalized to unity, so that path luminance was not considered.

5. CONCLUSION

A simple and reliable technique for evaluating the MTFs of atmosphere is introduced. An image of a low frequency Ronchi gratings can provide the MTF for several angular frequencies. It seems that by providing gratings with fine structures, that is, containing the details of frequencies higher than the cutoff frequencies of imaging systems.

Figure 3. The image of the Ronchi grating with a pitch D=2.6cm, installed at the distance 89m from the imaging system. Exposure time was $\frac{1}{250}ses$. The fundamental angular frequency is 3.4 Cycle/mrad.

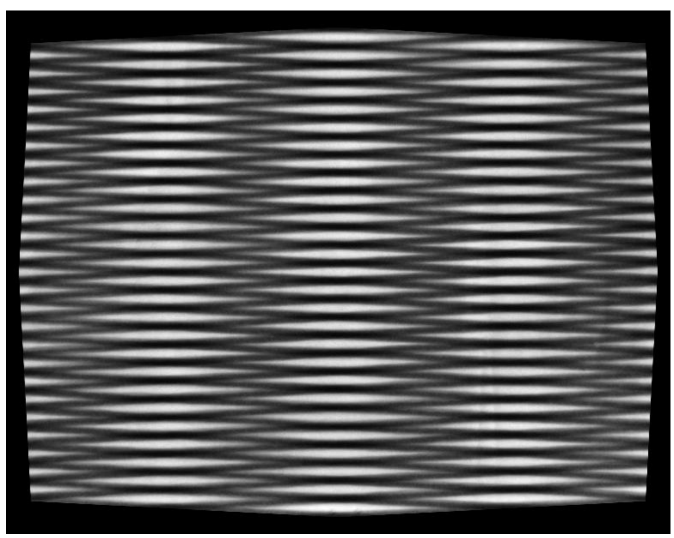

Figure 4. The moiré pattern formed by the multiplied images of the grating shown in Fig. 1. The angle between the rotated images is 6 degree.

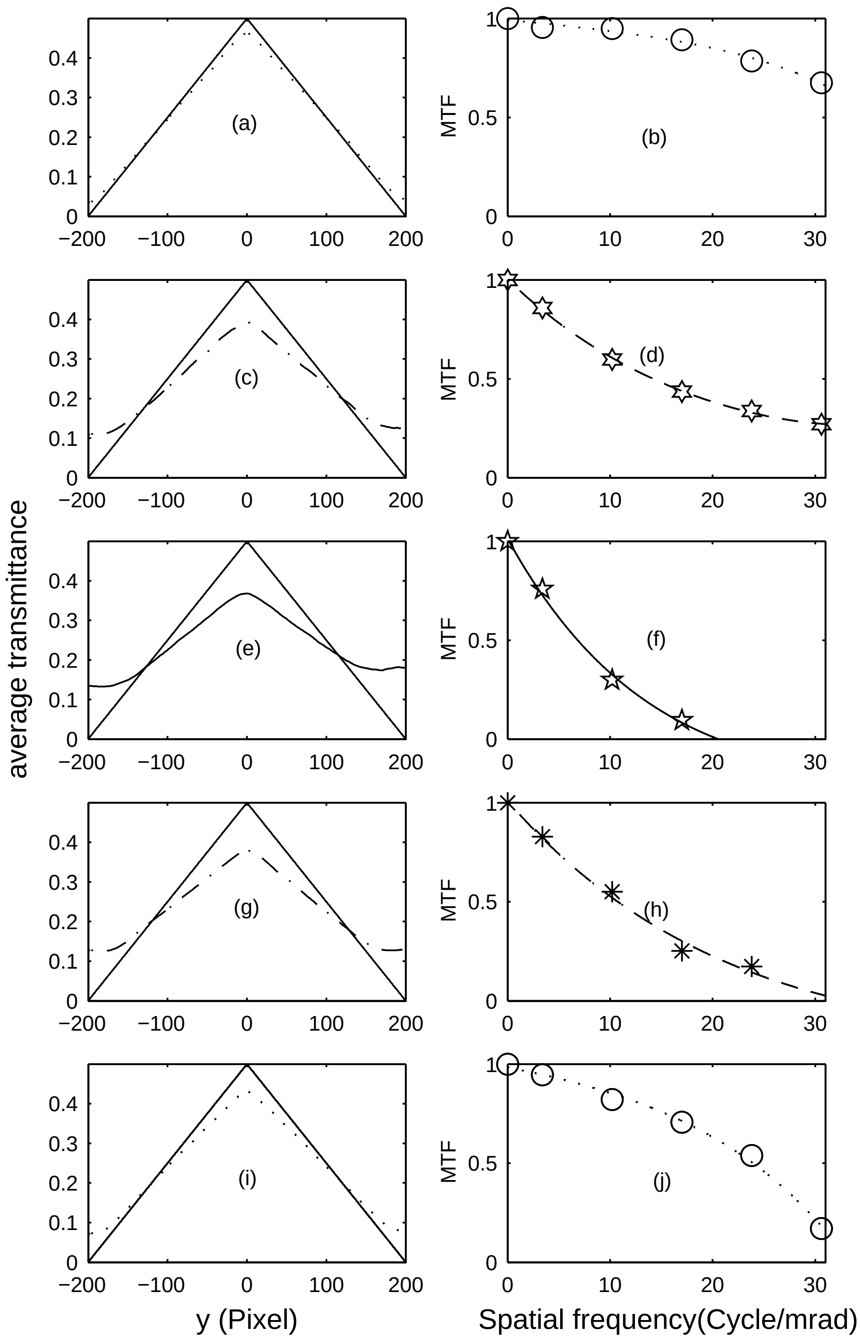

Figure 5. The experimental average transmittance functions in a moiré fringe interval are plotted for the images recorded at 7am, 9am, 11am, 4pm, and 5.5pm, - curves - in (a), (c), (e), (g), and (i), respectively. The triangle represents the average transmittance function of an ideal moiré pattern. The corresponding composed MTFs of the atmosphere, imaging system, and target are shown in (b), (d), (f), (h), and (j). The exposure times were $\frac{1}{250}sec$.

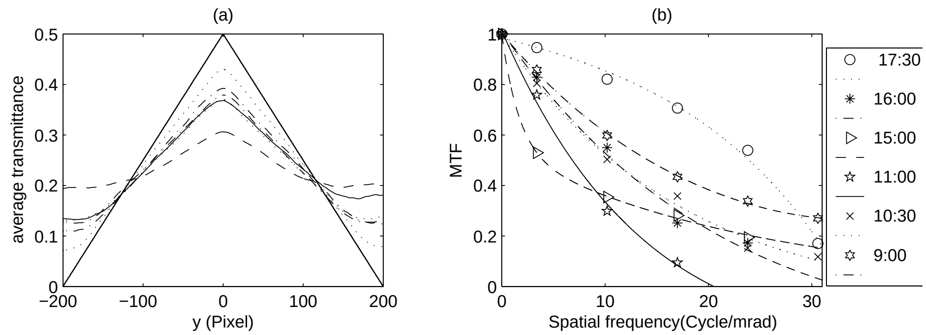

Figure 6. The experimental average transmittance functions in a moiré fringe interval at different day times and same exposure time are shown in (a). The corresponding composed MTFs of the atmosphere, imaging system, and target are shown in (b).

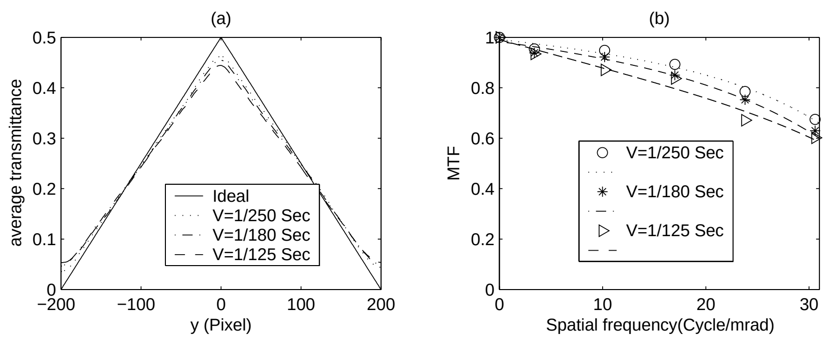

Figure 7. The experimental average transmittance functions in a moiré fringe interval at different exposure times and same turbulence condition are shown in (a). Corresponding images recorded at 7am. The corresponding composed MTFs of the atmosphere, imaging system, and target are shown in (b).

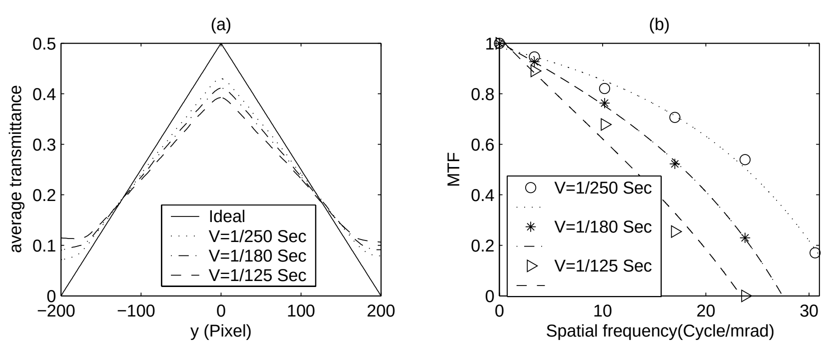

Figure 8. The experimental average transmittance functions in a moiré fringe interval at different exposure times and same turbulence condition are shown in (a). Corresponding images recorded at 5:30pm. The corresponding composed MTFs of the atmosphere, imaging system, and target are shown in (b).

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