Specification of vibrational modes and amplitudes in large-scale structure by time averaging moiré technique

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ABSTRACT

Specification of vibrational modes and amplitudes of structures are crucial issues in civil and mechanical engineering. Several techniques have been used for this kind of studies, including holographic interferometry, speckle interferometry and moiré technique. But, for large-scale structures modal analysis technique is usually used. In this work we have used time averaging moiré technique to study in plane vibrations of large structures. The study includes specification of vibrating modes and amplitudes of structures. The technique is applied by painting a suitable size linear sinusoidal reflectance pattern on the lateral surface of the structure. As the structure is put into vibration, using a wide-angle high-resolution digital camera, the image of vibrating pattern is recorded in an exposure time much larger than the vibration period. The visibilities of the image along a line parallel to the painted pattern line are derived by processing the reflectance distribution. By dividing the resulted visibilities by the visibility of the image of the static pattern we get the normalized visibility curve. The number of normalized visibilities equal to 1 provides the number of vibrational modes and the magnitudes of the visibility minima or the locations of the zero visibility give the amplitudes of vibration.

Keywords: moiré technique, in plane vibration, large-scale structures, vibrational mode and amplitude

1. INTRODUCTION

The study of vibrational modes and amplitudes in large-scale structures, like bridges, is a significant issue. Several mechanical and optical techniques are available for this kind of study[1-3]. Among the mechanical techniques the finite elements technique, which is called "modal analysis" technique, is more frequently applied. In this technique the structure responses to the external activation at several points of the structure are obtained then from the analysis of the results the vibrational modes are deduced. Optical techniques including holographic and speckle interferometry techniques have been used for specification of vibrational modes in small-scale structures[4-7]. In this work we apply time averaging moiré technique to measure amplitudes and vibrational modes of a large vibrating structure.

2. GENERAL DISCRIPTION

If we paint a linear periodic pattern on a vertical surface of a large structure, like a bridge, and let the structure to vibrate under the action of an external force, the image of the pattern in an imaging system, say a CCD camera, vibrates accordingly. If we record the time averaging intensity distribution on the image in a time-interval much larger than the vibration period, in general, we observe another periodical intensity distribution perpendicular to the pattern's strips, which is called "time average moiré pattern". In fact, the image appears as a grating with non-uniform visibility. The areas of the normalized visibility 1 corresponds to the nodes of the vibrational mode. The distance between two successive nodes divided by the magnification of the imaging system gives the half-wavelength of the vibrational mode. For evaluating the amplitude of the vibration, in practice, two different cases may be encountered.

a) A case with non-zero visibility, then, the amplitude can be evaluated by measuring the lowest visibility.

b) A case where between two successive nodes there are some areas of zero visibility, then, the distance between a node and the nearest zero visibility area provides the amplitude of the vibration.

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3. THEORETICAL APPROACH

The intensity distribution on the image grating for non-vibrating structure can be given by the following expression:

$$I(x,y) = 1 + aSin(\frac{2\pi}{p}y), \qquad (1)$$

where p and a stand for the grating pitch and visibility and y is along the direction perpendicular to the strips painted on the structure. By putting the structure into sinusoidal vibration the image grating also vibrates and the displacement of an arbitrary point on it can be given by

$$\vec{L}(x,y,t) = \vec{L}(x,y)Sin(\omega t)\,. \qquad (2)$$

The latter vibration changes the phase of the point by

$$\Delta\phi(x,y,t) = \vec{K}\cdot\vec{L}(x,y)Sin(\omega t) \qquad (3)$$

where $\vec{K}$ is a vector on the image plane of magnitude $2\pi/p$, and normal to the grating lines. For vibrating structure the intensity at the latter point is given by the following expression

$$I(x,y,t) = 1 + aSin[\frac{2\pi}{p}y + \Delta\phi(x,y,t)]\,, \qquad (4)$$

or by substituting $\Delta\phi(x,t)$ from Eq. (3) we get

$$I(x,y,t) = 1 + aSin[\frac{2\pi}{p}y + \vec{K}\cdot\vec{L}(x,y)Sin(\frac{2\pi}{T}t)]\,, \qquad (5)$$

or

$$I(x,y,t) = 1 + aSin(\frac{2\pi}{p}y)Cos[\vec{K}\cdot\vec{L}Sin(\frac{2\pi}{T}t)] + aCos(\frac{2\pi}{p}y)Sin[\vec{K}\cdot\vec{L}Sin(\frac{2\pi}{T}t)]\;, \qquad (6)$$

where ω is replaced by $2\pi/T$. The time average intensity at the given point can be evaluated from the following

$$\begin{aligned}\langle I(x,y)\rangle &= 1 + \frac{a}{T}\int_0^T Sin(\frac{2\pi}{p}y)Cos[\vec{K}\cdot\vec{L}(x,y)Sin(\frac{2\pi}{T}t)]dt \\ &+ \frac{a}{T}\int_0^T Cos(\frac{2\pi}{p}y)Sin[\vec{K}\cdot\vec{L}(x,y)Sin(\frac{2\pi}{T}t)]dt\end{aligned} \qquad (7)$$

Now considering that the zero order Bessel function is[8]

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} Cos(x\sin\theta)d\theta \qquad (8)$$

and

$$\frac{1}{2\pi}\int_0^{2\pi} Sin(xSin\theta)d\theta = 0, \qquad (9)$$

Eq. (7) can be given by

$$\langle I(x,y)\rangle = 1 + aSin(\frac{2\pi}{p}y)J_0(\Omega(x,y)), \tag{10}$$

where $\Omega(x,y) = \vec{K}\cdot\vec{L}(x,y)$. If we consider that the length of the structure is much larger than the width of the painted pattern we can assume that the vibrational displacement depends only on x. Using the definition of visibility:

$$V(x) = \frac{\langle I(x,y)\rangle_{\max} - \langle I(x,y)\rangle_{\min}}{\langle I(x,y)\rangle_{\max} + \langle I(x,y)\rangle_{\min}} \tag{11}$$

we get the normalized visibility, $V_n(x) = V(x)/a$, as

$$V_n(x) = \left|J_0[\Omega(x)]\right|. \tag{12}$$

In Fig. 1 the normalized visibility versus the vibration amplitude is plotted. For $V_n > 0.4$ we have single value amplitude, therefore the measurement of the visibility leads to the specification of the amplitude. But, for $V_n < 0.4$ other parameters should be also considered.

3.1. Specification of Vibrational Modes

Considering that the propagation speed of mechanical waves are high enough that the standing waves are formed in the vibrating structure in time interval much shorter than the time is spent for measurement, and assuming that the vibration of the structure in the direction of the painted strips is negligible, the displacement of an arbitrary point on the structure image can be given by

$$L_y(x,y) = ASin(\frac{2\pi}{\Lambda}x), \tag{13}$$

where A and Λ are the amplitude and the wavelength of the image vibration mode. The length of the structure image X and the wavelength of the mode, Λ_N, are related by

$$\Lambda_N = \frac{2X}{N}, \quad (N = 1,2,3,....) \tag{14}$$

where N is the mode number. Substituting form Eq. (13) in Eq. (12) we get

$$V_n(x) = \left|J_0[\frac{2\pi}{p}ASin(\frac{2\pi}{\Lambda}x)]\right|. \tag{15}$$

The argument of Bessel function J_0 in Eq. (15) is zero at $x = 0, \frac{\Lambda}{2}, \Lambda, ... \frac{n+1}{2}\Lambda$. At these points the visibility is 1, nodal points, and their number specifies the mode number.

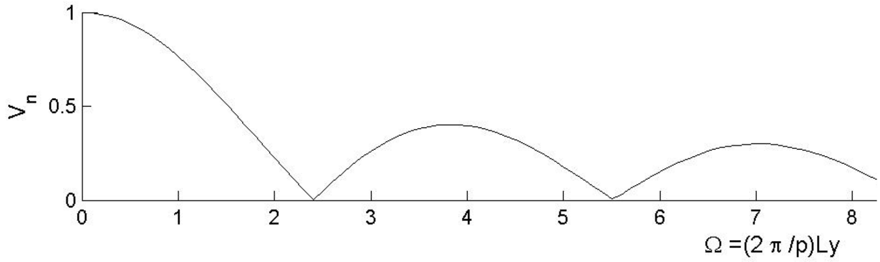


Figure 1. The normalized visibility on the image of a sinusoidal pattern painted on a lateral surface of a structure that vibrates, V_n, versus the vibration amplitude L_y.

3.2. Specification of the Vibration Amplitude

For specification of the vibration amplitude two different cases are distinguished. In the first case the minima of the visibilities are not zero. In this case each visibility minimum locates at halfway between two successive areas of $V_n = 1$. Measuring the minimum visibility and substituting x by $\Lambda_N/4$ in Eq. (15) we can get the amplitude from the following equation:

$$\frac{A}{p} = \frac{1}{2\pi} J_0^{-1}[(V_n)_{\min}], \tag{16}$$

in Fig. 2 the plot of Eq. (16), the ratio of the vibration amplitude to the pitch of painted pattern versus minimum visibility, is given.

In the second case there may be one or more zero visibilities between two successive areas of $V_n = 1$. These zeros are Bessel function zeros and the corresponding arguments are known quantities. Having the quantity of these arguments, obtaining Λ by mode specification, and measuring x, the distance between the required zero visibility area and the nearest area of normalized visibility 1, one can evaluate the amplitude A in the argument of Eq. (15) form the following equation

$$A = \frac{C_{Jm} p}{2\pi Sin(\frac{2\pi}{\Lambda} x_m)}, \tag{17}$$

where C_{Jm} is the m th argument of Bessel function for which the function is zero and x_m is the distance of m th zero visibility from the nearest area of visibility 1. For example, if the structure vibrates in fundamental mode and only one zero visibility appears in the image, we have $C_{J1} = 2.4048$, $x = \Lambda/4$ and form Eq. (17) we get $A = 0.38p$. For the amplitude larger than the latter, two or more zero visibility areas appear and symmetrically locate between areas of visiblity1. In Figs. 3 and 4 the vibrations of the painted structure are simulated for different amplitudes in the first and the second vibrational modes, respectively. The curves show the corresponding normalized visibility distributions across the image grating.

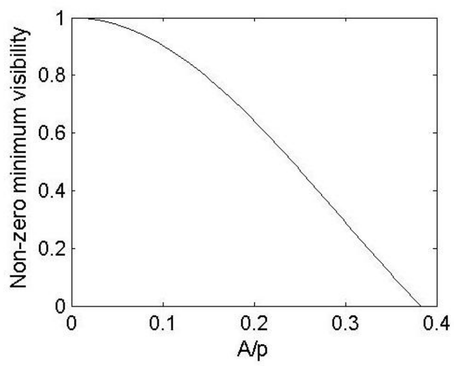


Figure 2. The non-zero minimum visibility of a sinusoidal pattern painted on a structure that vibrates versus the ratio of the vibration amplitude A to the pitch of the pattern p.

4. EXPERIMENTAL WORKS AND RESULTS

The sketch of the experimental set up is shown in Fig. 5. The sinusoidal pattern G of pitch $4.5mm$ is painted on an area of dimensions $300cm \times 4.5cm$ on the surface of the structure S. The imaging system $I.S.$, which is a Sony DSC-F717 camera snap the image of the pattern and feeds to the computer C for processing. The structure is an aluminum beam of length $3m$ and it is put in to vibration by hitting it by a hammer. After each hit it vibrates for several seconds. One or two seconds after each hit the camera's diaphragm opens for one second and the image of the vibrating pattern is recorded.

In Figs. 6-a and b the images of the pattern before and after hitting the beam are illustrated. In Fig. 6-c the recorded visibility, 1, the smoothed visibility, 2, and the normalized visibility, 3, are plotted. As can be seen from these plots and Fig. 6-b the beam vibrates in fundamental mode and the visibility has non zero minimum. Thus, for specifying the vibration amplitude one should measure the minimum visibility – which, according to graph 3 in Fig. 6-c, is about 0.25 – then, using the latter quantity in the graph of Fig. 2 and considering that $p = 4.5mm$ one gets the amplitude $A \cong 1.3mm$.

In Figs. 7-a and b the images of the pattern before and after hitting the beam are illustrated. In Fig. 6-c the recorded visibility, 1, the smoothed visibility, 2, and the normalized visibility, 3, are plotted. The image, in Fig. 7-b and normalized visibility curve, in Fig. 7-c, are similar to the simulation in Fig. 3 specified by $(A/p) = 0.55$. Thus, the beam vibrates in fundamental mode with two zero visibility minima. Therefore, the vibration amplitude can be obtained by the procedure described in Sec. 3.2.

5. CONCLUDING REMARKS

The presented technique is simple and powerful technique for vibration analysis. It can provide not only vibration mode and amplitude, but also the relaxation time and the damping coefficient of the vibration. The latter's can be measured by specifying the visibility curves at successive time intervals.

By choosing suitable amplitude to grating pitch ratio, a very wide range of vibration amplitudes can be covered which is a significant advantage over optical interferometry and holographic methods.

For very large structure it is better to paint the periodical pattern on rather small boards and install the boards at points with suitable spacing. During vibration the visibilities at these location are specified and then, by extrapolation the visibility over the entire structure is found.

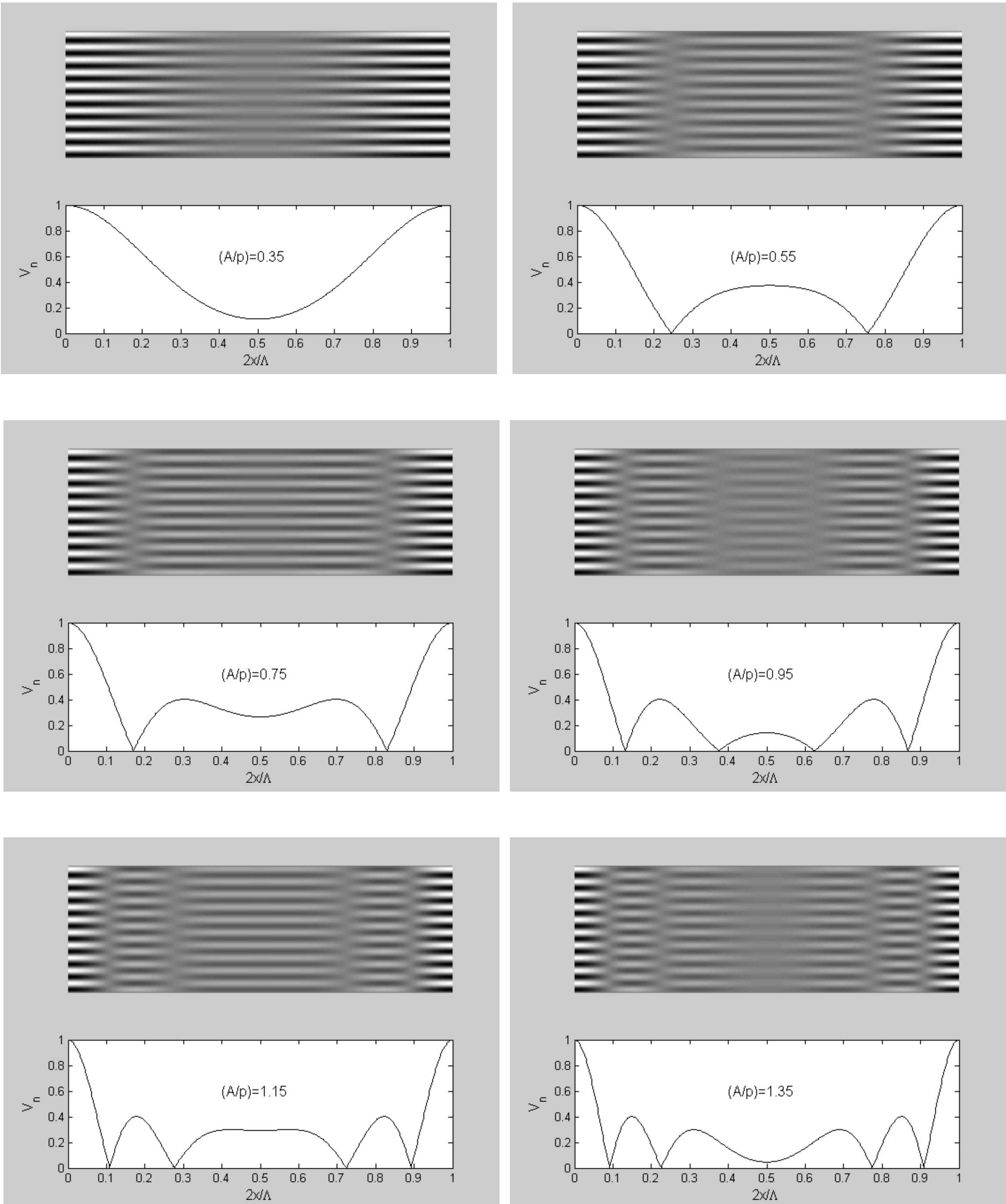


Figure 3. The simulations of the time average intensity distributions on the images of a sinusoidal pattern painted on a structure vibrating in fundamental mode for different amplitude to the pattern pitch ratios, A/p. The curves show the corresponding normalized visibilities across the pattern image.

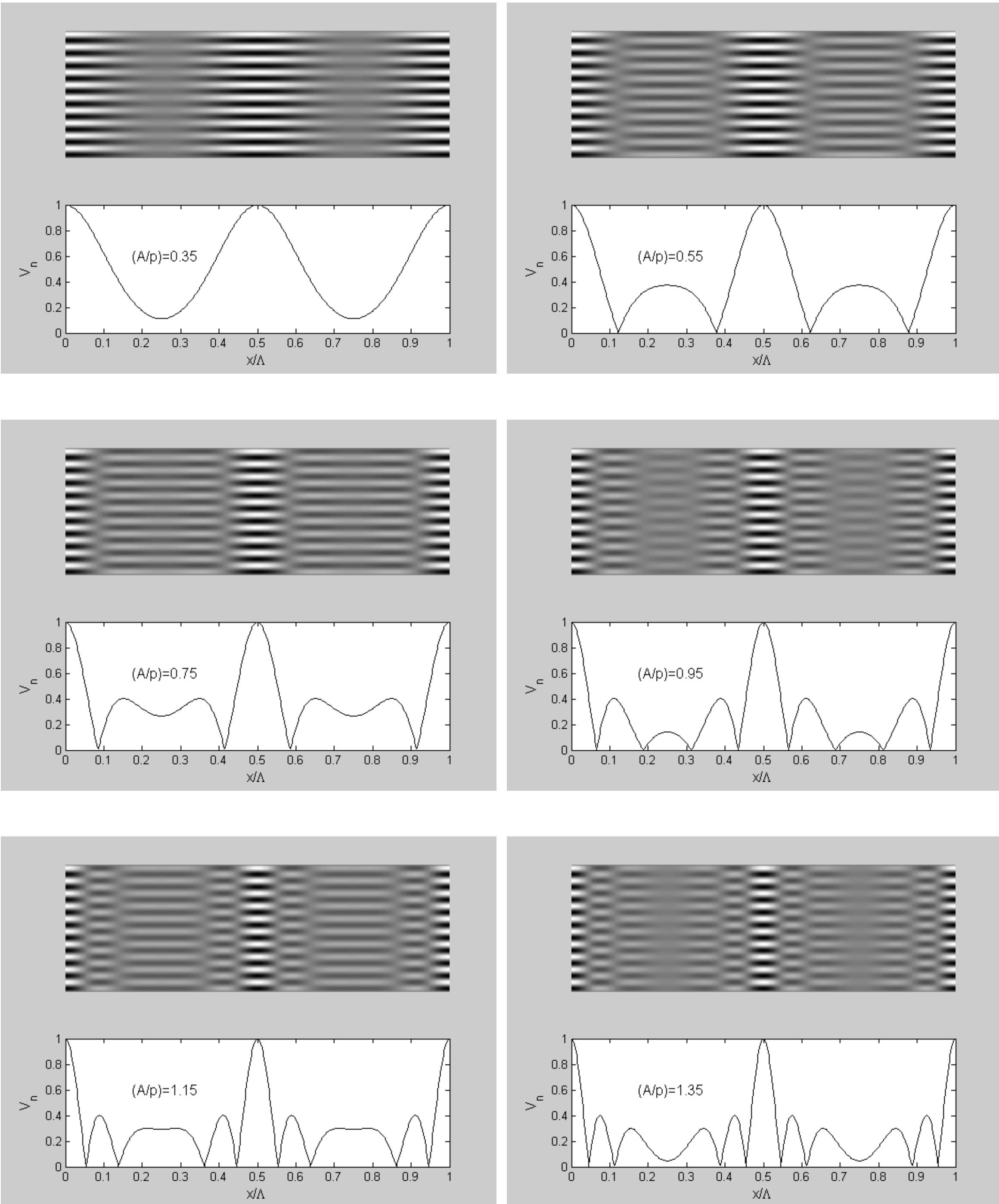


Figure 4. The simulations of the time average intensity distributions on the images of a sinusoidal pattern painted on a structure vibrating in the second mode for different amplitude to the pattern pitch ratios, A/p. The curves show the corresponding normalized visibilities across the pattern image.

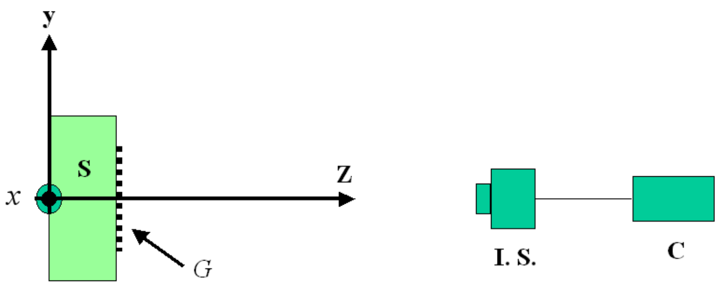
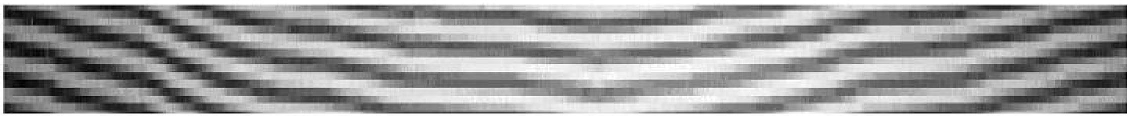
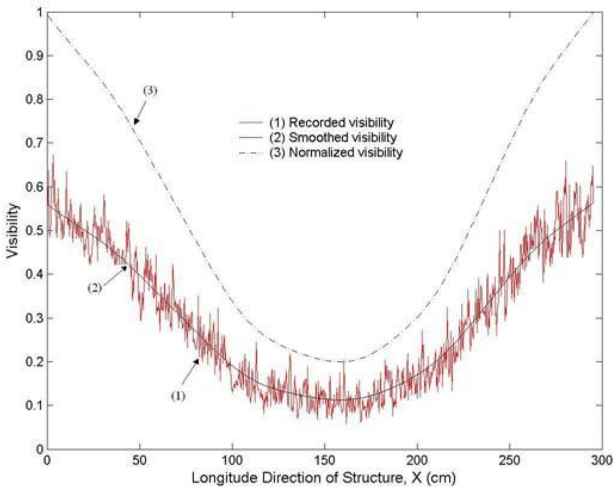


Figure 5. The sketch of experimental set up, where S, G, $I.S.$ and C stand for the structure, the painted sinusoidal pattern, the imaging system and computer, respectively.



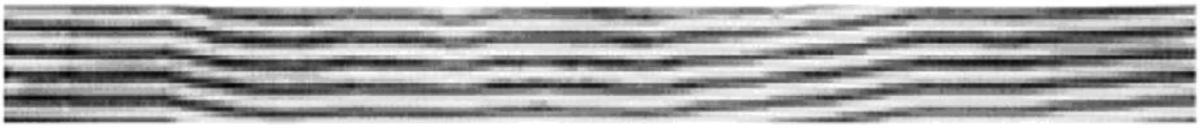
(a)

(b)



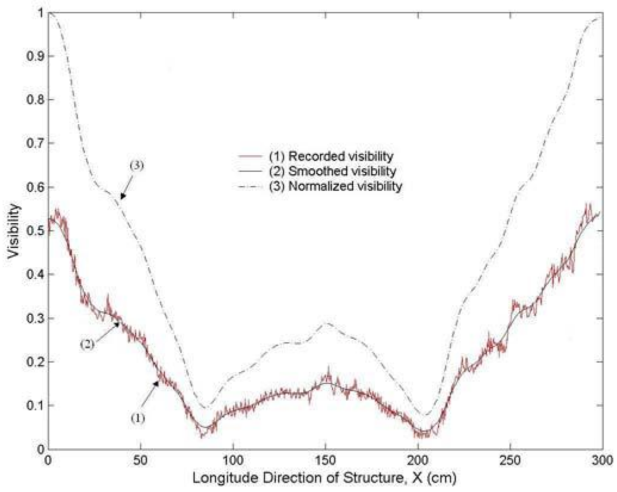
(c)

Figure 6. a) The image of the periodical pattern painted on the structure before putting it into vibration. b) The image of the same pattern after putting it into vibration. c) The corresponding recorded (1), smoothed (2), and normalized (3), visibilities.



(a)

(b)



(c)

Figure 7. a) The image of the periodical pattern painted on the structure before putting it into vibration. b) The image of the same pattern after putting it into vibration. c) The corresponding recorded (1), smoothed (2), and normalized (3), visibilities.

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