Use of a moiré deflectometer on a telescope for atmospheric turbulence measurements

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An instrument has been built for the study of the atmospheric turbulence by measuring the fluctuation of the angle of arrival across a telescope aperture using moiré deflectometry. A slightly divergent laser beam passes through a turbulent ground level atmosphere and enters the telescope aperture. The laser beam is recollimated behind the telescope's focal point by means of a collimator. The collimated beam passes through a moiré deflectometer. The fluctuating self-image of the first grating is formed on the second grating of the moiré deflectometer and fluctuating moiré fringes are formed. Using moiré fringe fluctuations we have calculated the fluctuations of the angle of arrival, the Fried's parameter r_0, and the atmospheric refractive index structure constant. Because of the magnifications of the telescope and moiré deflectometry, the precision of the technique can potentially be 1 order of magnitude more precise than previous methods.

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Recently, we have developed two methods, based on the moiré technique, for measuring the angle of arrival (AA) fluctuations of the light propagating in the turbulent ground level atmosphere [1,2]. The first method is based on incoherent imaging of a grating in turbulent atmosphere by a telescope [1]. We have used different configurations of this method to measure the atmospheric refractive index structure constant C_n^2 and the modulation transfer function of the ground level atmosphere [1,3,4]. The implementation of the method is not straightforward. The second method is based on coherent imaging. In this method a laser beam propagates through turbulent atmosphere and strikes perpendicularly a moiré deflectometer. In this work from the measurement of the relative displacements of moiré fringes we deduced the AA variance and C_n^2 [2]. Also we have used this method to wavefront sensing [5]. The major limitation of the second method is the small size of the gratings on the moiré deflectometer that can be used. Practically, constructing large area gratings, employing such gratings in a moiré deflectometer, and feeding the resulting moiré pattern to a CCD are not straightforward, even not possible for too large sizes.

Now, in this Letter we present a high-precision instrument that has been built for the study of atmospheric turbulence by measuring the fluctuation of the AA on the telescope aperture plane. A slightly divergent laser beam passes through a turbulent ground level atmosphere and enters to the telescope aperture. The laser beam is recollimated behind the telescope's focal point by means of a collimator. The collimated beam passes through a moiré deflectometer. A fluctuating self-image of the first grating is formed on the second grating of the moiré deflectometer and fluctuating moiré fringes are formed. Recording the successive moiré patterns by a CCD camera and feeding them to a computer, after filtering the higher spatial frequencies, produces highly magnified fluctuations of the laser beam. Compared to our previous methods, because of the large area of the telescope aperture, this instrument is more suitable for studying spatial and temporal properties of wavefronts. Using moiré fringe fluctuations, the fluctuations of the AA and the covariance of the AA components in the directions normal to the gratings ruling at any two points on one trace or on two different traces are evaluated. From these covariances, we have estimated the Fried's seeing parameter r_0. Also in this Letter we have calculated C_n^2. Because of the magnifications of the telescope and moiré deflectometry, the precision of the technique can potentially be 1 order of magnitude more precise than previous methods [1,2]. The schematic diagram of the experimental setup is shown in Fig. 1. The telescope aperture is reimaged behind the Cassegrain focus by means of collimating lens CL. Right behind the CL using a suitable bandpass filter F, the unwanted white lights are absorbed. One of the self-images of the first grating G1 is formed on a plane, where the second grating G2 and a diffuser D are installed. The lens PL projects the image of the latter plane on the CCD, which is connected to a computer.

In the theoretical considerations we assume that a wavefront enters a telescope aperture of diameter D

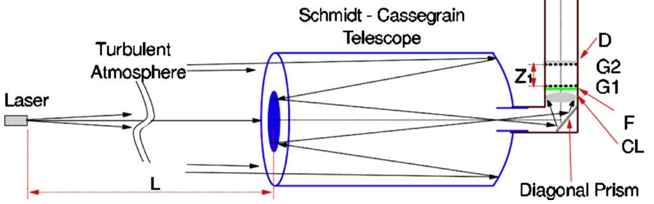

Fig. 1. (Color online) Schematic diagram of the experimental setup: CL, F, G1, G2, and PL stand for the collimating lens, bandpass filter, first grating, second grating, and the lens that projects the moiré pattern produced on the diffuser D on the CCD, respectively.

with a local AA on that plane, α. In the pupil image plane, we have a wavefront of diameter $D' = \gamma D$, and the corresponding AA is equal to

$$\alpha' = \frac{\alpha}{\gamma}, \tag{1}$$

where γ is the magnification of the optical system given by $\gamma = f'/f$, f is the telescope focal length, and f' is the focal length of the collimating lens. The collimated beam illuminates two gratings G1 and G2 of equal periods d, separated by Z_k along the optical axis, where Z_k is the kth Talbot distance of the first grating. Owing to the atmospheric turbulence the self-image of the first grating fluctuates on the second grating. The changes in the moiré patterns are related to the grating separation in the experimental setup, Z_k, the angle between gratings lines, $\theta = d/d_m$, in the moiré deflectometer, and the corresponding component of the AA fluctuations of the incident wavefront, where d_m is the moiré fringe spacing. According to the moiré formulation, for the displacement l of the self-image, in the direction normal to the grating lines, moiré fringes are shifted by s given by

$$s = \frac{d_m}{d} l. \tag{2}$$

Recalling that when the self-image of the first grating on the second grating is displaced by l the AA changes by $\alpha' = l/Z_k$. Thus, the component α' of the AA fluctuation on the direction perpendicular to the lines of the gratings (parallel to the moiré trace) on the first grating is given by

$$\alpha' = \frac{l}{Z_k} = \frac{1}{Z_k}\frac{d}{d_m} s. \tag{3}$$

Substituting from Eq. (3), in Eq. (1) we get

$$\alpha = \gamma\alpha' = \frac{f'}{f}\frac{1}{Z_k}\frac{d}{d_m} s. \tag{4}$$

According to Eq. (4), by increasing the gratings' distance or decreasing the period of the gratings, the measurement precision is improved. Compared to Eq. (8) of [2], here an improving factor f'/f appears. In this work, we have used $d = \frac{1}{100}$ cm, $f = 200$ cm, $f' = 13.5$ cm, and $Z_{k=1} = 3.7$ cm; by considering $s/d_m = \frac{1}{60}$, the minimum measurable AA fluctuation is 3.0×10^{-6} rad or 0.62 arc sec. It should be mentioned that, compared to [2], in this work we have used $d = \frac{1}{100}$ cm and $Z_1 = 3.7$ cm instead of $d = \frac{1}{150}$ cm and $Z_k = 39$ cm, respectively.

In the experiment the second harmonic of a cw diode pumped Nd:YAG laser beam passes through a turbulent surface layer of the atmosphere and enters the telescope (the Meade 8 in. Schmidt–Cassegrain model) aperture. The source and the experimental setup are installed at the height of 85 cm, over an asphalted area, at the distance of 360 m from each other. The CL with a focal length of 13.5 cm is used for the collimation of the beam. The gratings G1 and G2 with a period of 1/10 mm and dimensions of 20 mm × 20 mm are installed at the ends of a cylindrical tube with a changeable length to choose the desired Talbot distance. The PL with a focal length of 2.5 cm projects the image of the moiré patterns on a CCD (model DMK 21AU), which is connected to a computer. The moiré fringes were recorded by a sampling rate of 60 frames/s. Several sets of experimental data were recorded and digitized. In this Letter, we refer to the typical series of the measurements performed on January 11, 2010. That day was almost warm and sunny, but the night was cold and windy. Each set of data was collected in 33 s and contained 2000 frames. The typical recorded moiré pattern is shown in Fig. 2(a). In order to get more clear fringes the low frequency illumination of the pattern by spatial fast Fourier transform using the MATLAB software is obtained [Fig. 2(b)]. The traces of moiré fringe minima were specified. The traces of the minima could be determined to within 1 pixel accuracy. For more details see [1]. Once all 2000 data frames of the data set were processed, the moiré fringe displacement statistics—namely, the mean fringe trace, fringe displacement standard deviation, and displacement correlation function—were calculated. Using Eq. (4), the component α of the AA fluctuations was obtained. The plots in Figs. 3(a)–3(e) show the vertical components of AA fluctuations at five points on the telescope aperture versus time; all of the plots correspond to a vertical trace. The data sets were recorded at 21:32 (January 11, 2010). In the described experiments, the digitized frames consisted of 480 × 640 pixels, d_m was covered by 63 pixels, and the measurement precision of AA fluctuations was about 0.59 arc sec. By considering the maximum measurable displacement of a trace equal to half of the traces spacing, one can measure the AA fluctuations in range −18 to +18 arc sec without having any adjustment.

Often, the two-dimensional spatial covariance of AA fluctuations is used to investigate the properties of wavefronts distorted by the atmosphere. The theory is reviewed below, and the corresponding parameters are obtained from the experimental data. The covariance of the AA fluctuations in the aperture plane of the telescope can be defined by [6,7]

$$B_\alpha(\xi,\eta) = \langle \alpha(x,y)\alpha(x+\xi, y+\eta)\rangle, \tag{5}$$

where (x,y) and $(x+\xi, y+\eta)$ are coordinates of two arbitrary points of the distance $\sqrt{\xi^2+\eta^2}$ and $\langle\ \rangle$ denotes

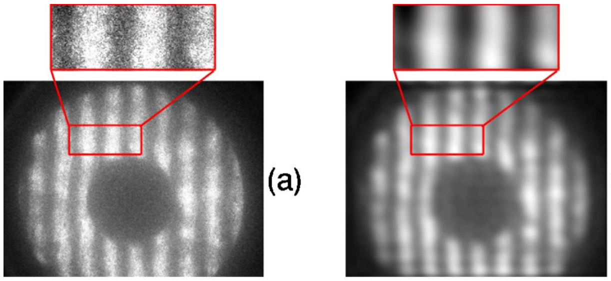

Fig. 2. (Color online) (a) Typical moiré pattern and (b) the corresponding low frequency illumination distribution.

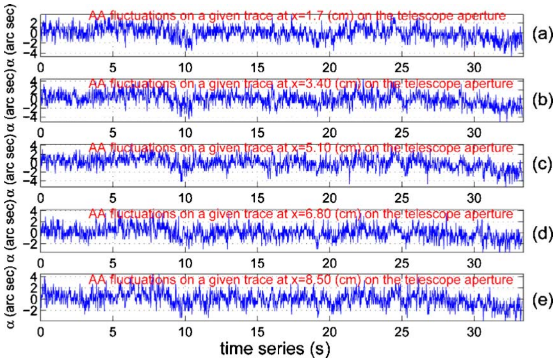

Fig. 3. (Color online) Typical vertical components of AA fluctuations at five different points on the telescope aperture versus time; all of the plots correspond to a given vertical trace.

statistical averaging. Here α is defined as the AA component in the x direction. For Kolmogorov turbulence, Fried's seeing parameter r_0 is related to the covariance $B_\alpha(\xi,\eta)$ of the AA fluctuation by [6,7]

$$B_\alpha(\xi,\eta) = 0.145\lambda^2 r_0^{-(5/3)} \times \left[(\xi^2+\eta^2)^{-(1/6)} - \frac{1}{3}\xi^2(\xi^2+\eta^2)^{-(7/6)} \right], \quad (6)$$

where λ is the wavelength. For $\eta=0$, we get the longitudinal covariance (in the direction of the AA fluctuation) as a function of the separation ξ of two points on a vertical trace [7],

$$B_\alpha(\xi,0) = B_l(\xi) = 0.097\left(\frac{\lambda}{r_0}\right)^{5/3}\left(\frac{\lambda}{\xi}\right)^{1/3}. \quad (7)$$

For $\xi=0$, we get the transverse covariance (in a direction perpendicular to the AA fluctuation) as a function of the separation η of two points of equal altitude on two different vertical traces [7],

$$B_\alpha(0,\eta) = B_t(\eta) = 0.145\left(\frac{\lambda}{r_0}\right)^{5/3}\left(\frac{\lambda}{\eta}\right)^{1/3}. \quad (8)$$

Using the moiré fringe displacements, the longitudinal covariance of the AA fluctuation for the two given points of different altitudes on one trace and the transverse covariance for the two given points of equal altitude on two different traces are calculated. Using Eq. (7), for the data sets (a) and (c) [(a) and (e)] of Fig. 3, values of r_0, 3.7 and 3.8 cm, are obtained, respectively, which are compatible with the typical values obtained by the differential image motion monitor technique. The relation between Fried's parameter r_0 and C_n^2, for a horizontal path of light, in a weak turbulence, has the form [8]

$$C_n^2 = 0.06L^{-1}\lambda^2 r_0^{-5/3}, \quad (9)$$

where L is the propagation length. Substituting the minimum value of r_0 in Eq. (9) the refractive index structure constant is calculated. The values of C_n^2 obtained for the mentioned data sets is $1.1\times 10^{-14}\ \mathrm{m}^{-2/3}$, which is compatible with the typical values obtained by other methods. This technique is also useful for calculating the longitudinal and transverse covariances as functions of ξ and η, respectively. Now, using the scaling forms of Eqs. (7) and (8) or the structure function of the AA fluctuation, the inner- and outer-scales of atmospheric turbulence can be determined. This work will be published elsewhere.

The application of the moiré deflectometry on a telescope in the measurement of the AA fluctuations magnifies the fluctuations by two factors: the telescope magnification and the moiré deflectometry magnification. Owing to the magnification of the telescope, the precision of the technique can potentially be 1 order of magnitude more precise than our previous methods. This instrument has a very good potential for wavefront sensing and adaptive optics applications in astronomy with more sensitivity. Also, compared to our previous reports, because of the large area of the aperture of the telescope, this instrument is more suitable for studying the spatial and temporal properties of wavefronts. The presented technique is very flexible and can be applied in a wide range of turbulence strengths, by choosing gratings of adequate pitch, size, and separation. By increasing the gratings' pitch and decreasing the distance between them, the AA fluctuations can be measured in a highly turbulent atmosphere, practically without a limit. Also, the instrument can be used with a white light source such as a celestial object or a laser guide star.

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