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EXCITATION OF THE NORMAL MODES OF A BINARY MEMBER BY ITS COMPANION

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Among the possibilities for close binary formation is the suggestion of Fabian, Pringle, and Rees (1975). They invoke a tidal process in which the energy from the relative orbital motions of the two unbound stars is transferred into the normal modes of non-radial oscillations of one or the other member. Press and Teukolsky (1977, hereafter PT) analyse this tidal process in some detail and give mathematical expressions for the energy transfer and the capture cross-section. Here we briefly summarize their work.

The energy transfer to stellar oscillations of the primary star having mass M_1 and radius R_1 due to a perturbing companion star having M_2 and R_2 can be written as

$$\Delta E = 2\pi^2 \Sigma_n \left| A_n (\omega_n) \right|^2 , \tag{1}$$

and

$$A_n (\omega_n) = \int_o^v \rho \underset{\sim}{\xi}_n \cdot \underset{\sim}{\nabla} \bar{U} (\omega) \, dv , \tag{2}$$

where the time integration has been effected from $-\infty$ to $+\infty$ and summation is over the normal modes. In equation (2) ρ is the density, $\bar{U}(\omega)$ is the time Fourier transform of $U(\underline{r}, t)$, the potential of the companion, and $\underset{\sim}{\xi}_n$ are normal modes. PT have also reduced the energy equation in terms of the product of a dimensional quantity and a dimensionless function.

$$\Delta E = \left(\frac{GM_1^{\,2}}{R_1} \right) \left(\frac{M_2}{M_1} \right)^2 \sum_{\ell=2,3,\ldots} \left(\frac{R_1}{R_{min}} \right)^{2\ell+2} T_\ell (\eta), \tag{3}$$

where ℓ is the spherical harmonic index and R_{min} is the periastron distance. The dimensionless parameter η is defined by

$$\eta = \left(\frac{M_1}{M_1 + M_2} \right)^{\frac{1}{2}} \left(\frac{R_{min}}{R_1} \right)^{3/2}. \tag{4}$$

The quantity η measures the duration of periastron passage, relative to the hydrodynamic time of the star: large η means slow passage. In equation (3) the dimensionless function $T_\ell(\eta)$ determines the energy deposition by the ℓ- pole tides. For mode analysis in equation (2) PT have used the method described by Robe (1968). Lee and Ostriker (1986), follow Cox (1980).

In this communication we decompose the eigendisplacements of a normal mode into irrotational and a solenoidal components (Sobouti, 1981)

$$\underset{\sim}{\xi} = \underset{\sim}{\xi}_p + \underset{\sim}{\xi}_g \quad , \tag{5}$$

where

$$\underset{\sim}{\xi}_p = - \underset{\sim}{\nabla} X_p(\underset{\sim}{r}) \quad , \tag{5a}$$

$$\underset{\sim}{\xi}_g = + \frac{1}{\rho} \underset{\sim}{\nabla} \times \underset{\sim}{A}_g = \frac{1}{\rho} \underset{\sim}{\nabla} \times \underset{\sim}{\nabla} \times (\hat{r} X_g(\underset{\sim}{r})) \ . \tag{5b}$$

Here $X_p(\underset{\sim}{r})$ and $X_g(\underset{\sim}{r})$ are two scalars, $\underset{\sim}{A}_g = \underset{\sim}{\nabla} \times (\hat{r} X_g)$ is a vector potential, and $\hat{r}$ is a unit vector in the radial direction. Substitution of equations (5) in equation (2) gives

$$A_n(\omega_n) = \int \rho \left(- \underset{\sim}{\nabla} X_p + \frac{1}{\rho} \underset{\sim}{\nabla} \times \underset{\sim}{A}_g \right) \underset{\sim}{\nabla} \bar{U} dv$$

$$= \int X_p \underset{\sim}{\nabla} \cdot \rho \underset{\sim}{\nabla} U) dv - \int \underset{\sim}{\nabla} \cdot (\underset{\sim}{\nabla} \times \underset{\sim}{A}_g) \bar{U} dv, \tag{6}$$

where each term is integrated by parts and the integrated terms have been put equal to zero. The second integral in equation (6) is obviously zero. The first integral simplifies further by noting that ρ is sphericaly symmetric and $\underset{\sim}{\nabla}^2 \bar{U} = 0$. For $\bar{U}$ is the potential of the companion at points within the primary and satisfies Laplace's equation. Thus

$$A_n(\omega_n) = \int X_p(\underset{\sim}{r}) \frac{d\rho}{dr} \frac{\partial \bar{U}}{\partial r} dv. \tag{7}$$

We conclude that the gravitational field of the companion excites the linear motions within the primary through their p-components. The accompanying g-motions are of course excited but through the intermediary of the p-motions. Thus, it should not be surprising to conclude at this premature stage that the contributions of the p-modes to the energy deposition, ΔE, of equation (1), is larger by far than those of the g-modes, a prediction well born out by our numerical calculations, and others.

Another noteworthy point: That the interaction is between $\underset{\sim}{\xi}_p$ and $\underset{\sim}{\nabla}\bar{U}$ is due to the fact that both fields are derived from scalar potentials. Had the perturbing force been derived from a vector potential (e.g. in magnetic interactions) then $\underset{\sim}{\xi}_g$-motions would have entered the play at the expense of the exclusion of $\underset{\sim}{\xi}_p$.

We show $T_2(\eta)$ and $T_3(\eta)$ v_s. η for polytropic indices, n = 1.5, 2, 2.5, 3, 3.25, 3.5, and 4 in Figure(1). Our results are nearly in agreement with those of Lee and Ostriker in the overlapping range of data. As regards the computations we calculate modes by a Rayleigh Ritz variational technique. Lee and Ostriker do it Dizembowski's (1971) way. PT, follow Robe (1968).

The capture cross-section, σ, or impact parameter, R_o, ($\sigma=\pi R_o^2$) is calculated for the above polytropes in term of the relative velocity of the binary members at infinity, V_∞. Results shown in Figure (2) are again in agreement with Lee and Ostriker for polytropes, n = 1.5, 2, and 3. Let the dependence of R_o on V_∞ be a power law $R_o=CV_\infty^{-\alpha}$. We find that for $V_\infty \approx 10$km/s, α increases form 1.06 for n = 1.5 to 1.09 for n = 3, and remains constant for N >3. This means that the cross-sections for n >3 polytropes show the same velocity dependence as n = 3.

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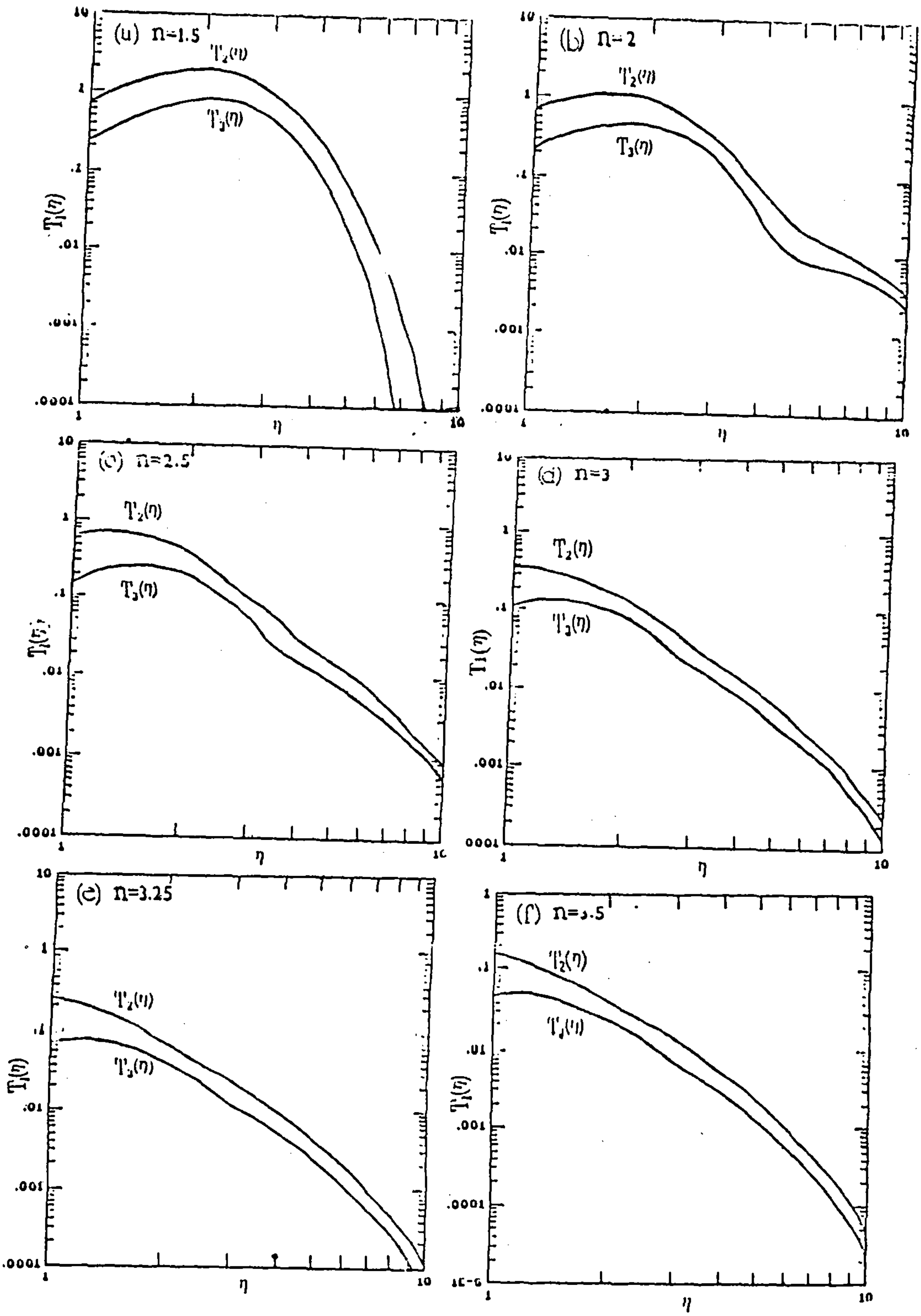

Fig. 1. The dimensionaless functions $T_2(\eta)$ and $T_3(\eta)$ which determine the amount of energy deposition during the two-body encounter by quadrupole and octupole tides, respectively, for (a) n=1.5, (b) n=2, (c) n=2.5, (d) n=3, (e) n=3/25, (f) n=3.5.

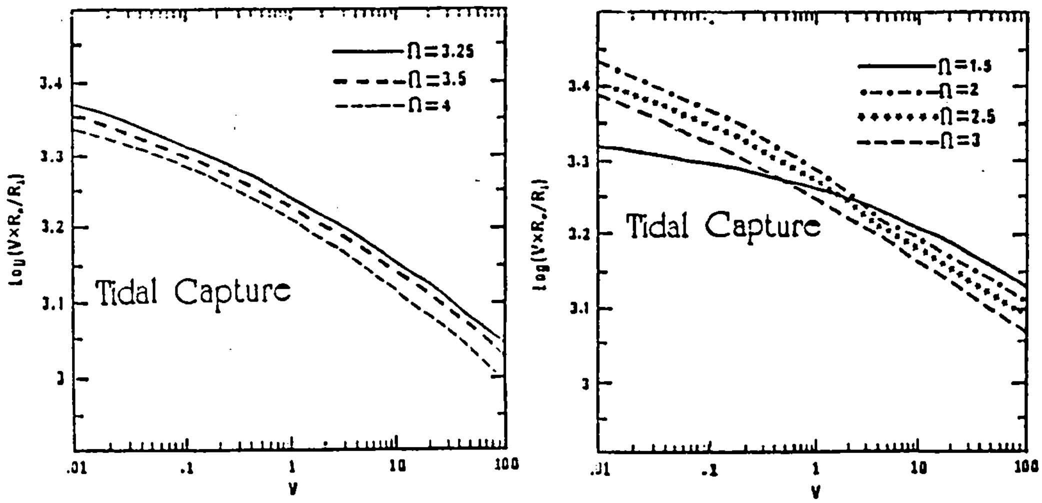

Fig.2. Tidal capture impact parameters in units of R_1 as a function of the relative velocity at infinity for the encounter between identical stars for polytropes n=1.5, 2, 2.5, 3, 3.25, 3.5, and 4.