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THE NORMAL MODES OF OSCILLATIONS OF FLUIDS IN THE PRESENCE OF MAGNETIC FIELDS

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The possible effect of a magnetic field on the structure and stability of stars was first analyzed be Chandrasekhar and Fermi (1953). Since then many aspects of the problem have been investigated by different authors. Interaction of toroidal fields with meridional motions and dynamo effects was studied in a number of papers by Mestel (Lüst 1965 and the references therein). Kovetz (1966) extended the variational formulation pioneered by Ledoux and Walraven (1958) and Chandrasekhar (1964), to include magnetized fluids. Kovetz's paper presents a careful analysis of the boundary conditions in the presence of magnetic fields of quite a general nature. Sobouti (1977c) considered a convectively neutral fluid immersed in a force-free field. He showed that the magnetic field removed the degeneracy of the neutral convective motions and the neutral toroidal displacements. Two sequences of modes developed. One mainly of toroidal nature and the other of poloidal, both with periods of the order of Alfven crossing times. Here we generalize the last work to convectively non-neutral fluids.

We assume a prefectly conducting and self-gravitating fluid, prevaded by a force-free magnetic field (Ferraro and Plumpton, 1966). A polytropic structure is assumed for the fluid.

Let ξ (r,t) denote a small Lagrangian displacement of a fluid element from its equilibrium position. The linearized equations of motion can be written as follows:

$$\rho \frac{\partial^2 \xi}{\partial t^2} = -F\xi \qquad (1)$$

where

$$F\xi = \nabla(\delta p) - \delta\rho \nabla U - \rho\nabla(\delta U) - \frac{1}{4\pi} \left| (\nabla \times \delta B) \times B + (\nabla \times B) \times \delta B \right| \qquad (2)$$

$$\delta\rho = -\rho\nabla\cdot\xi - \xi\cdot\nabla\rho \tag{3}$$

$$\delta P = -\gamma P\nabla\cdot\xi - \xi\cdot\nabla P \tag{4}$$

$$\delta B = \nabla\times(\xi\times B) \tag{5}$$

$$\nabla^2(\delta U) = -4\pi G\,\delta\rho \tag{6}$$

In equations (1) - (6) , δ denotes the Eulerian variation of a quantity, γ is the ratio of specific heats, U is the potenial due to self-gravity, and the other symboles have their usual meanings. On multiplying equation (1) by ξ^*, integrating over the volume initially occupied by the fluid, we get

$$W - \omega^2 S = 0 \quad \text{or } \omega^2 = \frac{W}{S} \tag{7}$$

where

$$S = \int dv\, \rho\xi^*\cdot\xi > 0 \tag{8}$$

$$W = W(1) + W(2) + W(3) + W(4) + W(5) \tag{9}$$

$$W(1) = \int dv\, \frac{1}{\rho}\frac{dp}{d\rho}\delta\rho^*\,\delta\rho \tag{10}$$

$$W(2) = \int dv\, \alpha p\, \nabla\cdot\xi^*\; \nabla\cdot\xi \quad ; \quad \alpha = \gamma - \frac{\rho}{p}\frac{dp}{d\rho} \tag{11}$$

$$W(3) = -G\int\int dv dv'\delta\rho^*(r)\delta\rho(r')|r-r'|^{-1} \tag{12}$$

$$W(4) = \frac{1}{4\pi}\int dv\, \delta B^*\cdot\delta B \tag{13}$$

$$W(5) = -\frac{1}{4\pi}\int dv\delta B^*\cdot(\xi\times B), \tag{14}$$

and ξ is assumed to have exponential time dependence, $e^{i\omega t}$. All the integrals in equation (8)-(4) are symmetric under the exchange of ξ and ξ^*. This property is a reflection of the symmetry of the F operator of equation (1) and guarantees the existance of an eigenvalue problem with real ω^2(Sobouti, 1977a) We use a gauged version of Helmholtz's theorem (Sobouti, 1981; 1986) to decompose the displacement field, ξ, into vectors derived from a scalar potential and two vector potentials. Each component in such a decomposition happens to be closely associated with the familiar p^-, g^- and toroidal modes of the fluid and greatly simplifies the task of mode classification and calculation. Further using the Rayleigh-Ritz variational technique we cast equation (1) into algebraic matrix form which is suitable for computational purposes. A direct consequence of the presence of the magnetic field is the modification of eigenfrequencies and eigenfuction of ever-present p^- and g^- modes. More importantly, however, the later modes acquire a toroidal component, which is absent in non-magnetized fluids. The effect of the magnetic field on p^- modes, as shown in Fig.1, is to increase the p^- eigenfrequencies. However, the p^- character of the modes is not destroyed. The stability of g^- modes in the presence of a magnetic field is shown in Fig.2. We see that the g-spectrum is destroyed by the magnetic field.

In these figures $\lambda = \gamma V_a^2/V_s^2$, in which V_a is the Alfven speed given by $V_a = B/\sqrt{4\pi\rho_c}$, and V_s is the sound speed given by $V_s = \gamma p_c/\rho_c$ where, p_c and ρ_c respectively denote the pressure and density at the center of the fluid. This dimensionless parameter stands for the strength of the magnetic field.

Eigenvalues and eigenfunctions for different polytropic index, n, are computed and some of these are given in Table 1, 2 and 3.

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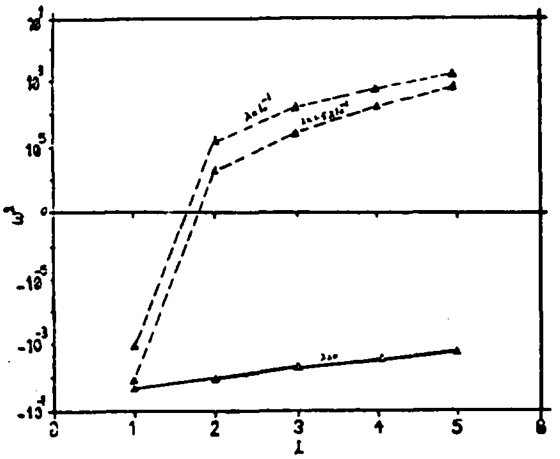

Figure 1. Variation of the eigenfrequency ω^2 with mode order, i, for $\bar{p}$ modes in units of $4\pi G\rho_c/n+1$, assuming n=1.0 and γ =5/3. The solid and dashed lines correspond to the non-magnetized and magnetized fluid, respectively.

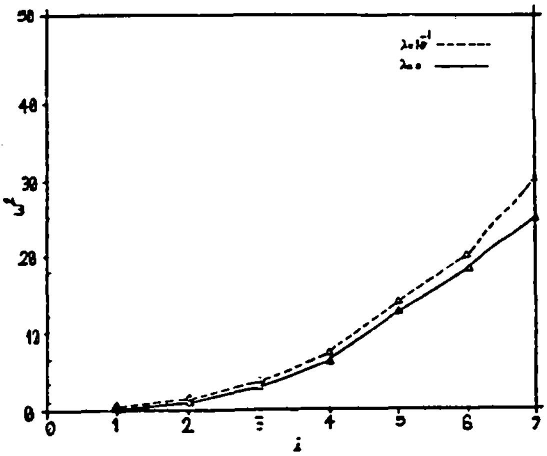

Figure 2. Frequency dependence of g-modes with i, for a convectively unstable fluid(α <0), assuming n=1.0 and γ=5/3. The solid line corresponds to B=0 and the dashed lines correspond to two different magnetic field strengths.

Table 1. The frequency ω^2 for $\bar{p}$ modes with different λ , assuming n=1.0 and γ=5/3.

λ	p_1	p_2	p_3	p_4	p_5	p_6	p_7
0	0	1.1549	3.6235	7.1309	1.1723×10^1	1.7619×10^1	2.5243×10^1
10^{-6}	2.1473×10^{-7}	1.1549	3.6239	7.1341	1.1741×10^1	1.7687×10^1	2.5467×10^1
0.5×10^{-1}	1.6898×10^{-2}	1.1866	3.7260	7.3518	1.2220×10^1	2.0128×10^1	4.3616×10^1
10^{-1}	1.3184×10^{-1}	1.2175	3.8[illegible]83	7.6078	1.2632×10^1	2.0128×10^1	4.3616×10^1

Table 2. The frequency ω^2 for $\bar{g}$ modes with different λ, assuming $n=1.0$ and $\gamma=5/3$.

λ	g_1	g_2	g_3	g_4	g_5	g_6	g_7
0	-3.4063×10^{-2}	-1.2773×10^{-2}	-6.8212×10^{-3}	-4.2994×10^{-3}	-2.9959×10^{-3}	-1.8616×10^{-3}	-1.4872×10^{-3}
10^{-6}	-3.4056×10^{-2}	-1.2724×10^{-2}	-6.8069×10^{-3}	-4.2591×10^{-3}	-2.8393×10^{-3}	-1.8339×10^{-3}	-1.2982×10^{-3}
$0.5\times10^{-}$	-1.6586×10^{-2}	-2.1466×10^{-6}	3.8532×10^{-5}	2.4680×10^{-4}	8.3826×10^{-4}	2.3201×10^{-3}	5.5631×10^{-3}
10^{-1}	-2.5431×10^{-3}	1.9616×10^{-5}	1.5346×10^{-4}	5.2854×10^{-4}	1.6762×10^{-3}	4.7611×10^{-2}	1.1291×10^{-2}

Table 3. The frequency ω^2 for t modes with different λ, assuming $n=1.0$ and $\gamma=5/3$.

λ	t_1	t_2	t_3	t_4	t_5	t_6	t_7
0	0	0	0	0	0	0	0
10^{-6}	5.9817×10^{-8}	1.7663×10^{-8}	4.7146×10^{-8}	1.1377×10^{-7}	2.7345×10^{-7}	5.6513×10^{-7}	1.1833×10^{-6}
0.5×10^{-1}	2.6408×10^{-2}	3.0443×10^{-2}	4.4782×10^{-2}	6.4759×10^{-2}	1.4066×10^{-1}	3.3795×10^{-1}	1.9209
10^{-1}	4.1023×10^{-2}	4.4632×10^{-2}	6.4799×10^{-2}	9.2838×10^{-2}	2.8391×10^{-1}	6.7674×10^{-1}	3.8459