

Dark companion of baryonic matter

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Flat or almost flat rotation curves of spiral galaxies can be explained by logarithmic gravitational potentials. The field equations of GR admit of spacetime metrics with such behaviors. The scenario can be interpreted either as an alternative theory of gravitation or, equivalently, as a dark matter paradigm. In the latter interpretation, one is led to assign a dark companion to the baryonic matter who's size and distribution is determined by the mass of the baryons. The formalism also opens up a way to support Milgrom's idea that the acceleration of a test object in a gravitational field is not simply the newtonian gravitational force g_N , but rather an involved function of (g_N/a_0) , a_0 MOND's universal acceleration.

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I. INTRODUCTION

The goal of the paper is to understand the idiosyncrasy of the rotation curves of spiral galaxies. The newtonian or the GR gravitation of the observable matter is not sufficient to explain the large asymptotic speeds of test objects in orbits around the galaxies, nor their slow decline with increasing distances. In search of the missing gravity, alternative theories of gravitation and/or of dark matter are proposed. In a recent work [1] we pointed out that no one has reported a case where there is no baryonic matter, but there is a dynamical issue to be settled. We argued that if the dark matter reveals itself only in the presence of the baryonic one, it is logical to assume that the two are twin companions. On the other hand, both dark matter scenarists and (at least some) alternative theorists explain the rotation curves of spirals equally satisfactorily. We argue, if two people give correct answers to the same question, they ought to be saying the same thing, albeit in different languages. And since in an alternative theory one gives a definite rule for the gravity field, there must be rules to govern the mutual companionship of the dark and baryonic matters.

We begin with a GR formalism and show that spacetime metrics with logarithmic behaviors are accommodated by Einstein's field equations and can adequately explain the anomalous features of the dynamics of the spirals. Conclusions are interpretable either in terms of an alternative theory of gravitation, or as a dark matter paradigm. With an advantage, however: the questions, how much dark matter accompanies a given baryonic mass, how it is distributed, and what is its equation of state, are also answered.

II. MODEL AND FORMALISM

We are concerned with the outer reaches of spiral galaxies (a baryonic vacuum), where the rotation curves display non classical features. Their asymptotic speeds do not have a Keplerian decline[4] and follow the Tully-Fisher relation[5]. We approximate the galaxy by a spherically symmetric distribution of baryonic matter. The spacetime around it will accordingly be spherically symmetric and static:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (1)$$

We adopt a dark matter language and assume that the galaxy possesses a static dark perfect gas companion of density $\rho_d(r)$, of pressure $p_d(r) \ll \rho_d(r)$, and of covariant 4-velocities $U_t = -B^{1/2}$, $U_i = 0$, $i = r, \theta, \varphi$. Einstein's field equations become.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu} = -[p_d g_{\mu\nu} + (p_d + \rho_d)U_\mu U_\nu], \quad (2)$$

where we have let $8\pi G = c^2 = 1$. To respect the Bianchi identities and the conservation laws of the baryonic matter, one must have $T^{\mu\nu}{}_{;\mu\nu} = 0$. The latter, in turn, leads to the hydrostatic equilibrium for the dark fluid, that is, if one wishes to attribute such notions to a hypothetical entity.

From Eq. (2) the two combinations $R_{tt}/B + R_{rr}/A + 2R_{\theta\theta}/r^2$ and $R_{tt}/B + R_{rr}/A$ give

$$\frac{1}{r^2} \left[\frac{d}{dr} \left(\frac{r}{A} \right) - 1 \right] = -\rho_d, \quad (3)$$

$$\frac{1}{rA} \left(\frac{B'}{B} + \frac{A'}{A} \right) = \rho_d + p_d, \quad (4)$$

respectively. Neglecting p_d in comparison with ρ_d and eliminating ρ_d between the two equations gives

$$\frac{B'}{B} = \frac{1}{r}(A - 1). \quad (5)$$

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We now assume that $A(r) - 1$ is a well behaved and differentiable function of r and has a series expansion in negative powers of r ,

$$A - 1 = \sum_{n=0} \frac{s_n}{r^n}, \quad s_n \text{ constant.} \quad (6)$$

Substituting this expansion in Eq. (5) and integrating the resulting expression gives

$$B = \left(\frac{r}{r_0}\right)^{s_0} \exp\left(-\sum_{n=1} \frac{s_n}{nr^n}\right) \approx \left[1 + s_0 \ln\left(\frac{r}{r_0}\right) - \frac{s_1}{r} - \dots\right] \quad (7)$$

We note that s_0 is dimensionless and s_n , $n \geq 1$, has the dimension (length) ^{n} . The right hand side expression is the weak field approximation and holds for $s_n/r^n \ll 1$, $\forall n$.

With A and B known, the density ρ_d can be calculated from either of Eqs. (3) or (4). Here, however, we adopt a weak field point of view, $B(r) = 1 + 2\phi_{grav}/c^2$, and calculate ρ_d from Poisson's equation. Thus,

$$4\pi G\rho_d = \frac{1}{2}c^2\nabla^2(B - 1) = \frac{c^2}{2r^2} \left[s_0 - \sum_{n=2} (n-1) \frac{s_n}{r^n} \right] \quad (8)$$

Hereafter, we restore the physical dimensions $8\pi G$ and c^2 for clarity. The pressure of the companion fluid is obtained from $T^{\mu\nu}{}_{;\nu} = 0$,

$$\frac{p'_d}{p_d + \rho_d} \approx \frac{p'_d}{\rho_d} = -\frac{1}{2r}(A - 1). \quad (9)$$

Integration is straight forward. The first two terms in the series are

$$p_d = \frac{c^2 s_0}{16\pi G r^2} \left[\frac{1}{2} s_0 + \frac{s_1}{3r} \right]. \quad (10)$$

Upon elimination of r between Eqs. (8) and (10) one obtains the equation of state, $p_d(\rho_d)$. It is barotropic. We conclude this section by writing down the dynamical acceleration of a test object circling the galaxy with the speed v

$$a_{\text{dyn}} = \frac{v^2}{r} = \frac{1}{2}c^2 B' = \frac{1}{2}c^2 \left[\frac{s_0}{r} + \frac{s_1}{r^2} + \dots + \frac{s_n}{r^{n+1}} + \dots \right] \quad (11)$$

III. WHAT ARE s_n 's

The s_1 term in Eqs. (6)-(11) represents the classic gravitation of the baryonic matter with a force range of r^{-2} . Magnitude-wise, s_1 , should be identified with the Schwarzschild radius of the spherical galaxy, $s_1 = 2GM/c^2$. The s_0 -term is not a classical term. It has a force range r^{-1} and dominates all other terms at large distances. It is responsible for the large asymptotic speeds and their non Keplerian decline at far reaches of the spirals. In [2] and [1] we resorted to the Tully-Fisher

relation (the proportionality of the asymptotic speed, $v_\infty = c(s_0/2)^{1/2}$, to the fourth root of the mass of the host galaxy) and arrived at

$$s_0 = \alpha \left(\frac{M}{M_\odot} \right)^{1/2}, \quad \alpha \text{ constant.} \quad (12)$$

In weak accelerations (less than certain 'universal acceleration' a_0), Milgrom's MOND [3] anticipates a force field $(a_0 g_N)^{1/2}$, instead of the newtonian gravitation, $g_N(\vec{r}) = GM/r^2$. The far distance limit of Eq. (11) with α given by Eq. (12) is of Milgrom's form. Comparing the two formalisms, one finds $\alpha = 2(a_0 GM_\odot)^{1/2} c^{-2}$. Either from this expression, with $a_0 \approx 1.2 \times 10^{-8} \text{cm/sec}^2$ [4], or from a direct statistical analysis of the asymptotic speeds of spirals [2] one finds

$$\alpha \approx 2.8 \times 10^{-12}, \quad \text{dimensionless 'universal constant'} \quad (13)$$

The remaining s_n -terms, $n \geq 2$, in Eqs. (6)-(11) are also nonclassical. The range of their force is $r^{-(n+1)}$ (not to be confused with the multipole fields of extended objects). There is no compelling observational evidence for their existence in regions external to a spherical distribution of matter. Nevertheless, we retain them for a possible formal support they may give to Milgrom's MOND, to be elaborated below.

A conjecture: There is a surprise in Eq. (11). Upon elimination of r in favor of $g_N = GM/r^2$, one may write it as

$$\frac{a_{\text{dyn}}}{a_0} = \left(\frac{g_N}{a_0} \right)^{1/2} + \left(\frac{g_N}{a_0} \right) + \dots + \alpha_n \left(\frac{g_N}{a_0} \right)^{(n+1)/2} + \dots \quad (14)$$

where α_n 's can be expressed in terms of s_n 's through a term-by-term comparison of Eqs. (11) and (14). One obtains

$$\alpha_n = \frac{c^2 s_n}{2a_0} \left(\frac{a_0}{GM} \right)^{(n+1)/2}, \quad n = 2, 3, \dots, \quad (15)$$

or

$$s_n = \frac{2a_0}{c^2} \alpha_n \left(\frac{GM}{a_0} \right)^{-(n+1)/2}. \quad (16)$$

All α_n 's are dimensionless. Apparently, Eq. (14) is an expansion of the dynamical acceleration in a power series of $(g_N/a_0)^{1/2}$. The coefficient of the first term is the 'universal constant' 1 because of the 'universal' Tully-Fisher relation. The coefficient of the second term is 1 because of the universal law of newtonian gravitation in the weak field regime. Now the conjecture: If there is any significance attached to the series expansion of Eq. (14) beyond the first two terms, is it possible that in the remaining terms

"All α_n 's are universal constants (not necessarily 1), and independent from the mass of the host baryonic

matter centered at the origin”?

The proof or disproof of the conjecture should come from observations. We recall, however, Milgrom’s stand that the dynamical acceleration of a test body is not simply proportional to g_N , but it is a involved function of g_N/a_0 , and vice versa, g_N is a function of a_{dyn}/a_0 . His suggestion for this function is through an, almost arbitrary, interpolating function. If the conjecture above holds, Eq. (14) can be considered as a series expansion of one such function, and a support for Milgrom’s idea.

IV. CONCLUDING REMARKS

That logarithmic potentials are natural solutions of Einstein’s field equations is the highlight of the paper. They enable one to arrive at a law of gravitation alternative to that of Newton and/or to those known to GR. Equivalently, one may choose to attribute dark companions to baryonic matters. In the case of a spherically symmetric baryonic mass, the size and distribution of the density and pressure of the companion, outside the baryonic mass, are given by Eqs. (8) and (10).

The spacetime is a baryonic vacuum but not a dark matter one. The consequences are noteworthy. For example:

The spacetime is not flat. Contraction of Eq. (2) gives

$$R = -(3p_d + \rho_d) \approx -\frac{s_0}{r^2} + O(r^{-4}).$$

The 3-space is not flat. Direct calculation with $g_{ij}^{(3)}$, $i, j = r, \theta, \varphi$, yields

$$R^{(3)} = -\frac{2}{r^2} \frac{d}{dr}(r\rho_d) \approx -2\frac{s_0}{r^2} + O(r^{-4}).$$

There is an excess lensing [7]. Contribution from the s_0 term alone is

$$\delta\beta = \frac{1}{2}\pi s_0.$$

Due to the smallness of both s_0 and Sun’s mass, effects in the scale of the solar system are immeasurably small[1].

That the dynamical acceleration of a test object in the external gravitational field of a spherical mass could have a series expansion in (g_N/a_0) , in accord with Milgrom’s idea, is an intriguing idea. The support for it should come from observations.

A word of caution: The paper relies heavily on observations pertaining to spiral galaxies. Its conclusions may be scale dependent, not applicable to systems with scales larger than galactic scales. In a recent paper Bernal et al [6] analyze weak lensing data from clusters of galaxies on the basis of the metric field of [2] (similar to those of Eqs. 6 and 9). They conclude, in the notation of this paper, $s_0 \propto M^{1/4}$, instead of $M^{1/2}$ of Eq. (12). This finding while raises an alarm against extrapolation to larger systems, clusters of galaxies and beyond, at the same time opens the question that deviations from the newtonian or GR gravitations may have a hierarchical structure depending on the size of the system under study.

Shortcomings of the paper and the open questions it leaves behind should also be mentioned. The theory developed here is for a spherical distribution of mass. Extension to extended objects and to many body systems is not a trivial task. It may require further assumptions not contemplated so far. The difficulty lies in the facts that a) the added s_0 - and s_n - terms, $n \geq 2$, are not linear in the mass of the baryonic matter. The nonlinearity is much more complicated than that of GR. b) In the parlance of a dark matter paradigm, the dark companion of a localized baryonic matter is not localized and extends to infinity. As a way out of the dilemma, we are planning to expand an extended object into its localized monopole and higher multipole moments, and see if it is possible to find a dark multipole moment for each baryonic one, more or less in the way done for the monopole moment.

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