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Three arguables: point particle singularity, asymmetry in EM and quantum waves, and the left out restricted Lorentz gauge from U(1), revised and abridged*

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We address three issues: 1) The point particle assumption inherent to non- quantum physics is singular and entails divergent fields and integrals. 2) In quantum physics electromagnetism (EM) plays an asymmetric roll. It acts on quantum wave functions (QW) but QW does not react back. We suggest to promote the one- sided action of EM on QW into a mutual action-reaction status. This enables QW to share its non-singular feature with EM and to remove the Coulomb singularity. 3) Quantum mechanics is U(1) symmetric. QW multiplied by an *arbitrary* phase factor and EM written in the same Lorentz gauge, leave both EM and QW invariant. The minimal coupling of QW to the EM 4- vector potential, A_μ, is a consequence of this arbitrary gauge. Symmetry under the restricted Lorentz gauge, is left out. We propose to enlarge U(1) to accommodate the restricted Lorentz gauge as well. This

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in turn invites in a coupling of QW to the derivatives of the vector potential, $\partial_\nu A_\mu$, in addition to the minimal coupling. We find that i) electron acquires a distributed charge, reminiscent of the QED- renormalized charge distributions; ii) because of its spin, electron acquires a self induced magnetic moment with the same g- factor as in QED but without relying on QED.

I. INTRODUCTION

1)The point particle singularity - Let us begin with the Coulomb potential $\frac{e}{r}$. There are two tacit assumptions in this basic law of EM; i) there is a physical entity of finite electric charge concentrated in zero volume, a notion that challenges the common sense; ii) one can assign a precise coordinate such as $r = 0$ to it and approach it infinitely closely, an assumption that contradicts the principles of the quantum mechanics. Heisenberg's principle of uncertainty tells us the particle cannot have a precise position and cannot be motionless. The best one can do, is to look for the position and/or the momentum of the particle with a probability $\psi^*\psi$ and the uncertainty $\Delta r \Delta p_r \geq \hbar$.

Singular concepts entail further singularities and divergences. It is true that in the scale-invariant and re-normalizable dynamical systems, one sweeps away most of the divergent terms and comes up with sensible recipes to do the everyday job. Nevertheless, sores persists. Even in the very prestigious QED, where the point particle notion creeps in through its ED- rather than its Q- component, after employing all re-normalization techniques, one resorts to arbitrary UV cut-offs or cannot rid oneself of logarithmic divergences. Moreover, there are non-scale invariant and non-re-normalizable systems to worry about. Isn't there any other way to circumvent the point particle concept?

Since the inception of Coulomb's law, inquisitive minds time and again have raised the issue of its singularity and have offered solutions, see e.g. [1], [2], [3], [4]. Even in recent years, long after the emergence of the scaling symmetry and the re-normalization techniques, [5] suggests constitutive properties to vacuum in the ultra-microscopic vicinity of charged particles. [6] expands on Podolsky's regularized electrodynamics and plasma-like vacuum.

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[7] propose a second gradient electromagnetostatics. The list is long and alternatives are often ad hoc. In this paper we argue that if the point-like charge is a non quantum concept, the solution to it should be a quantum mechanical one.

2) Asymmetric action of EM on Quantum wave fields - As examples, consider the Schroedinger equation for the hydrogen atom or the Dirac electron in the EM field. EM field *acts* on the quantum wave (QW) field of the electron, but QW of the electron does not *react* back. In [8]'s way of thinking, *"$\cdots$ it is contrary to the mode of thinking in science to conceive of a thing* (space time continuum in Einstein's case) *that acts itself, but which cannot be acted upon."*

3) Absence of restricted Lorentz gauge from U(1) - Quantum mechanics is said to be U(1) symmetric. That is, QW multiplied by an arbitrary phase factor, $\exp i\chi$, and EM written in the same Lorentz gauge, leave QW and EM invariant. The consequence is the minimal coupling of QW to EM through the vector potential, A_μ. One, however, has the option to require invariance under the restricted Lorentz gauge, ($\nabla^2\chi = 0$). This in turn invites in a coupling of QW to the derivatives $\partial_\nu A_\mu$. The U(1) symmetry gets enlarged.

In the paper we consider the Dirac electron. It possesses two fields, QW and EM. We stipulate, a la Einstein, a mutual *interaction* between the two by including an interaction term in the Lagrangian governing the dynamics of both fields. Quantum fields are distributed ones. Upon acting on other fields impart this feature to them, here the EM field. We show that the electric charge of the electron acquires a distribution with a finite density at the origin. The spinning electron, now with a distributed charge, develops a self induced magnetic dipole moment. Naturally the enlarged U(1) symmetry should have its conserved Noether charges. They show up in the anomalous g- factor of the electron, as even powers of the fine structure constant, α, because of the minimal coupling and, as odd powers because of the extra to minimal coupling.

II. DIRAC ELECTRON

The Lagrangian density to deal with is.

$$\begin{aligned} \mathcal{L} \; &= \; \bar{\psi}\left[\gamma_\mu\left(p_\mu - \frac{ze}{c}A_\mu\right) + \frac{1}{4}\kappa z\alpha^2 a_0\frac{e}{c}\gamma_\mu\gamma_\nu\partial_\nu A_\mu - imc\right]\psi \\ &\quad -\frac{1}{4}iz\frac{1}{c}F_{\mu\nu}F_{\mu\nu}, \end{aligned} \tag{1}$$

where γ_μ's are the Dirac matrices,

$$\bar{\psi} = \psi^\dagger\gamma_4,$$

$$x_\mu = (\mathbf{x}, x_4 = ict), \quad p_\mu = -ih\partial_\mu,$$

$$A_\mu = (\mathbf{A}, A_4 = iA_0), \;\; \mathbf{A} \text{ vector potential}, \quad A_0 \;\; \text{scalar potential},$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The first and fourth terms in (1) are the Lagrangian density of the free Dirac field. The last term is that of the free EM field. The second term is the conventional interaction responsible for the minimal coupling. The third term is yet another interaction. It is a legitimate 4- scalar constructed from γ's and derivatives of the vector potential. It is shown in [9] that it remains invariant under the restricted Lorentz gauge. z and κ are two, as yet unspecified, dimensionless coupling constants. They will be decided later by comparing the g- factor associated with the self induced magnetic moment of our electron with the laboratory measured one. The factor $\alpha^2 a_0$ in the fourth term and $1/c$ in the fifth term are so chosen to make all terms in $\mathcal{L}$ to have the same physical dimension.

Digression: The third term in (1), in the form of $\bar{\psi}\gamma_\mu\gamma_\nu\psi F_{\mu\nu}$, has precedence in the literature. In 1940's prior to QED, Pauli introduced it to account for the α- order term in the anomalous magnetic moment of the electron. Evidently, the restricted gauge symmetry of the added term was not known to Pauli and it was considered as an ad-hoc addition. In 1950's with the emergence of QED, Schwinger obtained the same gyromagnetic ratio and Pauli's addition went into oblivion. Here we promote Pauli's phenomenological term into a symmetry based interaction and talk of an enlarged U(1) symmetry with its own Noether charges.

The two Euler - Lagrange equations for ψ and $F_{\mu\nu}$ are,

$$\gamma_\mu \left(p_\mu - \frac{ze}{c} A_\mu \right) \psi - imc\psi + \frac{1}{4}\kappa z \alpha^2 a_0 e \frac{1}{c}\frac{\partial A_\mu}{\partial x_\nu} \gamma_\mu \gamma_\nu \psi = 0, \tag{2}$$

$$-\frac{\partial F_{\mu\nu}}{\partial x_\nu} = \partial^\alpha \partial_\alpha A_\mu = -ie\bar{\psi}\gamma_\mu \psi - i\frac{1}{4}\kappa \alpha^2 a_0 e \frac{\partial}{\partial x_\nu}\left(\bar{\psi}\gamma_\mu \gamma_\nu \psi \right). \tag{3}$$

See [9] for invariance of the κ-terms in (2) and (3) under the restricted Lorentz gauge. Noting that

$$\gamma_4 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{bmatrix} \quad \text{and} \quad \gamma_4\gamma_i = -i \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix},$$

one may split (3) into its scalar and vector components,

$$\begin{aligned} \partial^\alpha \partial_\alpha A_0 &= -e\psi^\dagger \psi \\ &\quad + i\frac{1}{4}\kappa\alpha^2 a_0 e \frac{1}{c}\frac{\partial}{\partial t}\left(\psi^\dagger \gamma_4 \psi \right) \\ &\quad - i\frac{1}{4}\kappa\alpha^2 a_0 e \frac{\partial}{\partial x_i}\left(\psi^\dagger \begin{bmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{bmatrix} \psi \right), \end{aligned} \tag{4}$$

$$\begin{aligned} \partial^\alpha \partial_\alpha A_i &= -e\psi^\dagger \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix} \psi \\ &\quad + i\frac{1}{4}\kappa\alpha^2 a_0 e \frac{1}{c}\frac{\partial}{\partial t}\left(\psi^\dagger \begin{bmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{bmatrix} \psi \right) \\ &\quad + \frac{1}{4}\kappa\alpha^2 a_0 e \; \epsilon_{ijk}\frac{\partial}{\partial x_j}\left(\psi^\dagger \begin{bmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{bmatrix} \psi \right). \end{aligned}$$

Equations (2) - (5) are mutually coupled and are non-linear. One may, however, employ an iteration scheme and obtain sensible solutions.

A. First order iteration carried out

For the starting EM field assume

$$A_0 = -\frac{e}{4\pi r}, \quad \mathbf{A} = 0, \quad \kappa = 0.$$

Equation (2) reduces to the classic equation for a hydrogen-like atom with relativistic spin 1/2 electron,

$$i\hbar \frac{\partial \psi}{\partial t} = \left[c\hbar \gamma_4 \gamma_i \frac{\partial}{\partial x_i} + \gamma_4 mc^2 - z\frac{e^2}{4\pi r} \right] \psi. \tag{5}$$

Equation (5) has been known since late 1920s. Its solutions can be found in the classical quantum mechanical and spectroscopic books, including [10] and [11]. Its ground state solution is

$$\psi = \left[\frac{N^2}{\pi(a_0/z)^3}\right]^{1/2} \exp\left(\frac{-zr}{a_0}\right)\left(\frac{2zr}{a_0}\right)^{-\beta^2/2}\left[\begin{array}{c}\chi \\ \frac{i\beta^2}{2z\alpha}\sigma_j\chi(\frac{x_j}{r})\end{array}\right], \tag{6}$$

where

$$\begin{aligned} \beta^2 &= 2\left(1-\sqrt{1-(z\alpha)^2}\right) = \alpha^2\left(1+\frac{1}{4}\alpha^2\right)+\mathcal{O}\left(\alpha^6\right), \\ N^2 &= \frac{2}{\Gamma(3-\beta^2)}\left(1-\frac{1}{4}\beta^2\right), \end{aligned}$$

The Pauli spinor χ is either

$$\chi^+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{or} \quad \chi^- = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The combinations $z\alpha$, a_0/z and $2zr/a_0$ appear frequently in the following calculations. To economize in writing we introduce the shorthand,

$$\bar{\alpha} = z\alpha, \quad \bar{a}_0 = \frac{a_0}{z}, \quad u = \frac{2zr}{a_0}, \quad \frac{\partial}{\partial x_j} = \frac{2}{\bar{a}_0}\frac{u_j}{u}\frac{\partial}{\partial u},$$

and rewrite (6) anew,

$$\psi = \left(\frac{N^2}{\pi\bar{a}_0^3}\right)^{1/2} e^{-u/2}u^{-\beta^2/2}\left[\begin{array}{c}\chi \\ \frac{i\beta^2}{2\bar{\alpha}}\sigma_j\chi(\frac{u_j}{u})\end{array}\right]. \tag{7}$$

Reduction of the scalar potential A_0

The ψ of (7) is an eigenfunction with an exponential time dependence. Therefore, the EM fields A_0 and A_i become time independent and $\partial^\alpha\partial_\alpha$ reduces to ∇^2. The first κ term in (4) vanishes because of the time independence of its argument. The second one survives. Upon substitution of (7) in (4), the remaining terms give,

$$\begin{aligned} \frac{\bar{a}_0^2}{4}\nabla^2 A_0 = \nabla_u^2 A_0 &= -\frac{\bar{a}_0^2}{4}e\psi^\dagger\psi \\ &= -\frac{N^2 e}{4\pi\bar{a}_0}\left[e^{-u}u^{-\beta^2} + \frac{1}{2}\kappa\alpha\beta^2\frac{\partial}{\partial u_j}\left(e^{-u}u^{-(1+\beta^2)}u_j\right)\right]. \\ &\approx -\frac{N^2 e}{4\pi\bar{a}_0}e^{-u}u^{-\beta^2} + \mathcal{O}(\alpha^3) \end{aligned} \tag{8}$$

Note that in the reduction of the κ term we have used the relation $\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}$. The right hand side of (8) is the charge density responsible for the generation of A_0. Its integral, the total charge, is of course $-e$. it tends smoothly to zero as $u \to 0$, a reminiscent of the rescaling technique practised in QED. Detail of the integration of (8) is given in [9]. The final result in terms of the Γ-functions is

$$A_0 = -\frac{(1+\bar{\alpha}^2)}{4\pi\bar{a}_0}e \times \left[\frac{1}{u}\Gamma(3-\bar{\alpha}^2, u) - \Gamma(2-\bar{\alpha}^2, u) + \Gamma(2-\bar{\alpha}^2) + 2\kappa\bar{\alpha}^2\frac{1}{u^2}\Gamma(3-\bar{\alpha}^2, u)\right]. \tag{9}$$

The far distance limit of (9) is

$$A_0 \Rightarrow -\frac{1+\bar{\alpha}^2}{4\pi\bar{a}_0}e\left[\frac{1}{u}\Gamma(3-\bar{\alpha}^2) + 2\kappa\bar{\alpha}^2\frac{1}{u^2}\Gamma(3-\bar{\alpha}^2)\right] \tag{10}$$

$$= -\frac{1}{4\pi\bar{a}_0}e\left(\frac{1}{u} + 2\kappa\bar{\alpha}^2\frac{1}{u^2}\right)$$

$$= -\frac{e}{4\pi r} - \kappa\bar{\alpha}^2\frac{e\bar{a}_0}{4\pi r^2} \qquad \text{as } r \Rightarrow \infty. \tag{11}$$

At far distances the potential is the conventional Coulomb monopole plus a small correction of $\mathcal{O}(\alpha^2)$.

The near distance behaviour of (9), is

$$A_0 \Rightarrow -\frac{1+\bar{\alpha}^2}{4\pi\bar{a}_0}e\left[\left(1 - \frac{2}{3}\frac{r^2}{\bar{a}_0^2}\right) + \bar{\alpha}^2\frac{2}{3}\frac{r^2}{\bar{a}_0^2}\left(-\ln(\frac{2r}{\bar{a}_0}) + \frac{5}{6}\right) + \kappa\frac{4}{3}\frac{r}{\bar{a}_0}\right].$$

$$\text{as } r \Rightarrow 0. \tag{12}$$

Near the center, at distances of the order of Bohr's radius there is a potential well of depth $-(1+\bar{\alpha}^2)e/4\pi\bar{a}_0$. Superimposed on the well, is the quadratic and the combination of quadratic-logarithmic terms. On letting $\bar{\alpha}^2 = 0$ one recovers the limiting behaviours of the non-relativistic spin-less particle case.

B. Reduction of the vector potential A_i

Preliminaries: We recall

$$u = \frac{2r}{\bar{a}_0}, \qquad \nabla^2 = \frac{4}{\bar{a}_0^2}\nabla_u^2, \qquad \frac{\partial}{\partial x_j} = \frac{2}{\bar{a}_0}\frac{u_j}{u}\frac{\partial}{\partial u},$$

where u_j is the jth component of $\mathbf{u} = 2\mathbf{r}/a_0$. In particular we note that, in the spherical polar coordinates,

$$\frac{u_1}{u} = \sin\theta\cos\phi = -\frac{1}{2}\left(\frac{8\pi}{3}\right)^{1/2}\left[Y_1^1(\theta,\phi) - Y_1^{-1}(\theta,\phi)\right],$$

$$\frac{u_2}{u} = \sin\theta\sin\phi = -\frac{1}{2i}\left(\frac{8\pi}{3}\right)^{1/2}\left[Y_1^1(\theta,\phi) + Y_1^{-1}(\theta,\phi)\right].$$

We also note the cartesian components of the unit vector $\hat{\phi}$,

$$\hat{\phi} = -\hat{x}\ \sin\phi + \hat{y}\ \cos\phi.$$

Returning to (5), the second term vanishes on account of its time independence. Substituting for ψ from (6) in the surviving terms one gets,

$$\nabla^2 A_i \;=\; \frac{N^2\beta^2}{\pi\bar{a}_0^3\bar{\alpha}}e\left[e^{-u}u^{-\beta^2} - \frac{1}{2}\kappa\bar{\alpha}\frac{\partial}{\partial u}\left(e^{-u}u^{-\beta^2}\right)\right]\epsilon_{ij3}\frac{u_j}{u}. \tag{13}$$

The term ϵ_{ij3} pops up in the course of reducing the following expression,

$$\chi^\dagger(\sigma_i\sigma_j)\chi = i\epsilon_{ijk}\chi^\dagger\sigma_k\chi = i\epsilon_{ij3}\chi^\dagger\sigma_3\chi = i\epsilon_{ij3}.$$

The last two equalities follow from the fact that χ's are chosen as the eigenspinors of σ_3. Noting that u_j is the jth Cartesian component of the vector $\mathbf{u}$, one finds

$$\epsilon_{123}\frac{u_2}{u} = \sin\theta\sin\phi \qquad \text{and} \qquad \epsilon_{213}\frac{u_1}{u} = -\sin\theta\cos\phi.$$

The vector potential has no z- component and in the (x, y)- plane lies in the ϕ- direction. Thus, we rewrite (13) as

$$\nabla^2\mathbf{A} = \frac{1}{c}\mathbf{J} = \frac{N^2\beta^2}{\pi\bar{a}_0^3\bar{\alpha}}e \times\left[\left(1 + \frac{1}{2}\kappa\bar{\alpha}\right)e^{-u}u^{-\beta^2} + \frac{1}{2}\kappa\bar{\alpha}\beta^2 e^{-u}u^{-(1+\beta^2)}\right]\sin\theta\hat{\phi}. \tag{14}$$

A detailed derivation of the self induced magnetic dipole field, $\mathbf{B} = \nabla\times\mathbf{A}$, and its close- and far- distance limits are spelled out in [9]. Here we only derive the magnetic dipole moment of the electron and the anomalous g- factor associated with it to compare its good agreement with the QED- derived and the laboratory measured values.

The 3-vector $\mathbf{J}$ defined in (14) is the current density responsible for the creation of the vector potential $\mathbf{A}$. It is a circular current in the (x, y)- plain. It of course satisfies the continuity equation, $\nabla.\mathbf{J} = \mathbf{0}$. The magnetic moment density associated with $\mathbf{J}$ is,

$$\begin{aligned}\mathbf{M}(u) &= \frac{1}{2c}\mathbf{r}\times\mathbf{J}(u) = \frac{\bar{a}_0}{4c}\mathbf{u}\times\mathbf{J}(u) \\ &= \frac{N^2\beta^2}{4\pi\bar{a}_0^2\bar{\alpha}}e\left[\left(1+\frac{1}{2}\kappa\bar{\alpha}\right)e^{-u}u^{-\beta^2}+\frac{1}{2}\kappa\bar{\alpha}\beta^2 e^{-u}u^{-(1+\beta^2)}\right]\sin\theta\ \mathbf{u}\times\hat{\phi}.\end{aligned} \tag{15}$$

Expressed in cartesian coordinates one finds,

$$\mathbf{u}\times\hat{\phi} = u(-\sin\theta\cos\theta\cos\phi\ \hat{x}+\sin\theta\cos\theta\cos\phi\ \hat{y}+\sin^2\theta\ \hat{z}).$$

The space integral of (15) is the total self induced magnetic moment of the electron. Its x and y components vanish on account of their ϕ- dependence. Its z component is,

$$\begin{aligned}\mu_e\hat{z} &= \int \mathbf{M}d^3x \\ &= \frac{1}{2}\bar{a}_0\bar{\alpha}e\left[1+\frac{1}{2}\kappa\bar{\alpha}-\frac{1}{3}\beta^2\right]\hat{z}, \\ &= \frac{1}{2}a_0\alpha e\left[1+\frac{1}{2}\kappa(z\alpha)-\frac{1}{3}(z\alpha)^2\right]\hat{z}.\end{aligned} \tag{16}$$

where we have expressed β^2 and N^2 in terms of α and z. The only approximation in (16) is on the right hand side of the last equality, where we have made the replacement,

$$\beta^2 \approx (z\alpha)^2\left(1+\frac{1}{4}(z\alpha)^2\right) \approx (z\alpha)^2.$$

1. *Determining the coupling constants, κ and z*

A glance at the the g- factor: There is a proportionality between the magnetic moment of a quantum particle and its total angular momentum. It is customary to measure the former in units of the Bohr magneton, $\mu_B = \frac{e\hbar}{2m_e c} = \frac{1}{2}a_0\alpha e$, and the latter in units of $\hbar$. Thus,

$$\frac{\mu}{\mu_B} = g\frac{S}{\hbar} \tag{17}$$

Laboratory measurements of [12] and theoretical QED derivations of [13] of the anomalous g-factor of electrons and muons agree to 12 decimal places and are considered as the stringiest test of the validity of QED and as a marvel of the experimental techniques. For our reference below, we quote the following, abstracted from [13] and [14]:

$$g = 2\left[1+\frac{1}{2}\left(\frac{\alpha}{\pi}\right)-0.328\ 478\left(\frac{\alpha}{\pi}\right)^2+1.181\ 241\left(\frac{\alpha}{\pi}\right)^3\cdots\right]. \tag{18}$$

The first order $\frac{1}{2}\left(\frac{\alpha}{\pi}\right)$ term is Schwinger's 1- loop calculation, (1948). The term $0.328(\alpha/\pi)^2 \approx 1.913 \times 10^{-6}$ comes mainly from the 2- loop QED calculations. There are, however, small contributions to it from the electroweak and hadronic light by light interactions, of the same order 10^{-6}.

Let us now look at the g-factor associated with the self- induced magnetic moment of our electron. By definition of (17) one gets,

$$\frac{\mu_e}{\mu_B} : \frac{1}{2} = g_e = 2\left[1 + \frac{1}{2}\kappa z \alpha - \frac{1}{3} z^2 \alpha^2\right].$$

It only suffices to let $\kappa = 1$ and $z = \frac{1}{\pi}$, and arrive at

$$g_e = 2\left[1 + \frac{1}{2}\left(\frac{\alpha}{\pi}\right) - \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2\right]. \tag{19}$$

Our g- factor in (19) is only a fisrt order partubative analysis. Its good concordence with (18), however, is striking:

- The odd power term $\frac{1}{2}(\frac{\alpha}{\pi})$ comes from the κ- coupling and is the same as the QED-derived term of Schwinger.

- The even power term $-\frac{1}{3}(\frac{\alpha}{\pi})^2$ is the contribution from the minimal coupling. It is the same as the 2- loop QED, but obtained here without the tedious loop calculations.

- The $-\frac{1}{3}(\frac{\alpha}{\pi})^2$ term in (19) differs from $-0.328(\frac{\alpha}{\pi})^2$ by 1.5%. One should, however, recall that there are contributions to latter from sources other than QED; see comments below (18).

- In further cycles of iteration we expect the pattern to repeat itself. That is, in an expansion of the g- factor in powers of $\frac{\alpha}{\pi}$, the κ- interaction to be responsible for the odd powers and the minimal coupling to give the even powers of $\frac{\alpha}{\pi}$ without interfering with each other. [1]

- Finally we note that each term in (19) is a Noether charge reflecting the symmetry associated with each term in the expansion of the gauge parameter of the minimal or the restricted gauge in powers of $\frac{\alpha}{\pi}$.

[1] The author recalls to have read (reference forgotten) a quotation that Feynman would have liked to see where the odd and even powers of α in the expansion of the anomalous magnetic moments of the particles come from.

III. CONCLUDING REMARKS

We have argued that there is a non- quantum concept in Maxwell's EM, the point charge assumption, which is the source of all EM and QED divergences. We have argued that a charged particle has two fields associated with it: an EM field and a QW one. We have conjectured a mutual action-reaction partnership between the two and have come up with a set of coupled EM and Dirac equations. The QW field shares its analyticity with the EM field.

The spinning electron acquires a distributed charge, a circular current and thereof a self-induced magnetic field. Deviations from Coulomb's law are noticeable at distances of the order of Bohr's radius. Its far distance limit remains Coulombian.

We have argued that the U(1) symmetry can be enlarged by requiring symmetry under the restricted Lorentz gauge in addition to the symmetry under an arbitrary gauge. This invites in a coupling of the QW to the derivatives of the EM vector potential in addition to the conventional minimal coupling to the vector potential itself.

We have attempted an iterative solutions of the coupled Dirac and Maxwell equations. In a first order analysis, we have derived the g- factor of the electron up to α^2 order, conformant with the QED derived ones but without the QED formalism.

In fact, at least in this limited scope of the anomalous g- factor, we are proposing an alternative to QED. And we think it is by far easier and more pedagogical than QED's loop analysis and Feynman's integrals. And by far much easier to communicate with novices in physics.

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