A&A manuscript no.
(will be inserted by hand later)

Your thesaurus codes are:
03 A&A Section 3: dark matter, missing mass, alternative dynamics, $f(R)$ gravitation

ASTRONOMY AND ASTROPHYSICS
March 16, 2006

An $f(R)$ gravitation instead of dark matter

Y. Sobouti

Institute for Advanced Studies in Basic Sciences, P. O. B. 45195-1159, Zanjan, Iran
email: sobouti@iasbs.ac.ir

14, March 2006

Abstract. We propose an action-based $f(R)$ modification of Einstein's gravity which admits of a modified Schwarzschild - deSitter metric. In the weak field limit this amounts to adding a small logarithmic correction to the newtonian potential. A test star moving in such a spacetime experiences an excess logarithmic potential leading to a constant asymptotic speed at large distances. This speed turns out to be proportional to the fourth root of the mass of the central body in compliance with the Tully-Fisher relation. A variance of MOND's gravity emerges as an inevitable consequence of the proposed formalism.

1. Introduction

Convinced of cosmic speed up and finding the dark energy hypotheses not a compelling explanation, some cosmologists have looked for alternatives to Einstein's gravitation (Deffayet et al, 2002, Freese et al, 2002, Ahmed et al, 2002, Dvali et al, 2003, Capozziello et al, 2003, Carroll et al, 2003, Norjiri et al, 2003, 2004, and 2006, Das et al, 2005, Sotiriou, 2005 and Woodard, 2006). There is a parallel situation in galactic studies. Dark matter hypotheses, intended to explain the flat rotation curves of spirals or the large velocity dispersions in clusters of galaxies, have raised more questions than answers. Alternatives to newtonian dynamics have been proposed but have had their own critics. The foremost among such theories, the Modified Newtonian Dynamics (MOND) of Milgrom (1983 a,b,c) is capable of explaining the flat rotation curves (Sandres et al, 1998 and 2002) and of justifying the Tully-Fisher relation with considerable success. But it is often criticized for the lack of an axiomatic foundation; see, however, Bekenstein's (2004 TeVeS theory where he attempts to provide such a foundation by introducing a tensor, a vector, and a scalar field into the field equations of GR.

Here we are concerned with galactic problems. We suggest to follow cosmologists and look for a modified Einstein gravity tailored to suit galactic environments.

Send offprint requests to: Y. Sobouti

2. A modified field equation

Alternative gravitations are often sought through modifications of the conventional Einstein-Hilbert action. Let us begin with the following

$$S = \int \left[\frac{1}{2}f(R) + L_m\right]\sqrt{-g}d^4x, \qquad (1)$$

where R is the Ricci scalar, L_m is the lagrangian density of the matter, and $f(R)$ is an, as yet, unspecified but differentiable function of R. Variations of S with respect to the metric tensor leads to the following field equation (Capozziello et al, 2003)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\frac{f}{h} = \left(h_{;\mu\nu} - h^{\lambda}_{;\lambda}g_{\mu\nu} + T_{\mu\nu}\right)\frac{1}{h}, \qquad (2)$$

where $h = df/dR$ and $T_{\mu\nu}$ is the energy momentum tensor of the matter field. The case $f(R) = R + constant$ and $h = 1$ gives the Einstein field equation with cosmological constant included in it. For the purpose of galactic studies we envisage a spherically symmetric static dust or perfect fluid with a standard Schwarzschild-like metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \qquad (3)$$

From Eqs (2) and (3) one obtains

$$\frac{B'}{B} + \frac{A'}{A} = -r\frac{h''}{h} + \frac{1}{2}r\left(\frac{B'}{B} + \frac{A'}{A}\right)\frac{h'}{h} + r(\rho+p)\frac{A}{h}, \qquad (4)$$

$$\frac{B''}{B} - \frac{1}{2}\left(\frac{B'}{B} - \frac{2}{r}\right)\left(\frac{B'}{B} + \frac{A'}{A} + 2\frac{h'}{h}\right) - \frac{2}{r^2} + \frac{2A}{r^2} = 0, \qquad (5)$$

$$\frac{B''}{B} - \frac{1}{2}\frac{B'}{B}\left(\frac{B'}{B} + \frac{A'}{A}\right) - \frac{2}{r}\frac{A'}{A} = -(\rho-p)\frac{A}{h} + A\frac{f}{h} - \left(\frac{B'}{B} + \frac{4}{r}\right)\frac{h'}{h}, \qquad (6)$$

$$R = 2\frac{f}{h} - \frac{3}{A}\left[\frac{h''}{h} + \left\{\frac{1}{2}\left(\frac{B'}{B} - \frac{A'}{A}\right) + \frac{2}{h}\right\}\frac{h'}{h}\right] - \frac{1}{h}(\rho - 3p). \qquad (7)$$

where $\rho(r)$ and $p(r)$ are the rest frame mass density and presume of the matter, respectively. Equation (4) is the combination $R_{tt}/B + R_{rr}/A$, Eq (5) is $R_{tt}/B - R_{\theta\theta}/r^2$, and Eq (6) is the rr-component of the field equation. Finally, Eq (7) is from the contraction of Eq (2). In principle, for a given ρ, p, and h (or$f$) one should be able to solve the four Eqs (4)-(7) for the four unknowns, A, B, R, and f (or h) as functions of r. We are interested in those solutions of Eqs (4)-(7) that differ from the classic solutions of Einstein by small amounts. To arrive at this we assume a parameterized function $h(r,\alpha)$, such that $h(r,\alpha\to 0)\to 1$. We then solve Eqs (4)-(7) up to first order in α.

3. Exterior solutions

Assume $\rho = p = 0$. If the combination $B'/B + A'/A$ is a well behaved differential expression, it should have a solution of the form $A(r)B(r) = g(r)$. Furthermore $g(r)$ should differ from 1 only by a small amount, in order to have the classic GR as a limit. There are a host of possibilities. For the sake of argument let us assume $g(r) = r^\alpha \approx 1 + \alpha \ln r$, α small. Equation (4) splits into

$$\frac{B'}{B} + \frac{A'}{A} = \frac{\alpha}{r}, \quad AB = r^\alpha, \tag{8}$$

$$h'' - \frac{1}{2}\frac{\alpha}{r}h' + \frac{\alpha}{r^2}h = 0. \tag{9}$$

Equation (9) has the solution $h = r^\beta$, $\beta = \alpha + O\left(\alpha^2\right)$ and $1 - \frac{1}{2}\alpha + O\left(\alpha^2\right)$. Of these, the solution $h \approx r^\alpha$ satisfies the requirement $h \to 1$ as $\alpha \to 0$. The second solution is discarded. Substituting $AB = h = r^\alpha$ in Eq (5) gives.

$$\frac{1}{A} = \frac{1}{(1-\alpha)}\left[1 - \left(\frac{s}{r}\right)^{(1-\alpha/2)} + \lambda\left(\frac{r}{s}\right)^{2(1-\alpha/2)}\right], \tag{10}$$

$$B = \left(\frac{r}{s}\right)^\alpha \frac{1}{A}, \tag{11}$$

where s and λ are constants of integration. For $\alpha = 0$, Eqs (10) and (11) are recognized as the Schwarzschild - deSitter metric. Therefore, s is identified with the Schwarzschild radius of a central body, $2GM/c^2$ and λ with a dimensionless cosmological constant. Also we have made B dimensionless by inserting s in Eq (11). This is always possible by re scaling the time coordinate in Eq (3) by any arbitrary constant factor. Substitution of Eqs (10) and (11) in Eqs (6) and (7) gives

$$f = \frac{3}{(1-\alpha)}\left[\alpha\left(\frac{s}{r}\right)^{2(1-\alpha/2)} + (2+\alpha)\lambda\right], \tag{12}$$

$$R = \frac{3}{(1-\alpha)}\left(\frac{s}{r}\right)^\alpha\left[\alpha\left(\frac{s}{r}\right)^{2(1-\alpha/2)} + (4-\alpha)\lambda\right]. \tag{13}$$

The Ricci scalar of the Schwarzschild space is zero. That of the de Sitter or the Schwarzschild - deSitter space is constant. For non zero α, however, R is somewhere between these two extremes. At small distances it increases as $(s/r)^{-2}$ and at large r's it behaves as $(s/r)^\alpha \approx 1 - \alpha \ln r/s$. Cosmologists may find this variable Ricci scalar of relevance to their purpose (see also Brevik et al, 2004, for a different modification of Schwarzschild - deSitter metric). Another point; we began with f as a function of R rather than r. Elimination of r between Eqs (12) and (13) provides one in terms of the other. Thus,

$$R = (3\alpha)^{-\alpha/2}[f + 6\lambda][f - 3(2+3\alpha)\lambda]^{\alpha/2}. \tag{14}$$

In the limit $\alpha \to 0$, one recovers the classical value, $f = R - 6\lambda$. For $\lambda = 0$, Eq (15 is easily invertible

$$f = (3\alpha)^{\alpha/2}R^{(1-\alpha/2)} \approx R[1 - \frac{\alpha}{2}\ln R + \frac{\alpha}{2}\ln(3\alpha)]. \tag{15}$$

Once more we observe the mild logarithmic correction to the classic GR.

4. Orbits in the spacetime of Eqs (10)-(13)

We assume a test star orbiting a central body specified by its Schwarzschild radius, $2GM/c^2$. We choose the orbit in the plane $\theta = \pi/2$. The geodesic equations for r, φ and t are

$$\frac{d^2r}{d\tau^2} + \frac{1}{2}\frac{A'}{A}\left(\frac{dr}{d\tau}\right)^2 - \frac{r}{A}\left(\frac{d\varphi}{d\tau}\right)^2 + \frac{1}{2}\frac{B'}{B}\left(\frac{dt}{d\tau}\right)^2 = 0, \tag{16}$$

$$\left(\frac{d\varphi}{d\tau}\right)^{-1}\frac{d^2\varphi}{d\tau^2} + \frac{2}{r}\frac{dr}{d\tau} = 0, \tag{17}$$

$$\left(\frac{dt}{d\tau}\right)^{-1}\frac{d^2t}{d\tau^2} + \frac{B'}{B}\frac{dr}{d\tau} = 0, \tag{18}$$

respectively. Equations (17) and (18) immediately integrate into

$$r^2 d\varphi/d\tau = J, \quad \text{constant}, \tag{19}$$

$$dt/d\tau = 1/B. \tag{20}$$

Substituting the latter in Eq (16) and assuming a circular orbit, $dr/d\tau = 0$, gives

$$\frac{J^2}{r^3} = \frac{1}{2}\frac{AB'}{B^3} = \frac{1}{2}\left(\frac{r}{s}\right)^\alpha \frac{B'}{B^4}, \tag{21}$$

where we have used Eq (11) to eliminate A. In galactic environment what one measures as the circular orbital speed is

$$v = \frac{r d\varphi}{\sqrt{B}dt} = \frac{r}{\sqrt{B}}\frac{d\varphi}{d\tau}\frac{d\tau}{dt} = \frac{\sqrt{B}J}{r}. \tag{22}$$

Eliminating J between Eqs (22) and (21) gives

$$v^2 = \frac{1}{2}\left(\frac{r}{s}\right)^\alpha \frac{rB'}{B^3}. \tag{23}$$

Further substitution for B from Eqs (11) and (10) yields

$$v^2 = \frac{1}{2}(1-\alpha)^2\left(\frac{r}{s}\right)^{-\alpha} \times$$

$$\frac{\left[\alpha + \left(1-\frac{1}{2}\alpha\right)\left(\frac{s}{r}\right)^{(1-\alpha/2)} + 2(1-\frac{1}{2}\alpha)\lambda\left(\frac{r}{s}\right)^{2(1-\alpha/2)}\right]}{\left[1-\left(\frac{s}{r}\right)^{(1-\alpha/2)} + \lambda\left(\frac{r}{s}\right)^{2(1-\alpha/2)}\right]^3} \quad (24)$$

To put Eq (24) in a tractable form:

– We neglect the λ term and substitute $s = 2GM/c^2$.
– We adopt the approximation $x^{-\alpha} = \exp(-\alpha \ln x) = 1 - \alpha \ln x + O\left(\alpha^2\right)$.
– The terms containing s are much small. We retain only the first order terms in the reduction of Eq (24).
– v is measured in units of c. We restor it hereafter.

With these provisions Eq (24) reduces to

$$v^2 = \frac{1}{2}\alpha c^2 + \frac{GM}{r}\left[1 + \frac{1}{2}\alpha\left\{1 + \ln\left(\frac{2GM}{c^2 r}\right)\right\}\right]. \quad (25)$$

A plot of v^2 as a function of r has the horizontal asymptote $\frac{1}{2}\alpha c^2$. This asymptote, however, cannot be a universal constant. For it will beat the intuition to imagine that a galaxy and a speck of dust dictate the same speed on faraway passerby objects. The parameter α should depend on the mass of the gravitating body residing at the origin. For any localized matter seen from far enough distances will betray no characteristics other than its mass. To find this mass dependence we resort to observations.

From Sanders and Verheijn (1998) and Sanders and Mc Gaugh (2002) we have compiled a list of forty spirals for which total masses, asymptotic orbital speeds, and velocity curves are reported. The figures in their papers contain the observed circular speeds and the newtonian ones derived from the observed mass of the stellar and HI components of the galaxies. We have selected those objects from Sanders et al. which a) have a noticeable horizontal asymptote and b) have fairly reduced newtonian speeds by the time the flat asymptote is approached. With the rough assumption that the mass of the spirals are distributed spherically symmetrically, we have calculated $\alpha = 2v_\infty^2/c^2$ and $\alpha(M/M_\odot)^{-1/2}$. These are reported in columns 5 and 6 of Table 1. Figure 1 is a histogram of the data for $\alpha(M/M_\odot)^{-1/2}$. On the left hand side, NGC6446 and NGC6946 have anomalously large HI masses and correspondingly low $\alpha(M/M_\odot)^{-1/2}$ values 2.30 and 2.45, respectively. On the right hand side, NGC3953, NGC3893, NGC4085, and NGC3972 have no (reasonably) declining newtonian velocity curves and have, anomalously large $\alpha(M/M_\odot)^{-1/2}$ values of 3.87, 3.60, 3.74, and 3.77, respectively. Discarding these exceptional cases we find a narrowly peaked maximum centered at $(3.05 \pm 0.19) \times 10^{-12}(M/M_\odot)^{1/2}$. The main sources of uncertainty in this value are a) the estimates of the total masses of the galaxies, b) the judgment whether what one measures as the asymptotic speed is indeed the orbital speed at the far outskirts of the galaxy and c) our heuristic assumption that the galaxies can be treated as spherically symmetric objects. Considering these sources of errors, the emergence of the narrow peak in the histogram with low dispersion of 0.19×10^{-12} is significant and is an indication of the fact that the dependence of the asymptotic speed on $M^{1/4}$ is robust. We dare conjecture that this mass dependence is exact. The proportionality constant may, however, be revised upon the availability of more accurate information. We will come back to this issue shortly.

Let us recapitulate the findings so far.

- We have demonstrated that a modified $f(R) = R^{(1-\alpha/2)}$ gravity can produce a flat rotation curve at faraway distances from a gravitating body. And
- The fourth power of the asymptotic speed is proportional to the mass of the central object. The latter is, of course, observation-based and is adopted as an empirical rule. The Tully-Fisher relation, the proportionality of v^4 and M, emerges as a consequence of these two features.

5. Kinship with MOND

The features just narrated are also shared by MOND theory. Below we show that some version of MOND can actually be derived from the present formalism. We recall that in the weak field approximation, newtonian dynamics is derived from the einsteinian one by writing the metric coefficient $B = \left(1 + 2\phi/c^2\right), \phi = \frac{GM}{r}$ and expanding all relevant quantities and equations up to first order in ϕ/c^2. In a similar way one may find a modified newtonian dynamics from the present modified GR by expanding $B(r)$ of Eq (11) up the first order in α and s/r. Thus

$$B(r) = 1 + \alpha + \alpha \ln(r/s) - s/r = 1 + 2\phi(r)/c^2, \quad (26)$$

where the second equality defines $\phi(r)$. Let us write $\alpha = \alpha_0(GM/GM_\odot)^{1/2}$ and find the gravitational acceleration

$$\begin{aligned} g &= |d\phi/dr| = (a_0 g_n)^{1/2} + g_n \quad (27)\\ &= g_n \quad \text{for} \quad g_n \gg a_0 \\ &= (a_0 g_n)^{1/2} \quad \text{for} \quad a_0 \gg g_n \to 0, \end{aligned}$$

where we have denoted

$$a_0 = \alpha_0^2 c^4/4GM_\odot \quad \text{and} \quad g_n = GM/r^2. \quad (28)$$

The limiting behaviors of g are the same as those of MOND. One may then comfortably identify a_0 as MOND's characteristic acceleration and calculate α_0 anew from Eq (28). For $a_0 = 1.2 \times 10^{-8} cm/sec^2$, one finds

$$\alpha = 2.8 \times 10^{-12}\left(M/M_\odot\right)^{1/2}. \quad (29)$$

It is gratifying how close this value of α is to the one obtained from the histogram and how similar MOND and the present formalism are, in spite of their totally different and independent starting points.

6. Anomalous acceleration of Pioneers 10 and 11

By Eq (27) a spaceship in a solar bound orbit should experience an excess acceleration $\left(a_0 G M_\odot / r^2\right)^{1/2}$ towards the sun. This comes from the logarithmic term of Eq (26) and its numerical values are

$$a_{ex}(20\text{au}) = 4.2 \times 10^{-6}\text{cm/sec}^2$$
$$a_{ex}(70\text{au}) = 1.2 \times 10^{-6}\text{cm/sec}^2$$

Pioneers 10 and 11 are reported (Anderson et al, 1998, and 2002) to have experienced an anomalous acceleration of 8.74×10^{-8}cm/sec^2 directed towards the sun and unaccounted for with the known dynamics of the solar system. It is also claimed that the anomaly is the same for both Pioneers and remains almost constant over the distances of 20-67 au develed by the spacecraft. The excess acceleration of $1.2-4.2 \times 10^{-6}$ cm/sec^2 is larger than the Pioneer anomaly by factors of 15-50. Its r^{-1} distance dependence is not in accord with the constancy of the latter either. However, keeping in mind the multitude and complexities of the factors involved in the determination of the orbits of spaceships, it is worth considering the possibility of an explanation in term of the formalism developed in this paper.

7. Concluding remarks

We have developed an $f(R) = R^{1-\alpha/2}$ gravitation which is essentially a logarithmic modification of Einstein Hilbert action. In spherically symmetric static situations the theory admits of a modified Schwarzschild - deSitter metric. The latter in the limit of weak fields gives a logarithmic correction to the newtonian potential. From the observed asymptotic speeds of galaxies we learn that the correction is proportional to the square root of the mass of the central body. Flat rotation curves, the Tully-Fisher relation and a version of MOND emerge as natural consequences of the theory.

There are two practices to obtain the field equations of $f(R)$ gravity, the metric approach, where $g_{\mu\nu}$ alone is considered as dynamical variables and the Palatini approach, where the metric together with the affine connections are treated as such. Unless $f(R)$ is linear in R, the resulting field equations are not identical. The metric approach is often shied away from, for its leading to fourth order differential equations. It is also believed to have instabilities in the weak field approximations (see e. g., Sotiriou, 2005 and also Amarzguioui et al, 2005). In the present paper we do not initially specify $f(R)$. Instead, at some intermediate stage in the analysis we adopt an ansatz for $df(R)/dR$ as a function of r and work forth to obtain the metric, R, and eventually $f(R)$. This enables us to avoid the fourth order equations. The trick should work in other contexts, cosmological, say.

The theory presented here is preliminary and further investigations are required in order to provide a more complete analysis; for instance, the matching of the modified exterior Schwarzschild - deSitter solution to proper interior solutions as well as the cosmological aspects of this theory need to be studied.

Acknowledgement: The author wishes to thank Professor B. Mashhoun for comments and helpful suggestions.

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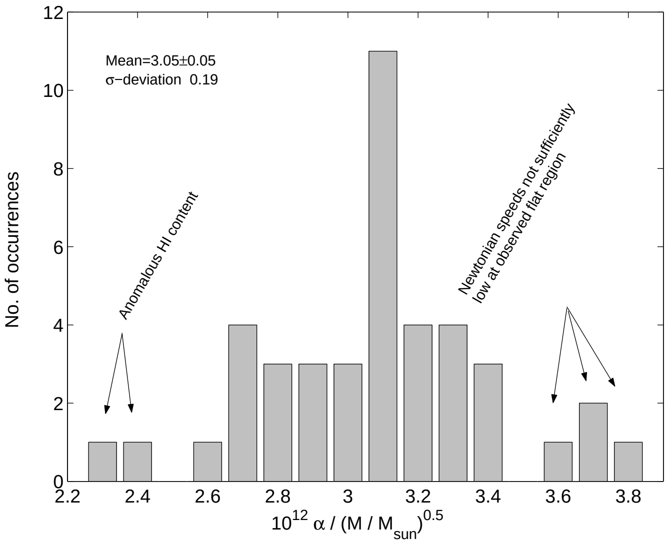

Fig. 1. A histogram of $\alpha(M/M_\odot)^{-1/2}$. Two cases on the left hand side and four on the right hand have exceptional velocity curves. With or without these six cases the mean value is 3.05. The dispersion 0.19, however, is calculated without them.

galaxy	r (kpc)	$M_{\rm tot} = M_{*+HI}$ ($10^{10} M_\odot$)	v_∞ ($\frac{km}{sec}$)	$\alpha = 2\frac{v_\infty^2}{c^2}$ (10^{-7})	$\frac{\alpha}{\sqrt{\frac{M}{M_\odot}}}$ (10^{-12})
NGC5533	72.0	22.0	250	13.9	2.96
NGC3992	30.0	16.22	242	13.0	3.23
NGC3953	11.7	8.17	223	11.1	3.89
NGC5907	32.0	10.8	214	10.2	3.10
NGC2998	48.0	11.3	213	10.1	3.00
NGC801	60.0	12.9	208	9.81	2.68
NGC5371	40.0	12.5	208	9.61	2.71
NGC3893	17.5	4.76	188	7.85	3.61
NGC4157	26.0	5.62	185	7.61	3.21
NGC4217	14.5	4.50	178	7.04	3.32
NGC4013	27.0	4.84	177	6.96	3.16
NGC4088	18.8	4.09	173	6.65	3.29
NGC3877	10.5	3.49	167	6.20	3.32
NGC4100	19.8	4.62	164	5.98	2.78
NGC3726	28.0	3.24	162	5.83	3.30
NGC6946	30.0	5.4	160	5.69	2.45
NGC4051	10.6	3.29	159	5.62	3.10
NGC4138	13.0	3.01	147	4.82	2.77
NGC3917	13.0	1.58	135	4.05	3.22
NGC4085	5.4	1.13	134	3.99	3.75
NGC2403	19.0	1.57	134	3.99	3.18
NGC3972	7.6	1.12	134	3.99	3.77
UGC128	40.0	1.48	131	3.81	3.13
NGC4010	9.0	1.13	128	3.64	3.42
NGC3769	33.0	1.33	122	3.31	2.87
NGC6503	21.8	1.07	121	3.25	3.14
NGC4183	18.0	0.93	112	2.79	2.98
UGC6917	9.0	0.74	110	2.69	3.13
UGC6930	14.5	0.73	110	2.69	3.15
M33	9.0	0.61	107	2.54	3.30
UGC6983	13.8	0.86	107	2.54	2.74
NGC7793	6.8	0.51	100	2.22	3.11
NGC300	12.4	0.35	90	1.80	3.05
NGC5585	12.0	0.37	90	1.80	2.97
NGC6399	6.8	0.28	88	1.72	3.1
NGC55	10.0	0.23	86	1.64	3.42
NGC6667	6.8	0.33	86	1.64	2.86
NGC6446	14.2	0.42	82	1.49	2.3
UGC6923	4.5	0.24	81	1.46	2.98
NGC7039	8.0	0.21	79	1.39	3.03
UGC6818	6.0	0.14	73	1.18	3.16
IC2573	8.0	0.077	66	0.97	3.49
NGC300	12.4	0.35	90	1.80	3.05
NGC5585	12.0	0.37	90	1.80	2.97
NGC6399	6.8	0.28	88	1.72	3.1
NGC55	10.0	0.23	86	1.64	3.42
NGC6667	6.8	0.33	86	1.64	2.86
NGC6446	14.2	0.42	82	1.49	2.3
UGC6923	4.5	0.24	81	1.46	2.98
NGC7039	8.0	0.21	79	1.39	3.03
UGC6818	6.0	0.14	73	1.18	3.16
IC2573	8.0	0.077	66	0.97	3.49