IAU 8th Asia-Pacific Regional Meeting
ASP Conference Series, Vol. 289, 2003
S. Ikeuchi, J. Hearnshaw & T. Hanawa, eds.

Flux Tube Oscillations and Coronal Heating

Y. Sobouti[1,2], K. Karami[1] and S. Nasiri[1,3]

1) Institute for Advanced Studies in Basic Sciences, P.O. Box 45195-159, Zanjan, Iran

2) Center for Theoretical Physics and Mathematics, AEOI, P.O. Box 11345-8486, Tehran, Iran

3) Department of Physics, Zanjan University, Zanjan, Iran

Abstract. Wave transmission in low β magnetic flux tubes has, mathematically, the same structure as the propagation of electromagnetic waves in optical fibers. In both cases the problem is reducible to a single wave equation for the longitudinal component of the perturbed field along the fiber/tube axis. We derive this equation, solve the dispersion relation associated with it, and assign three wave numbers to each mode. In cylindrical coordinates (r, φ, z), for a given φ-wave number, the plane of the r- and z- wave numbers is divided into one "mode zone" in which each grid point is a possible mode of the system and one "forbidden zone" in which no mode may dwell. The cutoff line, the boundary of the two zones, is given both analytically and numerically. Next we introduce weak resistive and viscous dissipation to the system, solve for the decay time of each mode and for the densities of heat generation rates by each dissipative process. The two densities have identical spatial dependencies, but different magnitudes. The resistive heat rate is inversely proportional to the Lundquist number, S, and the viscous one to the Reynolds number, R. The time decay exponent is proportional to the sum $(S^{-1} + R^{-1})$.

1. Introduction

The astronomical literature on waves in magnetic flux tubes has a back log of a quarter of a century. Ionson (1978), Wentzel (1979 a,b), Wilson (1979), Roberts (1981 a,b), Edwin and Roberts (1983), Hollweg (1984), Steinolfson et al. (1986), Davila (1987), Steinolfson and Davila (1993), and Ofman et al. (1994, 1995) have all addressed various aspects of the problem. The analysis of TRACE data by Nakariakov et al. (1999), however, has given a new impetus to such studies. Convincing evidence has emerged that the coronal loops can and do oscillate in matters of few hundreds of seconds and do heat up in the course of damping of the wave motions. Here, we are primarily interested in the mathematical and analytical properties of modes in magnetic flux tubes and of their decay by resistive and viscous processes.

2. Exposition of the Problem

Let the flux tube be a cylinder of radius $r = 1$, of length πL, and lie along the z-axis of a cylindrical coordinate system (r, φ, z). Under coronal conditions assume i) the plasma pressure is negligible in comparison with the magnetic one. ii) The scale height is much larger than the height of the flux tube, so that the density stratification can be neglected. iii) The space is pervaded by a constant magnetic field along the z-axis. iv) The plasma density, ρ, has the constant values ρ_i and ρ_e inside and outside of the cylinder, but varies discontinuously at $r = 1$. Let the system undergo a small perturbation about its equilibrium state. The perturbation induced velocity and magnetic fields, $\delta\mathbf{v}(r, \varphi, z)$ and $\delta\mathbf{B}(r, \varphi, z)$, respectively, are governed by the following equations.

$$\frac{\partial \delta\mathbf{v}}{\partial t} = \frac{1}{4\pi\rho}(\nabla \times \delta\mathbf{B}) \times \mathbf{B}, \qquad \frac{\partial \delta\mathbf{B}}{\partial t} = \nabla \times (\delta\mathbf{v} \times \mathbf{B}). \tag{1}$$

The first of Eq. (1) has no z-component. We write out the z- and transverse components of the second one and eliminate the transverse components of $\delta\mathbf{v}$ and $\delta\mathbf{B}$ in favor of δB_z. For an exponential z-, φ- and time dependence, $e^{i(lz/L+m\varphi-\omega t)}$, ℓ and m integers, one obtains the following Bessel's equation for δB_z:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + k^2 - \frac{m^2}{r^2}\right)\delta B_z(r) = 0, \qquad k^2 = \left(\frac{\omega}{v_A}\right)^2 - \frac{\ell^2}{L^2}, \tag{2}$$

where $v_A^2 = [B^2/4\pi\rho]^{\frac{1}{2}}$ is the Alfven speed. Both v_A and k are different inside and outside of the flux tube; for ρ is. For $k_i^2 = (\omega/v_{Ai})^2 - (\ell/L)^2 > 0$ in $r < 1$ and $k_e^2 = (\ell/L)^2 - (\omega/v_{Ae})^2 > 0$ in $r > 1$, solutions of Eq. (3) are

$$\begin{aligned} \delta B_z(r) &= J_m(k_i r), \quad & r < 1 \\ &= AK_m(k_e r), \quad & r > 1, \end{aligned} \tag{3}$$

where K_m is the modified Bessel function and A is a constant to be determined. In terms of δB_z the transverse components, $\delta\mathbf{B}_\perp$ and $\delta\mathbf{v}_\perp$ are

$$\delta\mathbf{B}_\perp = -\frac{\ell}{L\omega}B\delta\mathbf{v}_\perp = -i\frac{\ell}{Lk}\nabla_\perp(\delta B_z), \tag{4}$$

where $\nabla_\perp$is the gradient operator in the (r, φ) plane.

3. Boundary Conditions and Dispersion Relation

To avoid shock waves at $r = 1$, the Lagrangian change, or equivalently in this case of constant B, the Eulerian change in pressure, $\delta p \propto B\delta B_z$, must be continuous. ii) From $\nabla.\delta\mathbf{B} = 0$, δB_r must be continuous at $r = 1$. Applying these conditions to solutions of Eqs. (3) and (4), with a change in notation, $k_i = x$ and $k_e = y$, gives

$$\frac{1}{x}\frac{J'_m(x)}{J_m(x)} = \frac{1}{y}\frac{k'_m(y)}{k_m(y)}, \quad y^2 = c_l^2 - \frac{\rho_e}{\rho_i}x^2, \quad c_l^2 = \left(1 - \frac{\rho_e}{\rho_i}\right)\left(\frac{\ell}{L}\right)^2. \tag{5}$$

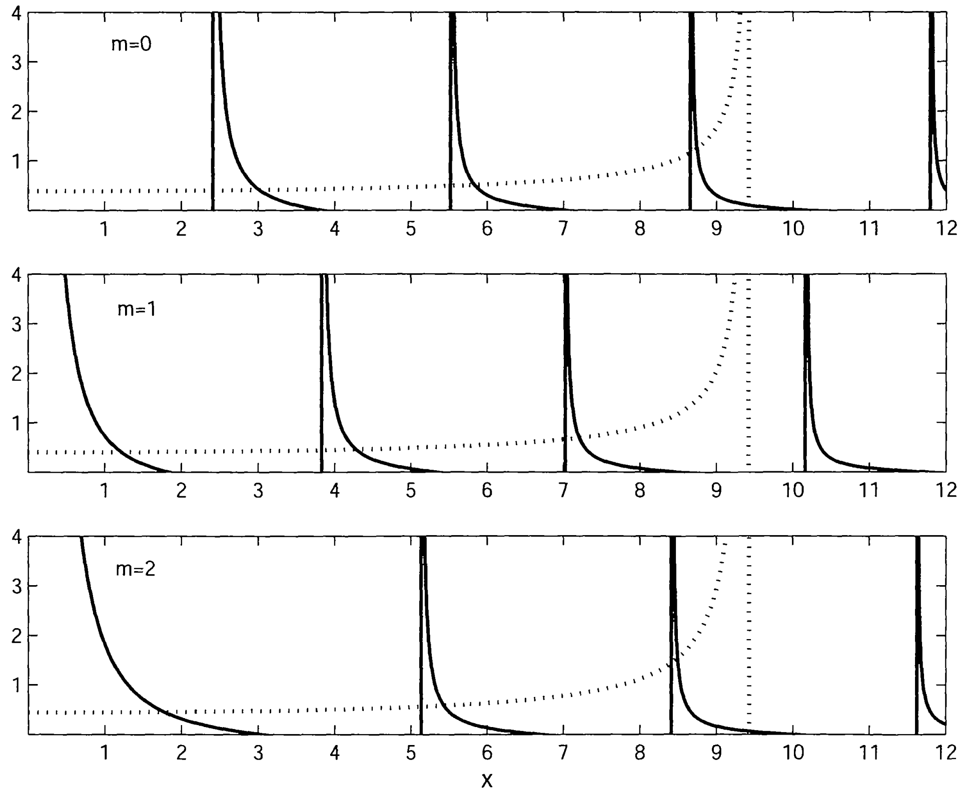

Figure 1. The plots of left and right sides of Eq. (5), solid and dotted curves, respectively, as functions of x for $m = 0, 1$ and 2. Auxiliary parameters are $l = 100$, $C_{100}^2 = 8.9$, radius $= 10^3$ km, height $= 10^5$ km, and $\rho_e/\rho_i = 0.1$. Intersections of solid and dotted curves, x_{nml}, are the eigenvalues.

This is the dispersion relation. Its solution for $x = k_i = [(\omega/v_{Ai})^2 - (\ell/L)^2]^{\frac{1}{2}}$ will give the eigen-frequencies, ω. In Fig. 1, the solid multi-branched curves are the left hand side of Eq. (5). They intersect the x-axis at zeros of $J_m'(x)$ and go to infinity at zeros of $J_m(x)$. The dotted curve is the right hand side of Eq. (5). It tends to infinity at $y = 0$. Intersections of the dotted curve with the multi - branched solid curves are the desired eigenvalues, x_{nml}. They sit between the nth zeros of $J_m'(x)$ and $J_m(x)$, γ_{nm}' and γ_{nm}, respectively. Thus

$$x_{nml} = \frac{1}{2}(\gamma_{mn-1} + \gamma_{mn}') + \alpha_\ell \pi/2 \approx (2n + m - 2 + \alpha_\ell)\pi/2 < x_{\max}, \qquad (6)$$

where $-\frac{1}{2} < \alpha_\ell < \frac{1}{2}$ is to be evaluated numerically, and $x_{\max} = \frac{\ell}{L}(\frac{\rho_i}{\rho_e} - 1)$ is the abscissa of the dotted asymptote in Fig. 1, where $y(x_{\max}) = 0$. The inequality in Eq. (6) utilizes the asymptotic values of the roots of J_m and J_m'.

Cutoffs: From the inequality of Eq. (6) one obtains

$$0 < n < 1 + \frac{2\ell}{\pi L}\sqrt{\frac{\rho_i}{\rho_e} - 1} + \frac{\alpha_\ell}{2} - \frac{m}{2}. \qquad (7)$$

For a given m and ℓ there is an upper cutoff to n. Vice versa, for a given n and m there is a lower cutoff for ℓ. Thus, for a given m, the wave number plane (n, ℓ) divides into two regions, a "mode zone" in which the possible modes of the flux tube reside, and a "forbidden zone" in which no mode can dwell. We recapitulate the findings of this sections. The eigenvalue problem for wave propagation in zero-β flux tube reduces to solving a Bessel's equation for the z-components of the perturbed magnetic field. To each mode of oscillation there corresponds a trio of wave numbers (n, m, ℓ) associated with the three directions (r, φ, z). For a given m, there is a lower cutoff for ℓ and an upper cutoff for n.

4. Dissipation

In the presence of viscous and resistive dissipations, the terms $(\eta/\rho)\nabla^2\delta\mathbf{v}$ and $(c^2/4\pi\sigma)\nabla^2\delta\mathbf{B}$ should be added to the right hand sides of Eqs. (1), where η and σ are the bulk viscosity and conductivity of the plasma, and c is the speed of light. The field components, undergo an exponential time decay. For weak dissipations the decay time $\tau_{nm\ell}$ of a mode (nml) is given by

$$\frac{1}{\tau_{nm\ell}} = \frac{\omega_A}{4\pi}\left(\frac{1}{S} + \frac{1}{R}\right)\left(x^2_{nm\ell} + \frac{\ell^2}{L^2}\right) \tag{8}$$

where $\omega_A = v_{Ai}$ is the Alfven frequency, $S = (\frac{4\pi\sigma}{c^2})/(\frac{2\pi}{v_{Ai}})$ and $R = (\frac{\rho_i}{\eta})/(\frac{2\pi}{v_{Ai}})$, Lundquist's and Reynolds' numbers, respectively, are the ratios of resistive and viscous time scales to the Alfven time required to cross the circumference of the tube of radius one. The density of heat generation rates by either process have identical r-dependence. They, however, differ magnitude wise. For the resistive process the heat rate is proportional to S and for the viscous one is proportional to R. More details are given in Karami, Nasiri and Sobouti (2002).

References

Davila, J. M. 1987, ApJ, 317, 514
Edvin, P. M., & Roberts, B. 1983, Sol. Phys., 88, 179
Hollweg, J. V. 1984, ApJ, 277, 392
Ionson, J. A. 1978, ApJ, 226, 650
Karami, K., Nasiri, S., & Sobouti Y. 2002, A&Ato Appear
Nakariakov, V. M. et al. 1999, Science, 285, 862
Ofman, L., Davila, J. M., & Steinolfson, R. S. 1994, ApJ, 421, 360
Ofman L., Davila, J. M., & Steinolfson, R. S. 1995, ApJ, 444, 471
Roberts, B. 1981a, Sol. Phys., 69, 27
Roberts, B. 1981b, Sol. Phys., 69, 39
Steinolfson, R. S. et al. 1986, ApJ, 526
Steinolfson, R. S., & Davila, J. 1993, ApJ, 415, 354
Wentzel, D. G. 1979a, ApJ, 227, 319
Wentzel, D. G. 1979b, ApJ, 227, 319
Wilson, P. R. 1979, A&A, 71, 9