Lines break automatically or can be forced with

Dark companion of baryonic matter-Beyond the point source

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I. INTRODUCTION

In two recent works [1] and [2], in order to explain the flat rotation curves of spiral galaxies, we approximated a galaxy by a point source and introduced an asymptotically logarithmic spacetime metric . The formalism was equivalent to assigning a "dark perfect gas companion" to the point mass with a size proportional to the square root of the point mass, and a density distribution fading away as r^{-2}. The conclusions, though sufficient for many practical purposes, were not generalizable to extended and many-body systems. For, a)The size of the dark companion was not proportional to the baryonic mass and b) the companion of a localized baryonic mass was not itself localized.

The way out of the dilemma that we have thought of, is to consider an extended system as a superposition of its mass multipole moments and see if one can assign a dark companion to each multipoles, separately.

II. MODEL AND FORMALISM

We consider the galaxy and its hypothetical dark matter companion to be an extended continuous and axially symmetric system. We also foresee it to be in a steady state of, generally differential, rotation. Because of the axial symmetry, the spacetime around the galaxy will also be axially symmetric. Because of the steady rotation the spacetime metric will further be static and Kerr-like. Thus, in a $(t, r, \vartheta, \varphi)$ coordinates, the latter three of which are the conventional spherical coordinates, we write

$$ds^2 = -Bdt^2 + Adr^2 + Cd\vartheta^2 + Dd\varphi^2 + 2Edtd\varphi, \quad (1)$$

where A, B, C, D, & E are functions of r & ϑ only. It will further be assumed that the rotation is non relativistic and the gravitational potential is weak everywhere. This enables one to write

$$A = 1 + \alpha(r, \vartheta), \quad B = 1 + \beta(r, \vartheta),$$

$$C = r^2[1 + \gamma(r, \vartheta)], \quad D = r^2 \sin^2\vartheta[1 + \delta(r, \vartheta)],$$

$$E = \frac{\sin^2\vartheta}{r}\eta(r, \vartheta), \quad (2)$$

where $\alpha,\ \beta,\ \gamma,\ \delta,\ \&\ \eta$ are dimensionless and of first order of smallness. In selecting E, we have been guided by Kerr's metric for which η is constant. To the first order in the small parameters, the covariant and contravariant metric tensors, the affine connections, and the Ricci tensors are calculated. They are partially tabulated in the appendix and partially displayed in the field equations below.

As to the baryonic content of the system we will decompose it into its mass multipoles, monopole, quadupole, etc. For its dark content we will assume a perfect gas model of density $\rho(r, \vartheta)$, pressure $p(r, \vartheta) << \rho$, and the 4-velocity $U^t = dt/d\tau$ (of order 1), $U^\varphi = d\varphi/d\tau$ ($<< 1$), and $U^r = U^\vartheta = 0$. The relevant energy momentum tensors of the dark matter will be

$$T_{\mu\nu} = pg_{\mu\nu} + (\rho + p)U_\mu U_\nu, \quad T^\lambda_\lambda = (3p + \rho). \quad (3)$$

Below, $T_{\mu\nu}$ appears on the right hand side of the field equations. In order to observe the Bianchi identities one should further have

$$T^{\mu\nu}{}_{;\nu} = 0.$$

This in turn leads to an equation of hydrodynamic equilibrium for the dark gas. The baryonic system is expected to have finite extensions and we will stay outside its boundaries. The dark companion, however, may extend to farther distances and not vanish in regions of interest to us.

The constrain $U_\lambda U^\lambda = -1$, to the first order of smallness, gives

$$U_t = -\left(1 + \frac{\beta}{2}\right), \quad U_\varphi \text{ a first order small.} \quad (4)$$

The field equation are

$$R_{\mu\nu} = \left(T_{\nu\mu} - \frac{1}{2}g_{\nu\mu}T^\lambda{}_\lambda\right), \quad (5)$$

where we have assumed $8\pi G = c^2 = 1$. From the material collected in the appendix and Eqs. (3) & (4) we calculate the left and right hand sides of Eq. (5) and obtain

$$\frac{R_{tt}}{1+\beta} = -\frac{1}{2}\beta_{,rr} - \frac{1}{r}\beta_{,r} - \frac{1}{2r^2}\left(\beta_{,\vartheta\vartheta} + \cot\vartheta\beta_{,\vartheta}\right)$$

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$$=-\frac{1}{2}\nabla^2\beta = -\frac{1}{2}(3p+\rho) \approx -\frac{1}{2}\rho, \tag{6}$$

$$\frac{R_{rr}}{1+\alpha}=+\frac{1}{2}\left(\beta_{,rr}+\gamma_{,rr}+\delta_{,rr}\right)+\frac{1}{r}\left(-\alpha_{,r}+\gamma_{,r}+\delta_{,r}\right)$$
$$+\frac{1}{2r^2}\left(\alpha_{,\vartheta\vartheta}+\cot\vartheta\alpha_{,\vartheta}\right)=\frac{1}{2}(p-\rho)\approx-\frac{1}{2}\rho, \tag{7}$$

$$\frac{R_{\vartheta\vartheta}}{r^2(1+\gamma)}=$$
$$\frac{1}{2}\gamma_{,rr}+\frac{1}{2r}\left(-\alpha_{,r}+\beta_{,r}+3\gamma_{,r}+\delta_{,r}\right)+\frac{1}{r^2}(-\beta+\gamma)$$
$$+\frac{1}{2r^2}\left[\alpha_{,\vartheta\vartheta}+\beta_{,\vartheta\vartheta}+\delta_{\vartheta\vartheta}+\cot\vartheta\left(-\gamma_{,\vartheta}+2\delta_{,\vartheta}\right)\right]$$
$$=\frac{1}{2}(p-\rho)\approx=-\frac{1}{2}\rho, \tag{8}$$

$$\frac{1}{r^2}\left[\frac{R_{\vartheta\vartheta}}{1+\gamma}-\frac{R_{\varphi\varphi}}{\sin^2\vartheta(1+\gamma)}\right]=$$
$$\frac{1}{2}\left(\gamma_{,rr}-\delta_{,rr}\right)+\frac{1}{r}\left(\gamma_{,r}-\delta_{,r}\right)$$
$$+\frac{1}{2r^2}\left[\left(\alpha_{,\vartheta\vartheta}+\cot\vartheta\alpha_{,\vartheta}\right)+\left(\beta_{,\vartheta\vartheta}+\cot\vartheta\beta_{,\vartheta}\right)\right]=0 \tag{9}$$

$$R_{r\vartheta}=\frac{1}{2}\left(\beta_{,r\vartheta}+\delta_{,r\vartheta}\right)$$
$$+\frac{1}{4}\cot\vartheta\left(\delta_{,r}-\gamma_{,r}\right)-\frac{1}{4r}\left(\alpha_{,\vartheta}+\beta_{,\vartheta}\right)=0, \tag{10}$$
$$R_{t\varphi}=\frac{\sin^2\vartheta}{2r}\left[\eta_{,rr}-\frac{2}{r}+\frac{1}{2}\left(\eta_{,\vartheta\vartheta}+3\cot\vartheta\eta_{,\vartheta}\right)\right]$$
$$=\frac{1}{2}\frac{\sin^2\vartheta}{r}\eta(p-\rho)+(p+\rho)U_\varphi$$
$$\approx\rho\left[-\frac{\sin^2\vartheta}{2r}\eta+U_\varphi\right]. \tag{11}$$

III. SPHERICALLY SYMMETRIC SOLUTIONS

These solutions are reported in details in [2]. They are reviewed for later references here. Equations (9) and (10) are satisfied by $\gamma=\delta=0$, and $\alpha(r)$ & $\beta(r)$ function of r only. Upon substitution in Eqs. (6-8), we find

$$\beta''+\frac{2}{r}\beta'=\rho,\quad \beta''-\frac{2}{r}\alpha'=-\rho,$$
$$\frac{1}{r}(\beta'-\alpha')-\frac{2}{r^2}\alpha=-\rho, \tag{12}$$

where we have denoted $d\beta/dr$ by β' for brevity. The three Eqs. (12) are linear in α, β, & ρ, and are not linearly independent. Their solution is

$$\beta'=\frac{\alpha}{r}\quad\text{and}\quad\rho=\frac{1}{r}\left(\alpha'+\frac{\alpha}{r}\right). \tag{13}$$

In [2], it was argued that if $\alpha(r)$ is a differentiable function and has a series expansion of the form

$$\alpha=\lambda+\frac{s_1}{r}+\sum_{n=2}\frac{s_n}{nr^n},\quad s_n'\text{s constants}, \tag{14}$$

then

$$\beta=\lambda\ln r-\frac{s_1}{r}-\sum_{n=2}\frac{s_n}{r^n}, \tag{15}$$

$$\rho=\frac{\lambda}{r^2}-\sum_{n=2}\frac{(n-1)}{n}\frac{s_n}{r^n}. \tag{16}$$

A test object in this spacetime will have an asymptotic circular speed about the origin $v_\infty^2=\lambda c^2/2$, where we have restored c^2 for clarity. In order to satisfy the Tully-Fisher relation we found

$$\lambda=\lambda_0\left(\frac{M}{M_\odot}\right)^{1/2},\ M\text{ mass monopole moment,}$$
$$\lambda_0\approx 2.8\times10^{-12},\ \text{universal constant.} \tag{17}$$

The dimensionless λ_0 was found from the inspection of the observed asymptotes of the spirals or, equivalently, from the 'universal acceleration' of Milgrom, $a_0\approx 1.2\times10^{-8}$ cm/sec^2. It is clear that the λ-term in Eqs. (14-16) is non classical. For no one expects the mass density of a galaxy to extend to infinity as r^{-2}, and its force field to have a range r^{-1}. At best, it could be attributed to a hypothetical dark matter accompanying the mass monopole moment of the system.

The s_1-term in Eqs. (14) and (15) is what one finds in Schwarzschild's metric or in Newtonian gravitation. s_1 should be identified with Schwarzschild's radius of the galaxy, $2GM/c^2$. For the remaining s_n-terms, $n\ge2$, we know of no supporting observational evidence. As a conjecture, however, if one assumes $s_n=\lambda_n\left(M/M_\odot\right)^n$ and λ_n's dimensionless universal constants, then the dynamical acceleration of a test object can be written as

$$a_{\rm dyn}=\sum_{n=0}\lambda_n\left(\frac{g_N}{a_0}\right)^{n+1/2},\quad g_N=GM/r^2 \tag{18}$$

This conclusion is in accord with Milgrom's idea that the dynamical acceleration is not simply proportional to g_N, but, in the language of this paper, is an involved function of (g_N/a_0), see [2] for details.

IV. AXIALLY SYMMETRIC SOLUTIONS

In spite of their complicated appearance, Eqs. (6)-(10) have simple analytical solutions. Equation (9) is satisfied by $\beta(r,\vartheta)=-\alpha(r,\vartheta)$, and $\gamma(r,\vartheta)=\delta(r,\vartheta)$+const. Equation (10) is satisfied if we further let $\beta(r,\vartheta)=-\delta(r,\vartheta)$+const. Now substituting

$$\beta(r,\vartheta)=-\alpha(r,\vartheta)=-\gamma(r,\vartheta)=-\delta(r,\vartheta)+\text{const.} \tag{19}$$

in Eqs. (6), (7), and (8) reduces all of them to

$$\nabla^2\beta(r,\vartheta)=\rho(r,\vartheta). \tag{20}$$

Instead of integrating Eq. (20) for a given ρ, we will do the opposite, calculate ρ in term of a preassigned

β. To choose the latter, we recall that in the weak field regime, β is essentially the gravitational potential. Here, it has contributions from the baryonic matter and its dark companion. The potential of the baryonic monopole moment is $-GM/r$. For its dark twin we found the term $\frac{1}{2}\lambda \ln r$ (this was dictated by the flat rotation curves of spirals). The leading terms in the force field of Eq. (15) were $GM/r^2 + \frac{1}{2}\lambda/r$. The dark force faded out at infinity slower than the baryonic one by a factor r. Let us generalize this behavior to higher multipoles. The lth baryonic moment has the gravitational potential $Q_l^{(b)} P_l(\cos\vartheta)/r^{(l+1)}$. Let us conjecture that there is a corresponding dark potential, a factor r slower than its baryonic counterpart, of the form $Q_l^{(d)} P_l(\cos\vartheta)/r^l$. Therefore, we propose the following expression for $\beta(r,\vartheta)$

$$\beta(r,\vartheta) = -\frac{s_1}{r} - 2\sum_{l=2} \frac{Q_l^{(b)}}{r^{l+1}} P_l(\cos\vartheta) + \lambda \ln r - \sum_{l=2} \frac{Q_l^{(d)}}{r^l} P_l(\cos\vartheta), \tag{21}$$

where the the terms in the first line are the gravitational potentials of the monopole and higher multipoles of the baryons. They satisfy Laplace's equation , for, we confine the analysis to distances beyond the visible boundaries of the galaxy. The terms, in the second line are due to the dark matter.

From Eq. (20), the corresponding density is

$$\rho(r) = \nabla^2\beta = \frac{\lambda}{r^2} - 2\sum_{l=2} l Q_l^{(d)} \frac{P_l(\cos\vartheta)}{r^{l+2}}. \tag{22}$$

What are $Q_l^{(d)}$'s? The Tully-Fisher relation helped to establish a relation between the monopole moment of baryons and its dark companion, Eq. (17). We are not aware of a similar observational evidence to write $Q_l^{(d)}$ in terms of $Q_l^{(d)}$. It is logical, however, to expect $Q_l^{(d)}$ to depend on $Q_l^{(b)}$ and should vanish as the latter vanishes. As to what the form of this dependence and its magnitude should be, has to await either the availability of further detailed observations, or a postulating hunch similar to that of Milgrom on the mutual dependence of dynamical accelerations and gravitational forces.

APPENDIX A

The material collected in this section is self explanatory. The coordinate system is $(t,\ r,\ \vartheta,\ \varphi)$, the latter three are the standard spherical polar coordinates. The assumption of axial symmetry and of steady motion of the material system makes all functions dependent on r & ϑ only. From Eqs. (1) and (2), the non vanishing elements of the metric tensor are

$$\begin{aligned} &g_{tt} = -(1+\beta), \quad g_{rr} = 1+\alpha, \\ &g_{\vartheta\vartheta} = r^2(1+\gamma), \quad g_{\varphi\varphi} = r^2\sin^2\vartheta(1+\delta), \\ &g_{t\varphi} = g_{\varphi t} = r^{-1}\sin^2\vartheta\eta, \quad \alpha,\beta,\gamma,\eta << 1. \end{aligned} \tag{A1}$$

The corresponding contravariant metric tensor, to the first order of smallness in $\alpha,\ \beta,\ \gamma,\ \delta,\ \&\ \eta$, is

$$\begin{aligned} &g^{tt} = -(1-\beta), \quad g^{rr} = 1-\alpha, \\ &g^{\vartheta\vartheta} = r^{-2}(1-\gamma), \quad g^{\varphi\varphi} = r^{-2}\sin^{-2}\vartheta(1-\delta), \\ &g^{t\varphi} = g^{\varphi t} = r^{-3}\eta. \end{aligned} \tag{A2}$$

In what follows a derivative $\partial f(x)/\partial x^\sigma$ is denoted as $f_{,\sigma}$. From

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}\left[g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}\right],$$

the non vanishing elements of the affine connections are

$$\begin{aligned} &\Gamma^t_{tr} = \frac{1}{2}\beta_{,r} \qquad \Gamma^t_{t\vartheta} = \frac{1}{2}\beta_{,\vartheta} \\ &\Gamma^t_{\varphi r} = \frac{\sin^2\vartheta}{2r}\left(\frac{3\eta}{r} - \eta_{,r}\right) \quad \Gamma^r_{\varphi\vartheta} = -\frac{\sin^2\vartheta}{2r}\eta_{,\vartheta} \end{aligned} \tag{A3}$$

$$\begin{aligned} &\Gamma^r_{tt} = \frac{1}{2}\beta_{,r} \qquad \Gamma^r_{t\varphi} = \frac{\sin^2\vartheta}{2r}\left(\frac{\eta}{r} - \eta_{,r}\right) \\ &\Gamma^r_{rr} = \frac{1}{2}\alpha_{,r} \qquad \Gamma^r_{r\vartheta} = \frac{1}{2}\alpha_{,\vartheta} \\ &\Gamma^r_{\vartheta\vartheta} = -r\left(1-\alpha+\gamma-\frac{1}{2}r\gamma_{,r}\right) \\ &\Gamma^r_{\varphi\varphi} = -r\sin^2\vartheta\left(1-\alpha+\delta+\frac{1}{2}r\delta_{,r}\right) \end{aligned} \tag{A4}$$

$$\begin{aligned} &\Gamma^\vartheta_{tt} = \frac{1}{2r^2}\beta_{,\vartheta} \qquad \Gamma^\vartheta_{t\varphi} = -\frac{\sin^2\vartheta}{r^3}\left(\cot\vartheta\eta + \frac{1}{2}\eta_{,\vartheta}\right) \\ &\Gamma^\vartheta_{r\vartheta} = \frac{1}{2}\left(1+\frac{1}{2}r\gamma_{,r}\right) \qquad \Gamma^\vartheta_{\vartheta\vartheta} = \frac{1}{2}\gamma_{,\vartheta} \\ &\Gamma^\vartheta_{\varphi\varphi} = -(1-\gamma+\delta)\sin\vartheta\cos\vartheta - \frac{1}{2}\sin^2\vartheta\delta_{,\vartheta} \end{aligned} \tag{A5}$$

$$\begin{aligned} &\Gamma^\varphi_{tr} = -\frac{1}{2r^4}(\eta - r\eta_{,r}) \qquad \Gamma^\varphi_{t\vartheta} = \frac{1}{r^3}\left(\cot\vartheta\eta + \frac{1}{2}\eta_{,\vartheta}\right) \\ &\Gamma^\varphi_{r\varphi} = \frac{1}{r}\left(1+\frac{1}{2}r\delta_{,r}\right) \qquad \Gamma^\varphi_{\vartheta\varphi} = \cot\vartheta + \frac{1}{2}\delta_{,\vartheta} \end{aligned} \tag{A6}$$

The following sums are frequently encountered in the calculations of the Ricci tensor

$$\begin{aligned} &\Gamma^\lambda_{r\lambda} = \frac{2}{r} + \frac{1}{2}\left(\alpha_{,r} + \beta_{,r} + \gamma_{,r} + \delta_{,r}\right) \\ &\Gamma^\lambda_{\vartheta\lambda} = \cot\vartheta + \frac{1}{2}\left(\alpha_{,\vartheta} + \beta_{,\vartheta} + \gamma_{,\vartheta} + \delta_{,\vartheta}\right) \end{aligned} \tag{A7}$$

The non vanishing elements of the Ricci tensor,

$$R_{\mu\nu} = \Gamma^\lambda_{\mu\lambda,\nu} - \Gamma^\lambda_{\mu\nu,\lambda} + \Gamma^\eta_{\mu\lambda}\Gamma^\lambda_{\nu\eta} - \Gamma^\eta_{\mu\nu}\Gamma^\lambda_{\mu\lambda},$$

are $R_{tt},\ R_{rr},\ R_{\vartheta\vartheta},\ R_{\varphi\varphi},\ R_{r\vartheta}$, and $R_{t,\varphi}$. They can be found in Eqs. (6)-(11).

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