Dark companion of baryonic matter - Logarithmic potentials are inherent to GR

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Flat rotation curves of spiral galaxies can be explained in terms of logarithmic gravitational potentials. The field equations of GR admit of spacetime metrics with such behaviors. The scenario can be interpreted either as an alternative theory of gravitation or, equivalently, as a dark matter paradigm. In the latter interpretation, one is led to assign a dark companion to the baryonic matter who's size and distribution is determined by the mass of the baryon. The formalism also opens up a way to support Milgrom's idea that the acceleration of a test object in a gravitational field is not simply the newtonian gravitational force g_N, but rather an involved function of (g_N/a_0), a_0 a universal acceleration.

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I. INTRODUCTION

The goal of the paper is to understand the idiosyncrasy of the rotation curves of spiral galaxies. The newtonian or the GR gravitation of the observable matter is not sufficient to explain the large asymptotic speeds of test objects in orbits around the galaxies. In search of the missing gravity, alternative theories of gravitation and/or of dark matter are proposed. In a recent work [2] we pointed out that no one has reported a case where there is no baryonic matter but there is a dynamical issue to be settled. We argued that if dark matter is always needed in the presence of the baryonic one, it is logical to assume that any baryonic matter has a dark companion. On the other hand, both dark matter scenarists and alternative theorists explain the rotation curves of spirals equally satisfactorily. We argue, if two people are giving correct answers to the same question, they ought to be saying the same thing, albeit in different languages. And since in an alternative theory one gives a definite rule for the gravity field, there must be rules to govern the mutual companionship of the dark and baryonic matters.

In [2], based on the general characteristics of the rotation curves, we proposed an empirical asymptotically logarithmic gravitational potential and constructed a spacetime metric consistent with the assumed potential. Here, we begin with a GR formalism and show that spacetime metrics with logarithmic behaviors are accommodated by Einstein's field equations. One need not resort to unorthodox notions to explain the dynamics of the spirals. The end result is, of course, interpretable either in terms of an alternative theory of gravitation, or as a dark matter paradigm. With an advantage, however: the questions, how much dark matter accompanies a given baryonic mass, how it is distributed, and what is its equation of state, are also answered.

II. MODEL AND FORMALISM

At a distance r, the ratio of the quadrupole to the monopole gravitational field of a flattened galaxy is of the order of $(R_{gyr}/r)^2$, where R_{gyr} is the gyration radius of the galaxy about one of the principle axes of the ellipsoid of inertia of the system. In a galaxy with R_{gyr} of the order of few tenth of $R_{visible}$, and at a distance r of about few times $R_{visible}$, this ratio can easily fall below a few part in thousand. Therefore, it is a reasonable approximation, to neglect the quadrupole field of the galaxy at its outer reaches and consider the spacetime around it to be spherically symmetric and static. Thus

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \quad (1)$$

We adopt a dark matter language and assume that the galaxy, itself approximated by a point mass at the origin, possesses a static dark perfect gas companion of density $\rho(r)$, of pressure $p(r) << \rho(r)$, and of covariant 4-velocities $U_t = -B^{1/2}, U_i = 0,\ i = r, \theta, \varphi$. Einstein's field equations become.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu} = -[pg_{\mu\nu} + (p+\rho)U_\mu U_\nu], \quad (2)$$

where we have let $8\pi G = c^2 = 1$. To respect the Bianchi identities and the conservation laws of the baryonic matter, one mast have ${T^{\mu\nu}}_{;\nu} = 0$. The latter, in turn, leads to the hydrostatic equilibrium of the dark fluid, that is, if one wishes to attribute such notions to a hypothetical entity.

From Eq. (2) the two combinations $R_{tt}/B + R_{rr}/A + R_{\theta\theta}/r^2 + R_{\varphi\varphi}/r^2\sin^2\theta$ and $R_{tt}/B + R_{rr}/A$ give

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$$\frac{1}{r^2}\left[\frac{d}{dr}\left(\frac{r}{A}\right)-1\right]=-\rho, \quad (3)$$

$$\frac{1}{rA}\left(\frac{B'}{B}+\frac{A'}{A}\right)=\rho+p, \quad (4)$$

respectively. Neglecting p in comparison with ρ and eliminating ρ between the two equations gives

$$\frac{B'}{B}=\frac{1}{r}(A-1). \quad (5)$$

We now assume that $A(r)-1$ is a well behaved function of r and has a series expansion in negative powers of r,

$$A-1=\lambda+\sum_{n=1}\frac{s_n}{r^n},\ s_n \text{ constant}. \quad (6)$$

Substituting this expansion in Eq. (5) and integrating the resulting expression gives

$$B=\left(\frac{r}{r_0}\right)^{\lambda}e^{-\sum s_n/nr^n}\approx\left[1+\lambda\ln\left(\frac{r}{r_0}\right)-\frac{s_1}{r}-\cdots\right].(7)$$

We note that λ is dimensionless and s_n has the dimension $(length)^n$. The right hand side approximation is the weak field assumption and holds for $\lambda << 1$ and $s_n/r^n << 1\ \forall\ n$.

With A and B known, the density ρ can be calculated from either of Eqs. (3) or (4). Here, however, we adopt a weak field point of view, $B(r)=1+2\phi_{grav}/c^2$, and calculate ρ from Poisson's equation. This allows the point mass of the galaxy to show up as a δ-function density centered at the origin. Thus,

$$\begin{aligned}4\pi G\rho &= \frac{1}{2}c^2\nabla^2(B-1)\\ &= \frac{1}{2}c^2\left[\frac{\lambda}{r^2}+s_1\delta(r)-\sum_{n=2}(n-1)\frac{s_n}{r^{n+2}}\right]. \quad (8)\end{aligned}$$

Hereafter, we restore the physical dimensions $8\pi G$ and c^2 for clarity. The pressure of the companion fluid is obtained from $T^{\mu\nu}{}_{;\nu}=0$,

$$\frac{p'}{p+\rho}\approx\frac{p'}{\rho}=-\frac{1}{2r}(A-1). \quad (9)$$

Integration is straight forward. The first two terms in the series are

$$p=\frac{c^2\lambda}{16\pi G}\left[\frac{\lambda}{2r^2}+\frac{s_1}{3r^3}\right]. \quad (10)$$

Upon elimination of r between Eqs. (8) and (10) one obtains the equation of state, $p(\rho)$. It is barotropic.

We conclude this section by writing down the dynamical acceleration of a test object circling the galaxy with the speed v

$$\begin{aligned}a_{\rm dyn} &= \frac{v^2}{r}=\frac{1}{2}c^2B'\\ &= \frac{1}{2}c^2\left[\frac{\lambda}{r}+\frac{s_1}{r^2}+\cdots+\frac{s_n}{r^{n+1}}+\cdots\right]. \quad (11)\end{aligned}$$

III. WHAT ARE λ AND s_n's

The s_1 term in Eqs. (6)-(11) represents the classic gravitation of the baryonic matter with a force range of r^{-2}. Magnitude-wise, s_1, should be identified with the Schwarzschild radius of the matter residing at the origin, $s_1=2GM/c^2$. The λ-term is not a classical term. It has a force-range r^{-1} and dominates all other terms at large distances. It is responsible for the flat asymptotes of the rotation curves of spirals. In [1] and [2] we resorted to the Tully-Fisher relation (the proportionally of the asymptotic speed , $v_\infty=c(\lambda/2)^{1/2}$, to the fourth root of the mass of the host galaxy) and arrived at

$$\lambda=\lambda_0\left(\frac{M}{M_\odot}\right)^{1/2}. \quad (12)$$

In the weak field regimes Milgrom's MOND anticipates an acceleration $(a_0g_N)^{1/2}$, instead of the newtonian gravitation, where a_0 is his universal acceleration. The far distance limit of Eq. (11) with λ given by Eq. (12) is of Milgrom's form. Comparing the two formalisms, one finds $\lambda_0=2(a_0GM_\odot)^{1/2}c^{-2}$. Either from this expression, with $a_0\approx1.2\times10^{-8}\text{cm/sec}^2$ [4], or from a direct statistical analysis of the asymptotic speeds of spirals [1] one finds

$$\lambda_0\approx2.8\times10^{-12}, \text{a dimensionless universal const.} \quad (13)$$

The remaining s_n-terms, $n\geq2$, in Eqs. (6)-(11) are also nonclassical. The range of their force is $r^{-(n+1)}$, (not to be confused with the multipole fields of extended objects). There are no compelling observational evidences for their existence. Nevertheless, we retain them for a possible formal support they may give to Milgrom's MOND, to be elaborated shortly.

A conjecture: There is a surprise in Eq. (11). Upon elimination of r in favor of $g_N=GM/r^2$, one may write it as

$$\frac{a_{\rm dyn}}{a_0}=\left(\frac{g_N}{a_0}\right)^{1/2}+\left(\frac{g_N}{a_0}\right)+\cdots+\lambda_n\left(\frac{g_N}{a_0}\right)^{(n+1)/2}+ \quad (14)$$

where λ_n's can be expressed in terms of s_n's through a term-by-term comparison of Eqs. (11) and (14). One obtains

$$\lambda_n=\frac{c^2s_n}{2a_0}\left(\frac{a_0}{GM}\right)^{(n+1)/2}, n=1,2,\cdots, \quad (15)$$

or

$$s_n=\frac{2a_0}{c^2}\lambda_n\left(\frac{GM}{a_0}\right)^{-(n+1)/2}. \quad (16)$$

All λ_n's are dimensionless. Apparently, Eq.(14) is an expansion of the dynamical acceleration in a power series of $(g_N/a_0)^{1/2}$. The coefficient of the first term is the *universal constant* 1 because of the *universal* Tully-Fisher relation. The coefficient of the second term is 1 because of the universal law of newtonian gravitation at intermediate distances. Now the conjecture: If there is any significance attached to the series expansion of Eq. (14) beyond the first two terms, is it possible that in the remaining terms

"All λ_n's are universal constants (not necessarily 1), and independent from the mass of the host baryonic matter located at the origin"?

The proof or disproof of the conjecture should come from observations. We recall, however, Milgrom's stand [3] that the dynamical acceleration of a test body is not simply proportional to g_N, but it is a involved function of g_N/a_0, and vice versa, g_N is a function of a_{dyn}/a_0. His suggestion for this function is through an (almost arbitrary) interpolating function. If the conjecture above holds, Eq. (14) can be considered as a series expansion of one such function, and a support for Milgrom's idea.

IV. CONCLUDING REMARKS

That logarithmic potentials are natural solutions of Einstein's field equations is the highlight of the paper. They enable one to arrive at a law of gravitation alternative to that of Newton and/or to those known to GR. Equivalently, one may choose to attribute dark companions to baryonic matters. In the case of a point baryonic mass, the size and the distribution of the density and pressure of the companion are given by Eqs. (8)and (10). The spacetime is a baryonic vacuum but not a dark matter one. The consequences are noteworthy. For example,

- The spacetime is not flat. Contraction of Eq. (2) gives

$$R = -(3p + \rho) \approx -\frac{\lambda}{r^2} + O\left(r^{-4}\right).$$

- The 3-space is not flat. Direct calculation with $g_{ij}^{(3)}, i, j = r, \theta, \varphi$, yields

$$R^{(3)} = -\frac{2}{r^2}\frac{d}{dr}(r\alpha) \approx -2\frac{\lambda}{r^2} + O\left(r^{-4}\right).$$

- There is an excess lensing, see [2] and [7]. Contribution from the λ_0 term alone is

$$\delta\beta = \frac{1}{2}\pi\lambda_0 (M/M_\odot)^{1/2}$$

.

- Due to the smallness of both λ_0 and Sun's mass, effects in the scale of the solar system are immeasurably small, see [2].

That the dynamical acceleration of a test object in the gravitational field of a point mass could have a series expansion in (g_N/a_0), in accord with Milgrom's idea, is an intriguing idea. The support for it should come from observations.

A word of caution: The paper relies heavily on observations pertaining to spiral galaxies. Its conclusions may be scale dependent, not applicable to systems with scales larger than the galactic scale. In a recent paper Bernal et al [6] analyze weak lensing data from clusters of galaxies on the basis of the metric field of [1] (similar to those of Eqs. (6) and (9)). They conclude, in the notation of this paper, $\lambda \propto M^{1/4}$, instead of $M^{1/2}$ of Eq. (12). This finding while raises an alarm against extrapolation to larger systems, clusters of galaxies and beyond, at the same time opens the question that deviations from the newtonian or GR gravitations may have a hierarchical structure depending on the size of the system under study.

Shortcomings of the paper and the open questions it leaves behind should also be mentioned. Strictly speaking, the theory developed here is applicable to a point source mass only. Extension to extended objects and to many body systems is not a trivial task. It may require further assumptions not contemplated so far. The difficulty lies in the facts that a) the added λ- and s_n- terms, $n \geq 2$, in Eq. (6) and thereafter, are not linear in the mass of the point source baryonic matter. The nonlinearity is much more complicated than that of GR. b) In the parlance of dark matter paradigms, the dark companion of a localized baryonic matter is not localized. As a way out of the dilemma, we are planning to expand an extended object into its localized monopole and higher multipole moments, and see if it is possible to find a dark multipole moment for each baryonic one, more or less in the way done for the monopole moment.

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