Dark companion of baryonic matter

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(Dated: January 27, 2009)

Whenever and wherever one talks of dark matter, one does so when and where there is a luminous matter and a dynamical issue to be settled. We promote this observation to the status of an axiom and assume that there is a dark companion to every luminous matter and there are orders to this companionship. To pursue the proposition in a formal and quantitative manner, we consider the anomalous rotation curves of spiral galaxies. From the available observations, we infer the gravitational potential prevailing in the outer parts of the galaxy and, thereof, construct the tt-component of the metric of the embedding spacetime. Next we examine a perfect fluid candidate as the dark companion and solve the relevant GR equations. We are able to determine the strength and the distribution of the dark fluid that accompanies a point baryonic mass. Finally, we argue that the whole paradigm can be explained just as well in terms of an alternative theory.

Keywords: Dark matter; Alternative GR; Spiral galaxies, rotation curves of

PACS numbers: 90

I. INTRODUCTION

That the baryonic matter of galaxies, clusters of galaxies, or for that matter, of the universe at large, does not provide sufficient gravitation to explain the observed dynamics of the systems, is an established fact. To solve the dilemma, dark matter/energy scenarios and/or alternative theories of gravitation have been speculated and debated. The fact, however, remains that the proponents of dark matter/energy have always looked for it in baryonic environments. No one has, so far, reported a case where there is no luminous matter and/or cosmic radiation, but there is a dynamical problem to be solved. In view of this negative observation, it is not unreasonable to hypothesize that any luminous matter has a dark companion and there are rules to this companionship as regards the magnitudes and the distributions of the companion and the matter itself.

On the other hand such a point of view, that denies the independent existence of the dark matter/energy, is equivalent to the assumption that the known theories of gravitation , based on baryonic matter alone, do not tell the whole story and there is room for amendments. This conclusion, in turn, reduces the distinction between the dark matter/energy hypotheses and alternative theories to the level of semantics. As long as the dark matter displays no other physical characteristics than gravitation, one will have the option, either to assume a dark component to every baryonic matter subject to certain rules, and account for its gravitation in a conventional way, or simply adhere to the baryonic matter but resort to an alternative law of gravitation. The two points of view should be equivalent.

With this perspective in sight, here we confine the discussion to the problem of spiral galaxies. There is substantial amount of information in the observed rotation curves of spirals to construct an empirical law of gravity. This step leads to a partial construction of the spacetime metric around the galaxy. Next we look for the required modification of the field equations of GR to ensure the self-consistency of the theory. The end results can then be interpreted, interchangeably, either in terms of a dark matter scenario or in terms of an alternative theory of gravitation

II. OBSERVED FACTS AND IMPLICATIONS

There are three main characteristics to the rotation curves of spirals.

- a) They often have a flat asymptote at far distances from the galaxy, see e.g. [1], [2], [3], [4], & [5].
- b) Their asymptotic speed is, more often than not, proportional to the fourth root of the mass of the galaxy, the Tully-Fisher relation [6].
- c) Deviations from the classical concepts (in this case gravitation) show up in large scale systems and at large distances.

These observed facts will be treated as axioms and will serve as the starting point of what follows.

The model: A test object orbits a galaxy at far distances from it, and has a constant distance-independent circular speed, item a above. To have such a speed one requires a force field that fades away as r^{-1} and, therefore, a gravitational potential as $\ln r$. In the GR perspective, the metric field surrounding the

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galaxy should also exhibit the same logarithmic behavior.

Let us view the galaxy from afar and approximate it by a point mass. The spacetime around will accordingly be spherically symmetric and isotropic:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2. \quad (1)$$

It is customary to write $B(r) = 1 + 2\phi(r)/c^2$ and, in the weak field regime, to consider $\phi(r)$ as the gravitational potential. One, however, knows that at close distances the gravitation is newtonian, item c above, and at far distances as we just learned should be proportional to $\ln r$. Thus, we let

$$B(r) = 1 - \frac{s}{r} + \lambda \ln r, \quad (2)$$

where $s = 2GM/c^2$ is the Schwarzschild radius of the galaxy, and λ is a dimensionless constant. It will emerge as part of the sought-after modification to the field equations.

What is λ? The answer is in the Tully-Fisher relation, item b above. From Eq. (2), the circular speed of an orbiting test object is

$$v^2 = \frac{1}{2}c^2 r\frac{dB}{dr} = \frac{GM}{r} + \frac{1}{2}\lambda c^2. \quad (3)$$

By Tully-Fisher, the asymptotic speed, $\sqrt{\lambda c^2/2}$, is proportional to the fourth root of the mass of the galaxy. This implies

$$\lambda = \lambda_0 \left(\frac{M}{M_\odot}\right)^{1/2}. \quad (4)$$

What is λ_0 ? In his theory of MOND, Milgrom [7] proposes a law of gravitation whose strong and weak limits are the newtonian gravity, $g_N = GM/r^2$, and $(a_0 g_N)^{1/2}$, respectively. From the inspection of the observed data Begeman et al. [5] find $a_0 \approx 1.2 \times 10^{-8}$ cm sec^{-2}. With λ of Eq. (4), Eq. (3) has the same strong and weak limits of MOND. We use this coincidence to find λ_0. We divide Eq. (3) by r, substitute for λ from Eq. (4), and identify the resulting term, $\frac{1}{2}\lambda_0 c^2 (GM/r^2 GM_\odot)^{1/2}$ with Milgrom's $a_0 g_N$. We obtain

$$\lambda_0 = \left[\frac{4a_0}{c^2}\frac{GM_\odot}{c^2}\right]^{1/2} \approx 2.8 \times 10^{-12}. \quad (5)$$

The problem is partially solved. We have constructed an empirical law of gravity, whose strong and weak limits are those of MOND. There remains to build the empirically constructed $B(r)$ into a consistent general relativistic formalism.

III. THE MODIFIED FIELD EQUATIONS

We seek this modification by adding a new tensor term to Einstein's field equations. To respect the Biancci identities and the conservation laws of the baryonic matter, this tensor should have a vanishing covariant divergence.This is best achieved by adopting a dark matter point of view. We assume the galaxy, approximated by a point mass, M, has a 'dark perfect fluid' companion, with the energy momentum tensor

$$T_d^{\mu\nu} = p_d g^{\mu\nu} + (\rho_d + p_d)U_d^\mu U_d^\nu, \quad (6)$$

$$T_d^{\mu\nu}{}_{;\nu} = 0, \quad U_d^t = B^{-1/2}, \quad U_d^i = 0, \quad (7)$$

where ρ_d and p_d are the density and the pressure of the dark fluid, respectively. The fluid is spherically symmetric and is at rest. Its 4-velocity, U_d^κ, is indicated in Eq. (7). The amended field Equations in the 'baryonic vacuum' of the galaxy now reads

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{d\mu\nu}. \quad (8)$$

The baryonic matter of the galaxy has a δ-function density distribution and is of zero pressure. It will show up as a constant of integration in the final stage of integrations. The spacetime metric is still spherically symmetric and isotropic as in Eq.(1). From Eq. (8), the two combinations, $R_{tt}/2B + R_{rr}/2A + R_{\theta\theta}/r^2$ and $R_{tt}/B + R_{rr}/A$, give

$$\frac{d}{dr}\left(\frac{r}{A}\right) = 1 - 8\pi G\rho_d r^2, \quad (9)$$

$$\frac{B'}{B} + \frac{A'}{A} = 8\pi G r A(\rho_d + p_d). \quad (10)$$

To this we add the time-component of Eq.(7),

$$\frac{p_d'}{\rho_d + \rho_d} = -\frac{1}{2}\frac{B'}{B}. \quad (11)$$

The space components of Eq. (7) are trivially satisfied on account of $U_d^i = 0$. Equation (9) immediately integrates into

$$\frac{1}{A} = 1 - \frac{s}{r} - 2G\frac{m_d(r)}{r}, \quad m_d(r) = 4\pi\int \rho_d r^2 dr, \quad (12)$$

where s is the integration constant and as in Eq. (2) should be identified with Schwarzschild's radius of the galaxy. We already have inferred $B(r)$ from observations, Eq. (2). There remains to solve Eqs. (10)-(12) for $A(r), \rho_d$ and p_d. Exact solutions are probably to be obtained numerically. Their weak field approximations are, however, analytically available and are inspiring. We consider the dimensionless quantities λ, s/r, & $Gm_d(r)/r$ much smaller than 1, and keep only their first order terms in all calculations. We also assume and verify later that $p_d \ll \rho_d$. With these provisions, Eqs. (10) and (12) gives

$$m_d(r) = \frac{\lambda}{2G}r, \qquad \rho_d(r) = \frac{\lambda}{8\pi G}\frac{1}{r^2}. \quad (13)$$

With this ρ_d, Eq. (11) integrates into

$$p_d(r)=\frac{\lambda}{32\pi G}\left[\frac{\lambda}{r^2}+\frac{2}{3}\frac{s}{r^3}\right] =\frac{\lambda}{4}\rho_d+\frac{(2\pi G\lambda)^{1/2}}{3}s\rho_d^{3/2}\ll\rho_d. \quad (14)$$

The second equality in Eq. (14) is obtained by eliminating r between $\rho_d(r)$ and $p_d(r)$. The inequality is on account of the smallness of λ. If one is allowed to use the terminologies and concepts of the real world's physics, one might say the dark fluid has a barotropic equation of states.

The goal set in the introduction is, at least partially, arrived at. For a point mass M (Schwarzschild's radius s) we have found a static dark fluid companion. Its strength and distribution is given by Eqs. (13), (14), (4), and (5). This parlance, however, is no more than borrowing a jargon from the physics of the observable world to explain the purpose. Equivalently, one may choose to say that the gravitation produced by a point mass is not newtonian and there is a logarithmic correction to it. Or, rather, the spacetime around a point mass is not that of Schwarzschild but that given by Eqs. (2), (12) and (13).

How much dark matter in a typical spiral? The question should be qualified by giving the radius inside which the mass is inquired. From Eq. (13), after inserting a factor c^2, which so far was suppressed, and substituting for λ from Eqs. (4) and (5), one finds

$$\frac{m_d(r)}{10^{10}M_\odot}=2.8\left[\frac{M}{10^{10}M_\odot}\right]^{1/2}\left[\frac{r}{10\ kpc}\right], \quad \text{or}$$
$$\frac{m_d(r)}{M_\odot}=1.4\times10^{-4}\left[\frac{M}{M_\odot}\right]^{1/2}\left[\frac{r}{a.u.}\right]. \quad (15)$$

The dark matter inside a sphere, centered on the baryonic point mass, is proportional to the radius of the sphere and to the square root of the mass residing at the center. For the Milky Way of total stellar + *HI* mass $\approx 6\times10^{10}M_\odot$ at $r=10$ and 50 kpc (the later is the distance to the Large Magellanic Clouds) the dark matter is 7 and 35 $\times10^{10}M_\odot$, respectively. They amount to 55% and 83% of the required dynamical mass. The dark matter accompanying Sun within the outermost reaches of the solar system, 100 a.u., say, is $\approx 1.4\times10^{-2}M_\odot$ and less by a factor of one hundred at Earth's distance.

Spacetime is not flat. From Eqs. (2), (12), and (13), one has

$$\frac{1}{A}=1-\lambda-\frac{s}{r}, \quad B=1+\lambda\ln r-\frac{s}{r}. \quad (16)$$

Contracting Eqs. (8) and (6) and using Eqs. (13), and (16), gives the scalar curvature of the 4-spacetime

$$R=8\pi G\left[3p_d-\rho_d\right]\approx-\frac{\lambda}{r^2}. \quad (17)$$

The scalar curvature of the 3-space, calculated from the 3-space metric, $g_{ij},\ i,\ j=r,\ \theta,\ \varphi$, turns out to be

$$R^{(3)}\approx-2\frac{\lambda}{r^2}. \quad (18)$$

Both curvatures are negative and fade away as r^{-2}. This is to be contrasted with Schwarzschild's spacetime, where the spacetime and the 3-space have zero scalar curvatures.

There is an excess lensing. This is to be expected on account of the excess gravitation of the dark companion. A light ray impinging on a lens from infinity and escaping to infinity bends by an angle [9]

$$\beta=2\int_{r_0}^{\infty}A^{1/2}\left[\left(\frac{r}{r_0}\right)^2\frac{B(r)}{B(r_0)}-1\right]^{-1/2}\frac{dr}{r^2}-\pi, \quad (19)$$

where r_0 is the distance of the closest approach to the lens. Substituting for A and B from Eq. (17), and keeping only the first order terms in s/r, s/r_0 and λ in the integral, gives

$$\beta=2\frac{s}{r_0}+\frac{1}{2}\pi\lambda=2\frac{s}{r_0}+\frac{1}{2}\pi\lambda_0\left(\frac{M}{M_\odot}\right)^{1/2}. \quad (20)$$

In this first order approximation, the excess deflection, $\frac{1}{2}\pi\lambda$, is independent of the impact parameter of the incident light ray. It is proportional to the square root of the mass of the lens, and could be large in large systems, clusters of galaxies, say.

Solar system implications: From Eq. (20), for a light ray grazing Sun's limb, the excess deflection amounts to $\approx 10^{-6}$ arcsec, negligible compared with the general relativistic value of 1.8 arcsec.
Precession of the perihelion of an orbit is obtained from [9]

$$\delta\phi=2\int_{r_-}^{r_+}\frac{A^{1/2}(r)}{J^2}\left[\frac{1}{B(r)}-E-\frac{1}{r^2}\right]^{-1/2}\frac{dr}{r^2}-2\pi$$
$$=3\pi\frac{s}{L}+\lambda(\frac{2s}{L})^{1/2}\left[1+\frac{3}{4}e\right], \quad (21)$$

where $r_\pm$ are the aphelion and the perihelion of the orbit, and $E, J, L,$ & e are its energy, angular momentum, semi latus rectum, & eccentricity, respectively. Again in view of the smallness of λ, the excess precession is much smaller than the conventional GR value.

Beyond the point mass: The metric coefficient of Eq. (16) are for a point mass. Galaxies at close and intermediate distances do not appear as such. In view of the smallness of λ and the proportionality of $m_d(r)$ to r, however, contributions of λ terms are significant only at far distances. Otherwise the gravitational potential is essentially newtonian and the spacetime as that of GR, item c above. Thus, considering the present-day accuracies of the observational data, one may generalize Eqs. (16) by replacing the point mass term, $-s/r$, by whatever GR requires for an extended object, and leave the λ-term as it is. In the weak field regime, the metric coefficients become

$$\frac{1}{A}=1-\lambda-2Gc^{-2}\int \mathbf{dr'}^3\rho(\mathbf{r'})|\mathbf{r}\text{-}\mathbf{r'}|^{-1}, \quad (22)$$

$$B=1+\lambda\ln r-2Gc^{-2}\int \mathbf{dr'}^3\rho(\mathbf{r'})|\mathbf{r}\text{-}\mathbf{r'}|^{-1}. \quad (23)$$

It should, however, be noted that this generalization does not follow from a founding principle. It can only serve practical exigencies.

Kinship with $f(R)$ gravity of [10]: In [10] we introduce an Einstein-Hilbert lagrangian, which is essentially $R^{1-\lambda/2}\approx R[1-\frac{1}{2}\lambda\ln r]$, and obtain a spacetime metric with logarithmic corrections to it. In the weak field regime, its near- and far- distance limits are the same as those of the present paper (and of those of MOND). For practical purposes the two theories are identical. There is, however, an axiomatic advantage to the present formalism. As noted earlier, the present formalism respects the Biancci identities and the conservation laws of the baryonic matter. No $f(R)$ formalism does so.

Mendoza et al [11] show that, in the spacetime of [10], the light and the gravitational waves propagate with the speed of light in vacuum. Their conclusion is also true in the present case, on account of the identical near- and far-distance limits of the present and the $f(R)$ formalisms of [10].

IV. CONCLUDING REMARKS

The proposed formalism is a modified GR paradigm or, equivalently, a dark matter scenario, to understand the anomalous rotation curves of the spiral galaxies. It is an inverse approach. From the available observations, the gravitational potential and, thereof, part of the spacetime metric is constructed. Next the GR formalism is called upon to infer what modifications to Einstein's field equations produces a cohesive and self-consistent picture. Naturally, the credibility of the proposition depends on how accurately the axiomatized model of section II describes the realities of the skies. For example, if the future observations reveal a decline in the rotation curves at very far distances, as some authors have pointed at such indications in the observed data [12] or entertained it on theoretical grounds [13], the model and the empirically inferred gravitational potential should be adjusted accordingly.

Ascription of a dark companion to every baryonic matter, the rules of the companionship, and the consequent equivalence of the scenario to an alternative GR theory are the highlights of the paper. However, only the case of a point mass is handled. An axiomatic generalization to many body systems and continuous distributions of luminous matter requires further deliberations and better and more extensive observational data. One might need further postulates. The difficulty lies in the fact that a) there is no superposition principle to resort to. One may not add the dark companions of two point baryonic masses, say; for, λ is not proportional to M but rather to its square root. b) The dark companion of a localized point mass is not itself localized. Certainly, more accurate rotation speeds, specially in orbits outside the plane of the galaxy will be helpful.

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