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Dynamics of Compact Objects

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Abstract:

It has been known for over 40 years that the velocity curves of spiral galaxies are not explained by Newton's laws of motion and of gravitation, if only the observed luminous matter and its distribution are taken into account. See e.g. Roberts and Rots (1937) and Einasto et. al. (1974). The velocity curves are much flatter than expected and require much more mass than that observed. Also, in clusters of galaxies, the virial masses required to explain the equilibrium state the system are much larger, by factors of the order 10, than the luminosity masses.

To explain the discordance some investigators have suggested some form or another of dark matter that shows only gravitational effects, but does not share any other properties with the ordinary laboratory and celestial matter that one knows of. See e.g. Ostriker and Steinhardt (2004) for a list of such suggestions. After examining a host of assumptions over the decades as to the nature of dark matter, its amount and distribution, its electromagnetic, weak and strong interaction properties, its possible interactions with the ordinary matter, no consensus of opinion has been reached among the experts. The hypothesised dark matter hides itself. The situation is not unlike the case of ether in late 19th century. Physicists had discovered the wave nature of the light, and out of the habit that any wave requires a medium for propagation had hypothesised ether as such medium. Yet ether escaped detection no matter how ingeniously the detection devices were designed. It was up to Einstein to leave ether to itself and suggest a new dynamics without it.

Perhaps inspired by such parallelism with ether, Milgrom (1983a, b, c) and others (see e.g. Sanders and McGaugh, 2002) suggested to modify Newton's gravitation at accelerations smaller than 10^{-8} cm/sec.sec. to explain the velocity curves of the galaxies. Their modified theory, however, leaves features to be desired.

We also wish to doubt the validity of Newtonian dynamics in galactic and extra galactic scales. We propose to revise both classical laws of motion and of gravitation by examining a new action integral. The system is assumed to consist of particles m_i at positions x_i, and of field $\phi(x)$. The action is taken to be

$$I = \int L \ dt$$

$$L = \sum_i \left\{ \frac{1}{2} m_i \ x_i^2 - m_i \ \phi(x_i) \right\}$$

$$-\frac{1}{8\pi G}\int \left|\nabla\,\phi(x)\right|^2\, d^3x$$
$$+\ \frac{1}{8\pi G}\sum_i R_i\int \left|\nabla\phi(x)\right|^2 \left|x_i - x\right|^{-1} d^3x$$

Where R_i is a length of the order of Schwarzchild radius of mass m_i. The last term contains specifications of both matter, x_i's and the field, $\phi(x)$. It guarantees the mutual interaction of the particles and the field.

Functional derivative $\delta I/\delta\phi(x)$ gives the revised Poisson's equation

$$\nabla.\left[\nabla\,\phi\left\{1-\sum_i \frac{R_i}{|x_i - x|}\right\}\right] = 4\pi G\sum_i m_i\delta(x_i - x).$$

For a single gravitating particle of mass M at position r the solution is

$$\phi(x) = \frac{GM}{R}\,\ell n\left(1-\frac{R}{r}\right) \approx -\frac{GM}{r} + ...$$

Functional derivative $\delta I/\delta x_i$ gives the revised equation of motion

$$\ddot{x}_i - \nabla\,\phi(x) + \frac{R}{8\pi G}\nabla_i\int \frac{|\nabla\phi(x')|^2}{|x_i - x'|}\,d^3x' = 0.$$

The circular orbital speed from this equation is

$$V^2 = \frac{GM}{r}\left\{1+u(s)+\frac{R}{r-R}\right\},$$

where

$$u(s) = \frac{3}{s}\left[1-\sqrt{\frac{2-3s}{s}}\,\arctan\sqrt{\frac{s}{2-3s}}\right]-\frac{1}{2},$$

where $0\langle s = R/\text{radius of the central object} \le \frac{2}{3}$.

The plot of $u(s)$ is given in Figure 1. For an object collapsed into its Schwarzshild radius R, $u(s) = 4.$ This means that a collapsed object displays a gravitational field five times more than the when it is in a non compact state.

Conclusion:

The proposed revision maintanes that one does not have to resort to dark matter hypotheses, at least not to the extent that is advocated by its proponents, to explain the dynamics of celestial systems. Taking collapsed objects into consideration might, at least partially, remedy the situation.

References

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