Point Mass Concept Cause of Singularities in GR

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We argue that the classical energy-momentum tensor in Einstein's field equation, regardless of how it is conceived, implicitly assumes the existence of point particles with exact positions and momenta, an assumption that contradicts Heisenberg's uncertainty principle. To resolve this inconsistency, we propose replacing each constituent of the classical energy-momentum tensor with a quantum mechanical counterpart: a *collection* of Klein-Gordon (KG) fields or their Yukawa-ameliorated versions (YKG). Unlike point particles, waves are inherently distributed entities, and thus, do not lead to spacetime singularities. Moreover, KG and YKG waves propagate to infinity, generating not only the familiar Newtonian r^{-2} force but also a non-Newtonian r^{-1} force. The latter can account for the flat rotation curves of spiral galaxies (in line with the Tully-Fisher relation) and may be interpreted as a dark matter scenario or an alternative gravity.

1. Introduction

In Newtonian gravitation, GM/r, or in Coulomb's law, Q/r, one implicitly assumes there are physical entities of finite mass or charge packed in zero volumes. One further assumes that, in odds with Heisenberg's principle of uncertainty, these point-likes can have precise coordinates and momenta. The point particle singularity is a common feature of all non-quantum physics, ranging from mechanics, electromagnetism, special and general relativity, to the physics of continuous media in its broadest sense. As long as the phase-space volume available to a dynamical system is spacious, classical approximations are adequate. Quantum deviations, however, develop when a system is forced to evolve in tighter phase space volumes. One example from astronomical realms is the case of neutron stars, an aggregate of neutrons compressed into exceedingly small volumes and cooled to about Fermi temperatures.

This paper is the abridged and modified version of.[1] It builds upon our ongoing effort to address limitations of the point-particle concept. Here we argue that the energy-momentum tensor in GR is that of a collection of point particles. We suggest to replace each of those gravitating particles by a quantum mechanical equivalent, a Klien-Gordon (KG) or the Yukawa-ameliorated version of it, YKG field. Quantum fields are distributed entities. They do not end up in spacetime singularities. Quantum fields are waves and can reach infinity. We show that their cumulative quantum relic creates r^{-1} gravity forces. The latter can in turn explain flat rotation curves of spiral galaxies and be interpreted as an alternative gravity, a dark matter paradigm or other.

Resorting to auxiliary fields in GR, for different needs and purposes, has a long history and rich literature.[2, 3, 4, 5, 6, 7, 8, 9, 10] What differentiates this paper from the ones cited here or not is our focus on the fact that particles shaping spacetime structure are not point-like but extended wave packets. We find that due recognition of this wave nature of individual gravitating particles not only removes the essential and coordinate singularities at individual spacetime points, but alters the spacetime structure at galactic distances. Evidently the cumulative quantum waves of a coherent and/or random collection of gravitating particles do not cancel out each other. At far outreaches of a galaxy, say, they create r^{-1} force and cause flat rotation curves. We will come back to this issue in sections 3 and 4.

2. Definition of the problem

A pair of neutrons coupled together through their isospin behave like an electrically neutral boson, a good approximation for Neutron stars. Neutrons aside, one knows that 99% of the matter in Universe is hydrogen and helium, either in interstellar and intergalactic spaces or as an electrically neural plasma in stars. With some condone we contend to mimic the gravitating matter of a galaxies, say, as the collection of N spin 0 bosons each (for the time being) represented by a KG field. For the Lagrangian density and the Euler-Lagrange equation of the model we write,

$$\mathcal{L} = N\left(\frac{\hbar^2}{m}g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi + mc^2\psi^*\psi\right) \tag{1}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\psi\right) + \frac{m^2c^2}{\hbar^2}\psi = 0, \quad \text{for each boson,} \tag{2}$$

where m is the mass of each constituent, of the order of one or two nucleon mass. And N is the number of nuclei in the gravitating matter, of the order $\approx 10^{69}$ for a Milky way type galaxy. The corresponding energy-momentum tensor of each constituent is,

$$\begin{aligned} T_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\partial}{\partial g^{\mu\nu}}\left(\sqrt{-g}\mathcal{L}\right) \\ &= -\frac{\hbar^2}{m}\partial_\mu\psi^*\partial_\nu\psi + \frac{1}{2}g_{\mu\nu}\left[\frac{\hbar^2}{m}\partial^\alpha\psi^*\partial_\alpha\psi - mc^2\psi^*\psi\right]. \end{aligned} \tag{3}$$

Einstein's field equation may now be written as,

$$\begin{aligned} R_{\mu\nu} &= \frac{8\pi G}{c^4}N\left[T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right] \\ &= \frac{4\pi GNm}{c^2}\left[g_{\mu\nu}\psi^*\psi - 2\frac{\hbar^2}{m^2c^2}\partial_\mu\psi^*\partial_\nu\psi\right]. \end{aligned} \tag{4}$$

For an exponential time dependence $\exp(-i\omega t)$, the two terms $\psi^*\psi$, $\partial_\mu\psi^*\partial_\nu\psi$ in (4) are time-independent. One may devise a static and spherically symmetric Schwarzschild-like metric,

$$ds^2 = -B(r)c^2dt^2 + A(r)dr^2 + r^2d\Omega^2, \quad \text{signature:}(-,+,+,+). \tag{5}$$

Reduction of (2) with (5) gives,

$$\frac{d^2\psi}{dr^2} + \left(\frac{2}{r} - \frac{1}{2A}\frac{dA}{dr} + \frac{1}{2B}\frac{dB}{dr}\right)\frac{d\psi}{dr} + \frac{m^2c^2}{\hbar^2}\left(1 + \frac{1}{B}\frac{\hbar^2\omega^2}{m^2c^4}\right)\psi = 0. \tag{6}$$

From the following combination of the components of (4) one finds

$$\begin{aligned}\frac{R_{tt}}{2B} + \frac{R_{rr}}{2A} + \frac{R_{\theta\theta}}{r^2} &= \frac{1}{r^2}\left(\frac{d}{dr}\Big(\frac{r}{A}\Big) - 1\right)\\ &= -\frac{2\pi N r_s}{C}\left(\Big(1 - \frac{1}{B}\frac{\hbar^2\omega^2}{m^2c^4}\Big)\psi^*\psi - \frac{1}{A}\frac{\hbar^2}{m^2c^2}\frac{d\psi^*}{dr}\frac{d\psi}{dr}\right),\end{aligned} \tag{7}$$

$$\frac{R_{\theta\theta}}{r^2} = \frac{1}{r^2}\Big(\frac{1}{A} - 1\Big) + \frac{1}{2rA}\frac{d}{dr}\ln\left(\frac{B}{A}\right) = \frac{2\pi N r_s}{C}\psi^*\psi.\ r_s := 2Gm/c^2, \tag{8}$$

where C is a constant to be discussed later. In (6) and (7), $(\hbar^2\omega^2/m^2c^4)$ is the ratio of the square of the oscillation energy to the rest mass energy of the KG constituent. And $\frac{\hbar^2}{m^2c^2} \approx 10^{-31}\text{m}^2$ is the square of the Compton wavelength of the constituent nucleon. Both are extremely small. One may safely ignore them without compromising the essential role of the wave nature of the KG field. Furthermore, it is preferable to work with dimensionless radius, $x = r/r_s$, $d/dx = r_s d/dr$, $r_s = 2Gm/c^2$. On applying these simplifications to (6), (7) and (8) one arrives at.

$$\psi'' + \left(\frac{2}{x} - \frac{1}{2}\frac{A'}{A} + \frac{1}{2}\frac{B'}{B}\right)\psi' + k^2\psi = 0, \quad k = r_s\frac{mc}{\hbar} \ll 1, \tag{9}$$

$$\frac{1}{x^2}\left(\frac{d}{dx}\left(\frac{x}{A}\right) - 1\right) = -\frac{2\pi N}{C}\psi^*\psi, \tag{10}$$

$$\frac{1}{x^2}\left(\frac{1}{A} - 1\right) + \frac{1}{2xA}\frac{d}{dx}\ln\left(\frac{B}{A}\right) = \frac{2\pi N}{C}\psi^*\psi, \tag{11}$$

The coupled and non-linear equations (9) - (11) can be solved by iteration. As an initial guess we adopt $B = A^{-1} = (1 - x^{-1})$, substitute them into (9) and solve it for ψ, substitute the result into (10) and solve it for A, and finally substitute the results into (11) and solve it for B.

I. For near origin, $(x \to 0)$, we expand all functions as Taylor series in x and find

$$\psi(x) = 1 - \frac{1}{4}k^2x^2 + \cdots, \tag{12}$$

$$B(x) = \frac{1}{A(x)} = 1 + \frac{2\pi N}{C}\Big(\frac{1}{3}x^2 - \frac{1}{10}k^2x^4 + \cdots\Big), \tag{13}$$

Essential singularity is removed. Existence of singular blackholes is ruled out There could, however, be non-singular ones if a fraction of the gravitating KG bosons become degenerate. Gravitational forces will not be able to crush them into singularity without violating the uncertainty principle. Moreover, spacetime is asymptotically flat. A test particle, the nearer to origin, will experience the lesser gravitational force and the freer it will be. This reminds one of the asymptotic freedom of quarks confined within nuclei. We will come back to this analogy in section 4, where we elaborate on the commonalities of gravitational and strong nuclear forces.

II. For $x \to 1$, we expand all functions as Taylor series in $(x-1)$. To the order $(x-1)^2$ for ψ, and the order $(x-1)$ for A and B we find

$$\psi = 1 - \frac{1}{4}k^2(x-1)^2, \tag{14}$$

$$B = \frac{1}{A} = 1 + \frac{1}{3}\frac{2\pi N}{C}x^2\left(\left(1-\frac{1}{2}k^2\right) - \frac{3}{4}k^2\left(x-1\right)\right). \tag{15}$$

Horizon singularity is removed. Both A and B keep their spacelike and timelike nature before and after $x = 1$.

III. The far distance solutions are the most interesting. As $x \to \infty$ both A and $B \to 1$. (9) reduces to the standard Helmholtz equation with analytical solutions,

$$\psi'' + \frac{2}{x}\psi' + k^2\psi = 0, \quad \psi = \frac{1}{x}e^{\pm ikx}, \quad \textit{not normalizable!} \tag{16}$$

But ψ of (16) is not normalizable. At $x \to \infty$, it does not fall off steeply enough to have finite $\int \psi^*\psi$. To circumvent the difficulty we suggest a Yukawa-ameliorated Klein-Gordon (YKG) field, $\frac{1}{x}e^{-(\kappa \pm ik)x}$, where we will shortly see that $\kappa \ll 1$. Such an amelioration is logical and perhaps imperative; for it is inconceivable to imagine the quantum wave of a single particle to extend to infinity and alter the spacetime structure at universal scales. With the inclusion of κ, (16) changes accordingly,

$$\psi'' + \frac{2}{x}\psi' - (\kappa \pm ik)^2\psi = 0, \tag{17}$$

$$\psi = \sqrt{\frac{\kappa}{2\pi}}\frac{1}{x}e^{-(\kappa \pm ik)x}, \quad \int \psi^*\psi dx^3 = 1. \tag{18}$$

Substitution (18) in (10) and (11) gives.

$$B = A^{-1} = 1 + \frac{N}{2x}e^{-2\kappa x} = 1 + \frac{Nr_s}{2r}e^{-2\kappa r/r_s}. \tag{19}$$

Spacetime is essentially flat, except for a small static and exponentially dying out gravitational ripple $\frac{Nr_s}{2r}e^{-2\kappa r/r_s}$.

To appreciate the physics behind (19), let us restore the variable $r = xr_s$, and examine the gravitational force in the weak field approximation,

$$\begin{aligned}\frac{1}{2}c^2\frac{dB}{dr} &= -\frac{1}{2}Nc^2\left(\frac{r_s}{r^2} + \frac{2\kappa}{r}\right)e^{-2\kappa r/r_s} \\ &= -\left(\frac{NGm}{r^2} + \frac{Nc^2\kappa}{r}\right)e^{-2\kappa r/r_s}. \end{aligned} \tag{20}$$

The $1/r^2$ term is the Newtonian force, albeit modulated by the Yukawa falloff factor. The $1/r$ term is non-Newtonian and non-GR. We will shortly see that it is responsible for the flat rotation curves of spiral galaxies.

2.1. *Determination of κ - Tully-Fisher relation (TFr):*

Abstracted from a forage of observational data,,[11] TFr states:

a. The observed rotation speeds, $v^2 = rg_{grav}$, of distant stars and HI clouds in spiral galaxies show a much gentler decline than Newton's GM/r.
b. The observed v^2, more often than not, has the flat asymptotic $v^2 \propto \sqrt{M}$, rather than $\propto M$ in the Newtonian gravitation.

In his Modified Newtonian Gravity (MOND), Milgrom encapsulates[12] TFr by noting that:

$$\begin{aligned} g_{effective} &= g_{New} \quad \textit{if large}, \\ &= \sqrt{g_{New}a_0}, \quad \textit{if small}, \quad a_0 = 1.2 \times 10^{-10} m.sec^{-2}. \end{aligned}$$

Leveraging TFr and MOND, the $1/r$ term in (20) may now be written as,

$$Nc^2\frac{\kappa}{r} = \sqrt{\frac{NGm}{r^2}\left(\frac{Nc^4\kappa^2}{Gm}\right)} = \sqrt{g_{New}\left(\frac{Nc^4\kappa^2}{Gm}\right)}.$$

It only suffices to identify $\left(\frac{Nc^4\kappa^2}{Gm}\right)$ with Milgrom's a_0 and obtain,

$$\kappa = \sqrt{\frac{Gm}{Nc^4}a_0} \approx 4.06 \times 10^{-41}\frac{1}{N^{1/2}} = 4.06 \times 10^{-41}\left(\frac{m}{M}\right)^{\frac{1}{2}}, \tag{21}$$

where for m we have used the mass of a nucleon. For a Milky Way type galaxy of about $1.5 \times 10^{12} M_\odot$, one has

$$\kappa_{MW} \approx 9.6 \times 10^{-76}.$$

Note how small κ is and how gently the factor $e^{-2\kappa x}$ falls off as $x \to \infty$.

3. Massive Graviton

Yukawa introduced his field in 1935 to model the *residual* strong forces that bind nucleons together. The forces are attractive. There are two masses in Yukawa paradigm, the masses of u and d quarks as the sources of strong interactions, and the masses of three mediating pions ($\pi^{\pm},\ \pi^{0}$).

YKG of (17) and (18) is the gravitational analog of Yukawa field albeit in galactic scales. YKG forces are also attractive. There are also two masses in YKG; the masses of gravitating YKG waves, m, seen in $k = \frac{2Gm^2}{ch}$, and masses of the mediating gravitons associated with κ. One may have already guessed (and we will show in a forthcoming communication) that the massive YKG gravitons come in three polarizations in one to one correspondence with the three pions.

The Compton wavelength and mass of YKG massive gravitons are:

$$\lambda_g = \frac{r_s}{2\kappa} = \frac{Gm}{c^2\kappa} = 3.04 \times 10^{-14} \left(\frac{M}{m}\right)^{1/2} \text{meter}, \tag{22}$$

$$m_g = \frac{\hbar}{c\lambda_g} = 1.16 \times 10^{-29} \left(\frac{m}{M}\right)^{1/2} \text{ kg} = 6.49 \times 10^{4} \left(\frac{m}{M}\right)^{1/2} \text{ev}\,. \tag{23}$$

It can be easily verified that at $r = \lambda_g$, a) the two r^{-2} and r^{-1} forces become equal, that is the non-newtonian gravitational force becomes dominant at distances farther than λ_g. And b) at $r = \lambda_g$ the exponential factor $e^{-2\kappa r/r_s}$ reduces to e^{-1}. From Table(1), for the Milky Way we find,

$$\lambda_g^{MW} \approx 1.2 \times 10^{21}\text{m} \approx 38 \text{ kpc}, \qquad m_g^{MW} \approx 2.8 \times 10^{-64} \text{ kg} \approx 9.5 \times 10^{-29} \text{ ev}.$$

In Table (1) we have calculated the mass and the wavelength of gravitons for the Milky Way, the Sun, the Earth, one kg mass, the nucleons, and the u and d quarks

- In the Milky Way, transition from Newtonian to the non-Newtonian gravitation takes place at 38 kpc, far outside of the visible disc of the galaxy of radius 26.8 kpc.
- For the Sun (or rather the whole solar system mass) transition takes place at 7100 AU, somewhere within the Oort clouds. The latter are believed to have an inner edge of 2000 to 5000 AU and an outer edge of 10000 to 100000 AU, see https://science.nasa.gov/solar-system/oort-cloud/.
- For the Earth transition distance is longer than Jupiter's orbital radius, 9.54 AU.
- For a one kg weight, $\lambda_g = 74$ cm is small enough to think of a table top setup to check deviations from Newtonian gravitation.
- The last four lines for p, n, u, d are included as a matter of curiosity. They should not taken seriously. It is interesting, however, to note that how the gravitons mediating the gravitational interaction of u and d, and pions mediating the residual strong interaction of the same particles have almost the same masses.

There are further parallelism between our YKG gravitation and the Yukawa field for the residual strong interactions. In the concluding section, 4, we elaborate on these parallels and prudently dare suggest that both might be the two extremes of one and the same physical notion.

Table 1. Compton wavelength and mass of massive gravitons that mediate the YKG gravitation of a wide range of masses, from celestial bodies to quarks.

	Mass (kg)	$\lambda_g(m)$	m_g
MW	1.5×10^{42}	$1.2 \times 10^{21} \approx 38$ kpc	9.5×10^{-29}eV
Sun	2×10^{30}	$1.06 \times 10^{15} \approx 7100$ AU	1.7×10^{-22} eV
Earth	6×10^{24}	$1.7 \times 10^{12} \approx 11.3$ AU	1.06×10^{-19}eV
1 kg	1	0.74	26×10^{-6}eV
p & n	1.673×10^{-27}	30×10^{-15}	6.4 MeV
d quark	8.54×10^{-30}	2.2×10^{-15}	85.8 MeV
u quark	3.56×10^{-30}	1.4×10^{-15}	134.6 MeV

4. Conclusion

We argue that the point particle concept in classical physics is singular and contradicts Heisenberg's uncertainty principle. In GR, the energy-momentum tensor consists of a collection of point particles. We propose replacing each member of that collection with a Yukawa-ameliorated Klein-Gordon (YKG) wave.

In an spherically gravitating system, waves being distributed entities, eliminate essential and horizon singularities from spacetime. However, non-singular black holes may exist if a fraction of the YKG waves enters a degenerate state.

YKG waves extend to infinity and produce a non-Newtonian r^{-1} gravitational force. This feature can explain the flat rotation curves of spiral galaxies, offering a potential alternative to dark matter scenarios and other gravity theories.

There are a number of interesting parallelism between the YKG gravitation and the residual strong interactions:

- To begin with, both forces are attractive and exhibit asymptotic freedom. As two interacting particles move closer to one another, the strength of both gravitational and strong forces diminishes to zero (see the comment below equation (13)).
- In the standard GR, gravitation is mediated by massless gravitons, which have two polarization states. YKG gravitons are massive and (we will demonstrate in a forthcoming publication that) possess at least three polarization.
- These polarization states can be put in one to one correspondence with the three Yukawa pions that mediate residual strong interactions.
- The mass of YKG gravitons, for the gravitational interaction between nucleons and the d and u quarks, is nearly identical to the mass of π mesons

(see Table 1).

Given these similarities, is it reasonable to speculate that YKG gravitation and at least the residual strong interactions are the two extremes of one and the same underlying principle?

Comment. References

1. Y. Sobouti and H. Sheikhahmadi, [arXiv:2404.18954 [physics.gen-ph]].
2. R. Ruffini and S. Bonazzola, “Systems of selfgravitating particles in general relativity and the concept of an equation of state,” Phys. Rev. **187**, 1767-1783 (1969)
3. H. A. Buchdahl, “General Relativistic Fluid Spheres,” Phys. Rev. **116**, 1027 (1959)
4. K. S. Virbhadra, “Janis-Newman-Winicour and Wyman solutions are the same,” Int. J. Mod. Phys. A **12**, 4831-4836 (1997) [arXiv:gr-qc/9701021 [gr-qc]].
5. J. W. Moffat, “Scalar-tensor-vector gravity theory,” JCAP **03**, 004 (2006) [arXiv:gr-qc/0506021 [gr-qc]].
6. J. W. Moffat and V. T. Toth, “Rotational velocity curves in the Milky Way as a test of modified gravity,” Phys. Rev. D **91**, no.4, 043004 (2015) [arXiv:1411.6701 [astro-ph.GA]].
7. J. D. Bekenstein, “Alternatives to Dark Matter: Modified Gravity as an Alternative to dark Matter,” [arXiv:1001.3876 [astro-ph.CO]].
8. P. Jetzer, P. Liljenberg and B. S. Skagerstam, “Charged boson stars and vacuum instabilities,” Astropart. Phys. **1**, 429-448 (1993) [arXiv:astro-ph/9305014 [astro-ph]].
9. M. Bezares, M. Bošković, S. Liebling, C. Palenzuela, P. Pani and E. Barausse, “Gravitational waves and kicks from the merger of unequal mass, highly compact boson stars,” Phys. Rev. D **105**, no.6, 064067 (2022) [arXiv:2201.06113 [gr-qc]].
10. A. R. Liddle and M. S. Madsen, “The Structure and formation of boson stars,” Int. J. Mod. Phys. D **1**, 101-144 (1992)
11. R. B. Tully and J. R. Fisher, “A New method of determining distances to galaxies,” Astron. Astrophys. **54**, 661-673 (1977)
12. Y. Sobouti, ‘Dark companion of baryonic matter,” [arXiv:0810.2198 [gr-qc]].