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Massive Gravity as an Alternative Gravity

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Abstract—The Newtonian gravity force is massless and decreases as $1/r^2$, too steeply to explain the flat rotation curves of spiral galaxies. Massive gravity, on the other hand, drops as $1/r$ and is capable of doing the job. Massive fields have a respected record in the history of field theories. We follow the suit and add a "mass term" to the field equation of Newtonian gravity, which, to begin with, is static. Next, we use the observation-based Tully-Fisher relation to determine the nature and characteristics of the added mass term. We are able to produce the rotation curves flat enough to justify the observational data up to several optical radii of the galaxies, where observations are both abundant and reliable. At very far distances, however, massive gravity goes through a sequence of intermittently attractive and repulsive phases. This is a welcome novelty. It may enable one to address the wavy fluctuations and patchy voids that are not uncommon in the archives of observed rotation curves. With a stretch of imagination, massive gravity may find an observational support in the Oort clouds, a stipulated spherical shell of debris at farthest outreaches of the Solar system.

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1. INTRODUCTION

Zwicky [1] is credited for a decisive inference that the Newtonian gravity of the observable matter in galaxies is much too weak to explain the flat rotation curves of spiral galaxies, or the large velocity dispersions in elliptic galaxies and clusters of galaxies. Since then, vast literature has followed. Attempts to resolve the puzzle range from a variety of alternative theories of gravity to a multitude of dark matter scenarios.

What we propose here might be categorized as an alternative theory of gravity. We argue that the Newtonian massless, $1/r^2$, force dies away too steeply to explain the dynamics of galaxies at far distances. A massive field, on the other hand, behaves as $1/r$ and can produce flat asymptotes. The proposed alternative adequately justifies the presently available body of observations.

At still father distances, massive gravity predicts alternately attractive and repulsive forces. This means alternately regions of stable and unstable orbits, wavy fluctuations and/or patchy voids on rotation curves. Rotation curves with wavelike variations on them are not uncommon, but, to the best of the author's knowledge, are seldom addressed in the literature. We think that massive gravity has the capacity to provide an answer.

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The Tully-Fisher relation, that rotation curves, more often than not, have flat asymptotes, and the asymptotic speeds are more or less proportional to the fourth root of the luminosities (masses) of the spirals, plays a pivotal role in determining the characteristics of the mass of massive gravity.

The paper is organized as follows. In Section 2, for pedagogical reasons and in order to see the lineage of what we propose here to what already exists in the literature, we give a brief review of massive gravity waves. In Section 3 we introduce our static massive gravity, extract the mass parameter of the field from the Tully-Fisher relation, and demonstrate that the proposed static massive gravity is indeed capable of giving rotation curves flat enough to explain the bulk of the archived data. In Section 4 we explore possible observational implications of the theory in the Solar system and its possible tests in galactic environments. In Section 5 we summarize our conclusions and list an agenda items for future follow-up.

2. A BACKGROUND REVIEW OF MASSIVE GRAVITY WAVES

In this section we consider certain historical developments leading to massive gravity waves and massive gravitons. We will not return to them in the rest of the paper. One may wish to leave out the section altogether, or find it to be of interest to see the parities/disparities between a propagating massive wave and a static massive field.

The earliest record of massive gravity goes back to Fierz and Pauli [2]. Years later, particularly in 1970–80 and the 2000s, the theory was refined. and nowadays one has a consistent theory of massive spin 2 gravitons that propagate with 5 polarizations.

Let us consider a spacetime metric $g_{\mu\nu}$ and expand it about the flat Minkowsky metric $\eta_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, |h_{\mu\nu}| \ll 1. \tag{1}$$

In harmonic coordinates, in which the contracted Christoffel connections vanish, $\Gamma_\lambda = \Gamma^\mu{}_{\mu\lambda} = 0$, Einstein's field equation reduces to

$$\partial^\lambda \partial_\lambda h_{\mu\nu} = 0, \tag{2}$$

$$\partial_\lambda h^\lambda{}_\mu - \frac{1}{2}\partial_\mu h = 0, \quad h = h^\lambda{}_\lambda. \tag{3}$$

In principle, there are 10 components of the symmetric $h_{\mu\nu}$. But the four harmonic coordinate conditions, or any other equivalent set, reduce them to 6. To have massive waves, one supplements the Einstein-Hilbert Lagrangian with a mass term [2],

$$m^2(h^{\mu\nu}h_{\mu\nu} - h^2). \tag{4}$$

Fierz and Pauli note that any other quadratic combination of $h_{\mu\nu}$ and h induces unwanted ghost modes.

The added mass term, however, breaks the diffeomorphism invariance of the theory. To restore the invariance, Stueckelberg fields are invited in, an electromagnetic 4-vector, A_μ; and a scalar χ. By doing so one, eventually, arrives at a theory of massive gravity waves, or gravitons, propagating with five polarizations; ± 2 in common with the massless case, ± 1 excited by the A_μ vector and 0 associated with the χ field. Of the six degrees of freedom, to choose $h_{\mu\nu}$, one gets discarded by the particular choice of the mass term (4), to avoid awakening a ghost. We will shortly return to this suppressed degree of freedom. Some of the later but monumental refinements of the theory are as follows:

- The vDVZ discontinuity [3, 4]: The Fierz-Pauli theory does not uniformly converge to the massless case as $m \to 0$. There is a discordance in light bending between the Fierz-Pauli theory and the classical GR in the limit of $m \to 0$.

- The Vainshtein screening [5]: The vDVZ discontinuity is an artifact of the linear theory. It can be removed by including higher powers of $h_{\mu\nu}$ in the mass terms.

- The Boulware–Deser ghosts [6]: Higher powers of $h_{\mu\nu}$ come with their own ghosts.

- Finally, in refined massive gravity, ghosts of the higher order theories are packed away [7–9].

The interested reader may find a comprehensive review of massive graviton and its further generalizations and alternatives in [10]. A massive graviton is also suggested as a testable cold matter candidate in [11].

In the last decade or so, a number of static massive gravities have appeared in the literature. Of relevance to this paper are, e.g., [13] discussing a 3D static massive gravity, [12] developing a spherically symmetric and asymptotically flat static gravity, [14] classifying six derivative Lagrangians in pursuit of static spherically symmetric solutions, [15] investigating the stability of the static Einstein universe with massive gravity, and [16] entertaining a 3D static gravity minimally coupled to a massive scalar field.

3. STATIC MASSIVE GRAVITY

What is said above concerns the propagating massive gravity waves. But a gravity field can be static. In the weak field approximation, one assumes that of all $h_{\mu\nu}$'s, only $h_{00} = -2\Phi(\mathbf{r})/c^2$ matters and can be time-independent. The harmonic conditions (3) reduce to $0 = 0$, and the GR field equations give $\nabla^2\Phi = 4\pi G\rho$. This is the familiar Poisson equation for Newtonian gravity sourced by the density distribution $\rho(\mathbf{r})$. We add a "mass term" to it,

$$\nabla^2\Phi + k^2\Phi = 4\pi G\rho, \quad k = \frac{c}{\hbar}m, \tag{5}$$

where k has the dimension of $(length)^{-1}$ with a corresponding mass equivalent, $m = \hbar k/c$. This mass term is the only choice we have to resort to. It is different from that of (4), which is identically zero in the present approximation. In a way it is an equivalent of the sixth degree of freedom that Fierz and Pauli had left out to avoid ghosts. Had we allowed a time-dependent $\Phi(\mathbf{r}, t)$ and had we insisted on restoring the diffeomorphism invariance of the theory by inviting in some sort of Stuekelberg field, we would perhaps have had a propagating ghost. By sticking to a static $\Phi(\mathbf{r})$, shall we say, the would-be ghost is immobilized!

We will solve (5) by a Green function technique. For a point mass at the origin, $\rho = M\delta(\mathbf{r})$, one finds

$$\Phi(r) = -GM\frac{1}{r}\cos(kr - \alpha), \tag{6}$$

where α is a dimensionless adjustable constant of integration introduced for later convenience. The gravitational force and the circular orbital velocity are

$$\Phi' = \frac{GM}{r^2}[\cos(kr - \alpha) + kr\sin(kr - \alpha)], \tag{7}$$

$$v^2{}_{\rm circ} = r\Phi' = \frac{GM}{r}[\cos(kr-\alpha) + kr\sin(kr-\alpha)]. \quad (8)$$

Equations (6)–(8) for small r behave as Newtonian ones, but for $(kr-\alpha) \to \pi/2$ they give a $1/r$ force and a constant amplitude $v^2{}_{\rm circ}$. But before proceeding further, let us see what one may say about the added mass term.

3.1. What is k and How Far is $(kr-\alpha) = \frac{\pi}{2}$

The Tully—Fisher relation (TFR), first published in 1977, states that the orbital velocities of stars and HI clouds at far outreaches of spirals remain almost constant, and are more or less proportional to the 3.5—4th root of the luminosities of the spirals [17]. The luminosities themselves are, empirically, assumed to be proportional to the masses of the spirals. Nowadays, striped away from its technical and formal astronomical jargon, the TFR is agreed upon to mean the followings:

- The rotation curves of spirals have, more often than not, flat asymptotes.
- The asymptotic speed is proportional to the fourth root of the observable baryonic mass of the galaxy,
$$v^2_{\rm asymp} \propto M^{1/2}.$$

These two observation-based statements are in discordance with the Newtonian prediction that the square of the orbital velocities die away as M/r while remaining proportional to the mass of the body residing at the center.

Based on the TFR and analysis of a large number of observed rotation curves, Milgrom [18] was able to design his theory of "MOND," where he argued that the effective gravitational force operating at the galactic scales behaves in a Newtonian way, g_N, at large accelerations, but drops to $\sqrt{g_N a_0}$ at $g_N \ll a_0$, where a_0 has the dimension of acceleration. To say the least, MOND's limits of strong and weak acceleration are well in line with the observations from which the TFR is concluded. In particular, fitting the weak acceleration, $\sqrt{g_N a_0}$, to the flat asymptotes of the rotation curves of spirals, Milgrom and others, e.g., in [19] and [20], find

$$a_0 \approx 1.2\times 10^{-10}\ {\rm m/s}^2. \quad (9)$$

This number seems to be universal and is commonly used in devising a variety of alternative gravities such as MOND itself, $f(R)$ gravities, etc.

Returning to our massive gravity, let us interpret the amplitude GMk/r in the second term on the right-hand side of (7) as

$$\frac{GMk}{r} = \sqrt{\frac{GM}{r^2}a_o} \Rightarrow k = \sqrt{\frac{a_0}{GM}}, \quad (10)$$

$$\frac{1}{k} = \sqrt{\frac{GM}{a_0}} = 7000\sqrt{\frac{M}{M_\odot}}\ {\rm au} = 34\sqrt{\frac{M}{10^{12}M_\odot}}{\rm kpc}. \quad (11)$$

For one solar mass, 7000 au is far outside the solar planetary orbits and the Kuiper belt, reaching almost the Oort clouds. For an Andromeda size galaxy of about $10^{12}M_\odot$, 34kpc is more than two or three times the optical radius of the galaxy. These huge distances only emphasize the fact that deviations from Newtonian gravity should be expected at much farther distances from the gravitating bodies than in their immediate surroundings

3.2. Massive gravity of extended systems

Knowing the Green function of (6), the general solution of (5) is

$$\Phi(\mathbf{r}) = -G\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\cos(k|\mathbf{r}-\mathbf{r}'|-\alpha)d^3\mathbf{r}', \quad (12)$$

$$\nabla\Phi(\mathbf{r}) = G\int \rho(\mathbf{r}')\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\Big[\cos(k|\mathbf{r}-\mathbf{r}'|-\alpha) + k|\mathbf{r}-\mathbf{r}'|\sin(k|\mathbf{r}-\mathbf{r}'|-\alpha)\Big]d^3\mathbf{r}'. \quad (13)$$

It is easy to see that for $r \gg r'$, (12) and (13) reduce to the point source case of (6) and (7), respectively. In the immediate and intermediate neighborhood of the system, however, where r and r' are of the same order of magnitude and $k|\mathbf{r}-\mathbf{r}'| \ll 1$, they behave in the Newtonian way. One may Taylor expand the trigonometric functions in powers of $k|\mathbf{r}-\mathbf{r}'|$ and find

$$\Phi(\mathbf{r}) = -G\cos\alpha\Big[\int\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}' - \frac{1}{2}k^2\int\rho(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|d^3\mathbf{r}'\Big] - G\sin\alpha[Mk + \mathcal{O}(k^3)],$$

$$M = \text{ total mass of the system}, \quad (14)$$

$$\nabla\Phi(\mathbf{r}) = +G\cos\alpha\int\rho(\mathbf{r}')\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \times \Big[1-\frac{1}{2}k^2|\mathbf{r}-\mathbf{r}'|^2\Big]d^3\mathbf{r}' + \mathcal{O}(k^3). \quad (15)$$

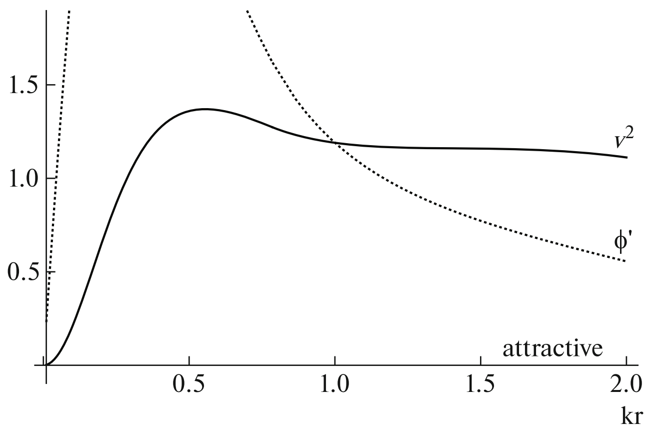

Fig. 1. Flat rotation curve in massive gravity. For a galaxy of $10^{12} M_\odot$, the number $kr = 2$ corresponds to $r \approx 70$ kpc, 4–5 times the optical radius of the galaxy, and for one solar mass to $r \approx 15000$ au, far outside the solar planets and the Kuiper belt, reaching the Oort clouds. The numbers are adjustable by changing α.

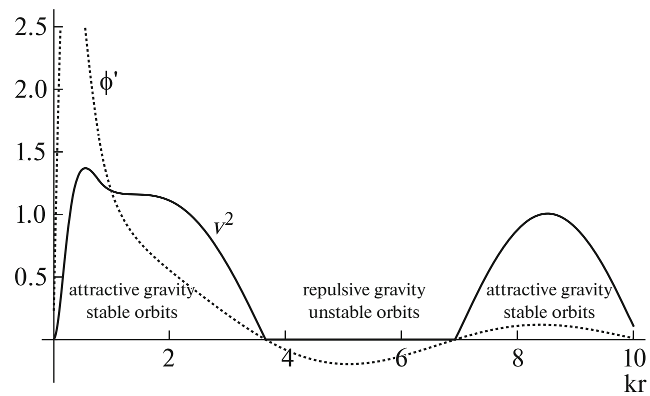

Fig. 2. Wavy and patchy features on rotation curves in massive gravity. In the interval $kr \approx (3.7–6.9)$, massive gravity becomes repulsive. Orbits in this interval are unstable. Voids in the interval should not come as a surprise. The corresponding distances are ≈125–235 kpc for a galaxy of $10^{12} M_\odot$, and 25.000–50.000 au for one solar mass. The numbers are adjustable by changing α.

We note that the k-th order term in (14) is constant and does not contribute to the force field. For a spherically symmetric distribution, (14) reduces to

$$\Phi(r) = -G\cos\alpha\left[\frac{M(r)}{r} - 2\pi k^2 \int \rho(r') \times (r^2 + r'^2)^{1/2} r'^2 dr'\right] + \mathcal{O}(k^3). \qquad (16)$$

We are now in a position to demonstrate how our proposed massive gravity is capable of generating flat rotation curves. We work out a toy model; the Plummer Sphere [21] has the mass density

$$\rho(r) = \frac{3M}{4\pi a^3}\left(\frac{a^2}{a^2 + r^2}\right)^{5/2}, \qquad (17)$$

where M and a are the total mass and the characteristic size of the sphere, respectively. For this density, we have calculated Φ' and $v^2 = r\Phi'$ from Eqs. (12)–(16) and plotted them in Fig. 1. For all practical purposes, the velocity remains flat up to $kr = 2$. For one solar mass, this corresponds to a distance of $\approx 2 \times 7000 = 14.000$ au, far outside the Solar planetary system and the Kuiper belt, bordering the inner regions of the stipulated Oort clouds. For the Milky Way or the Andromeda of about $10^{12} M_0$, this distance is ≈68 kpc, about 4 and 2.5 times of the optical radii of the Milky Way or of M31, respectively. Let us note that by playing with α, one can increase or decrease these distances to some extent to one's desire.

In Fig. 2 we extend the plots up to $kr = 10$. The gravitational force, Φ', eventually goes through a sequence of repulsive (negative) and attractive (positive) phases. Naturally one does not expect stable orbits where gravity is repulsive. Therefore, gaps and voids in the rotation curves should not come as a surprise.

4. POSSIBLE OBSERVATIONAL IMPLICATIONS

4.1. Solar System Implications

According to NASA Solar System Exploration, the Oort clouds are not yet observed. Since Jan Oort (1953), however, there are educated opinions that they must be there not only in the Solar system bur also around other stars. The Oort clouds are believed to be at a distance of about 8.000–100.000 au from the Sun and are described as a spherical halo, see Fig. 3.

In Fig. 2. the interval $kr \approx (7–10)$ is marked as a region of attractive gravity and of stable orbits. Debris passing through this region should stay there for long times. Before and after this stable zone, gravity is repulsive. Any stray by-passer should eventually drift away, leaving voids behind. For one solar mass this stable interval corresponds to ≈50.000–70.000 au, not too far from where the Oort clouds are believed to exist. Note that the numbers are adjustable by adjusting α. Could the Oort clouds serve as a Solar system test of what we are proposing here as massive gravity? Could the voids in the NASA picture of Fig. 3, before and after the Oort clouds, be identified with repulsive gravity regions of Fig. 2? Should one expect series of Oort-like clouds at farther outreaches of the Sun and of stars, or for that matter around any reasonably stabilized stellar system?

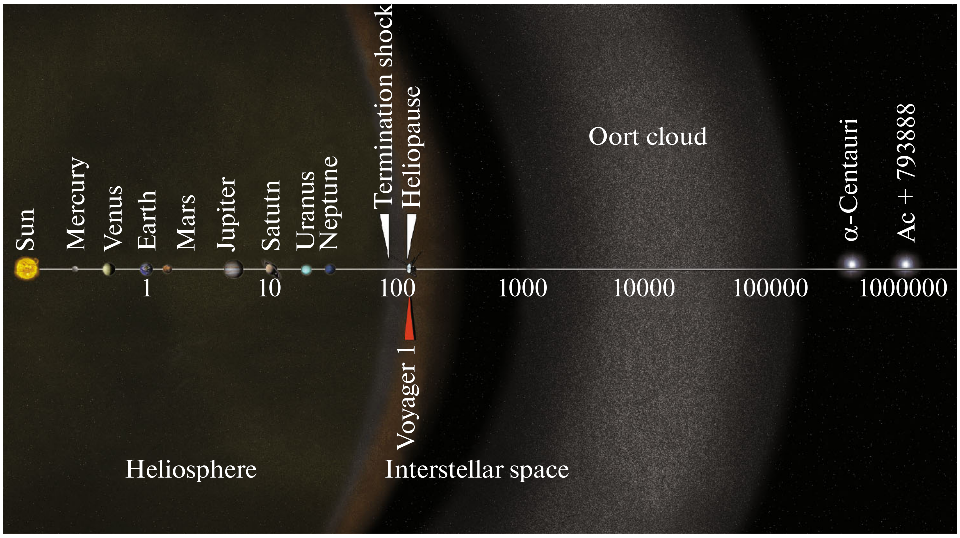

Fig. 3. The Oort clouds, extending over a distance of several thousands to several hundred thousands au. Note the logarithmic distance scale. Source: NASA science, Solar System Exploration.

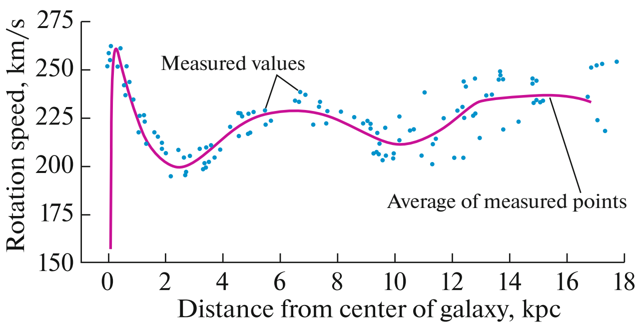

Fig. 4. Rotation curve of the Milky Way, Clemens [22]. No explanation why it is wavy.

4.2. Galactic and Extragalactic Tests

Most galaxies, including the Milky Way, show wavelike features superimposed on their otherwise flat rotation curves. More often than not, amplitudes of the wavy fluctuations are much too large to be attributed to the spiral structures of the galaxies. Massive gravity might have an explanation, and might even serve as a mean to learn more about the masses of the galactic bulges; a rotation curve of a spiral can be considered as a superposition of contributions from the disk and the bulge of the galaxy. Disks are extended and massive components of the galaxies, while bulges, both size- and masswise, are smaller components. This means, while the characteristic, $1/k$, length scales of the disks are of the order of a few tens of kpc, those of the bulges might be ten to hundred times smaller. Then the superposition of the contributions from the disk and the bulge might look like the rotation curve of the Milky Way, Fig. 4, or of NGC 6503, Fig. 5.

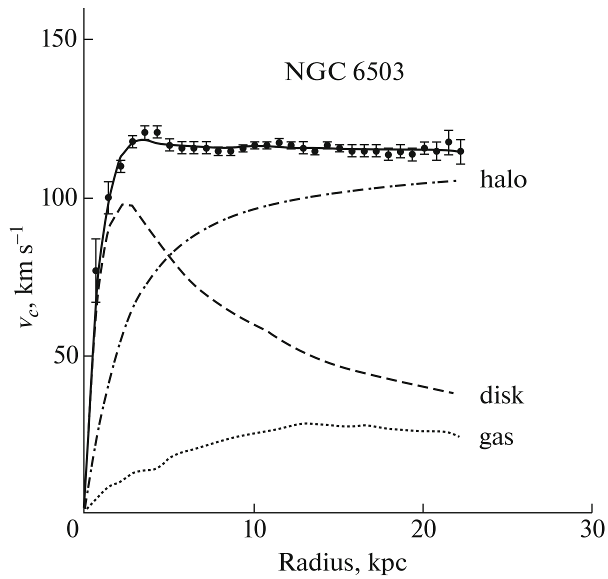

Fig. 5. Rotation curve of NGC 6503 dwarf spiral galaxy. Note the minute wavelike feature on the otherwise flat asymptote. They are often swept away as observational uncertainties or something other. We, however, think they deserve a thought. Image credit Katherine.

5. CONCLUDING REMARKS

We are proposing a massive static gravity hypothesis by adding a mass term to the otherwise massless Newtonian gravity. By doing so we are able to provide a longer range $1/r$ force, capable of giving asymptotically flat rotation curves. The altered force is sourced only by the baryonic content of the galaxy, without invoking any dark matter notion.

We use the TFR and find that the added mass term is inversely proportional to the mass of the gravitating source sitting at the center, $k^2 = a_0/GM$. The proposed massive gravity is capable of producing rotation curves, flat enough up to distances of several optical radii of typical galaxies. At still farther distances, the theory predicts regions of intermittently attractive and repulsive forces, indicative of regions of stable and unstable orbits. This in turn results in wave-like fluctuations and/or patchy voids on the otherwise flat asymptotes. Wavy and patchy rotation curves are not uncommon occurrences. Massive gravity can have an explanation for them.

A consensus of opinion among experts is that the Solar system is enveloped by Oort clouds, a huge spherical shell of debris, extending over a range of 8.000–100.000 au from the Sun. We suggest this as a Solar system test of the proposed massive gravity. The first region of stable orbits is where the planets and the Kuiper belt are located, then comes a region of a relative void, then the Oort clouds, then a second void. The massive gravity does predict such a sequence of alternating orbits and voids.

An Agenda for Future Follow-Up

In (12)–(15) we have given integral equations for the massive gravity generated by a general mass distribution $\rho((r))$ and have reduced them up to $\mathcal{O}(k^2)$ in the immediate neighborhood of the gravitating body. We invite the interested readers to consider the following generalizations:

- Massive gravity of a spheroid, inside and outside,
- Massive gravity of a circular plate or disk of a given thickness.
- Massive gravity of toy galaxies, consisting of small spherical bulges embedded in large and extended disks.

The law of motion in massive gravity is that of Newton, $acceleration \propto force$. This can be derived from an action principle written down for a dynamical system consisting of a field and particles, in which a minimal coupling between the field and the particles ensures their mutual interaction. The motion of a test body of insignificant mass can be carried out straightforwardly. There are, however, complications to be sorted out:

- Two-body problem with comparable masses in massive gravity.
- A virial theorem in massive gravity, its relevance to the fundamental plane of elliptical galaxies, and the Faber-Jackson relation.

On the observational side one may think of:

- Re-analysis of the available seemingly flat rotation curves for minute wavelike fluctuations on them , see e.g., the rotation curve of NGC 6503, Fig. 5. These minute variations are often swept away as observational uncertainties or something other.
- Re-analysis of the available rotation curves with prominent wavy features, e.g., the rotation curves of the Milky Way and M31, for an explanation in terms of the superposition of the gravity fields of large bulges and larger disks

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