Black Holes from Stars to Galaxies – Across the Range of Masses
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An $f(R)$ gravitation for galactic environments

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Abstract. We propose an action-based $f(R)$ modification of Einstein's gravity which admits of a modified Schwarzschild-deSitter metric. In the weak field limit this amounts to adding a small logarithmic correction to the Newtonian potential. A test star moving in such a spacetime acquires a constant asymptotic speed at large distances. This speed turns out to be proportional to the fourth root of the mass of the central body in compliance with the Tully-Fisher relation. A variance of MOND's gravity emerges as an inevitable consequence of the proposed formalism.

1. Introduction

Convinced of cosmic speed up and finding that the dark energy hypotheses is not a compelling explanation, some cosmologists have looked for alternatives to Einstein's gravitation. There is a parallel situation in galactic studies. Dark matter hypotheses, intended to explain the flat rotation curves of spirals or the large velocity dispersions in ellipticals and clusters of galaxies have raised more questions than answers. Alternatives to Newtonian dynamics have been proposed but have had their own critics. The foremost among such theories, the Modified Newtonian Dynamics (MOND) of Milgrom (1983 a,b,c) is capable of explaining the flat rotation curves of spirals (Sandres *et al.*, 1998 and 2002) and of justifying the Tully-Fisher relation with considerable success. But it is often criticized for the lack of an axiomatic foundation.

Here we are concerned with galactic problems. We suggest to follow cosmologists and look for a modified Einstein gravity tailored to suit galactic environments.

2. A modified field equation

The model we consider is an isolated point mass embedded in an asymptotically flat spacetime. As an alternative to the Einstein-Hilbert action we assume the following $S = \frac{1}{2}\int f(R)\sqrt{-g}d^4x$, where R is the Ricci scalar and $f(R)$ is an, as yet, unspecified but differentiable function of R. Variations of S with respect to the metric tensor leads to the following field equation (Capozziello *et al.*, 2003)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\frac{f}{h} = \left(h_{;\mu\nu} - h^{\lambda}_{;\lambda}\, g_{\mu\nu}\right)\frac{1}{h}, \tag{2.1}$$

where $h = df/dR$. The case $f(R) = R + constant$ and $h = 1$ gives the Einstein field equations. For the purpose of galactic studies we envisage a spherically symmetric static metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{2.2}$$

3. Kinship with MOND

The features of this model are also shared by Milgrom's MOND. Below we show that some version of MOND can actually be derived from the present formalism. We recall that in the weak field approximation, Newton's dynamics is derived from that of Einstein by writing $B = \left(1 + 2\phi/c^2\right), \phi = GM/r$, and expanding all relevant quantities and equations up to first order in ϕ/c^2. In a similar way for our modified Newtonian dynamics we find

$$B(r) = 1 + \alpha + \alpha \ln(r/s) - s/r = 1 + 2\phi(r)/c^2, \tag{3.1}$$

where the second equality defines $\phi(r)$. Let us write $\alpha = \alpha_0(GM/GM_\odot)^{1/2}$ and find the gravitational acceleration

$$g = |d\phi/dr| = (a_0 g_n)^{1/2} + g_n, \;\; a_0 = \alpha_0^2 c^4/4GM_\odot, \;\; \text{and} \;\; g_n = GM/r^2. \tag{3.2}$$

The limiting behaviors of g are the same as those of MOND. One may comfortably identify a_0 as MOND's characteristic acceleration and calculate α_0 anew from Eq (3.2). For $a_0 = 1.2 \times 10^{-8} cm/sec^2$, one finds

$$\alpha = 2.8 \times 10^{-12} \left(M/M_\odot\right)^{1/2}. \tag{3.3}$$

It is gratifying how close this value of α is to the one obtained above, and how similar MOND and the present formalism are, in spite of their totally independent starting points.

4. Concluding remarks

We have developed an $f(R) \approx R^{1-\alpha/2}$ gravitation which is essentially a logarithmic modification of Einstein- Hilbert action. In spherically symmetric static situations the theory admits of a modified Schwarzschild-deSitter metric. The latter in the limit of weak fields gives a logarithmic correction to the Newtonian potential. From the observed asymptotic speeds of galaxies we learn that the correction is proportional to the square root of the mass of the central body. Flat rotation curves, the Tully-Fisher relation and a version of MOND emerge as natural consequences of the theory.

There are two practices to obtain the field equations of $f(R)$ gravity, the metric approach, where $g_{\mu\nu}$'s are considered as dynamical variables and that of Palatini, where the metric together with the affine connections are treated as such (see Magnano, 1995 for a review). Unless $f(R)$ is linear in R, the resulting field equations are not identical (see Ferraris *et al.*, 1994). The metric approach is often shied away from, for its leading to fourth order differential equations. In the present paper we do not initially specify $f(R)$. Instead, at some intermediate stage in the analysis we adopt an ansatz for $df(R)/dR$ as a function of r and work forth to obtain the metric, R, and eventually $f(R)$. This enables us to avoid the fourth order equations. The trick should work in other contexts (cosmological, say). Further details can be found in Sobouti (2005).

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