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# A Potential Flow Pertaining to Binary Systems

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Gas flow arising from the orbital motion of stars in a close binary system is a consequence of equations of fluid motion and is inevitable. The velocity of flow is of the order of and less than the orbital velocities. It has the same magnitude and topological structure as most radial velocities derived from observations of spectral lines whose origin lies in the gaseous envelope.

*Key words:* Gas Flow in Binary Systems

## I. Introduction

This is an attempt to seek a systematic approach to the motion of gas surrounding close binary systems. Kuiper (1941), Kopal (1957) and Gould (1959) sought solutions of this problem by considering the motions of individual gas particles as restricted three-body orbits. Prendergast (1960) attempted to integrate the relevant hydrodynamic equations. He, essentially, considered a two dimensional flow confined to the orbital plane of the binary.

As a main theme of this work we wish to make the following proposition. Motion of gas in the envelope of binary systems is not necessarily that of jet streams or particles ejected from the stellar components. The gas enclosing the binary can and does maintain motion even if it has not been ejected from stars. This statement does not exclude ejection of matter or undermind its role in the evolution of the binary. It merely denies to it the role of being the major contributor to the gas motion in the inner regions of the envelope.

Thus, from the point of view taken in this paper, gas flow in a binary system may consist of a part generated by the orbital motion of the stars and another resulting from a possible outflow of matter from the stars. The former flow should always be present whenever a binary system has a gaseous envelope and should be derivable from a velocity potential. In the following we derive an expression for this potential and show that the calculated velocity has the proper magnitude and streamlines have the proper geometry to be of relevance to some gas flow observations. It should, however, be clear that the problem solved here is a mathematical one that may or may not be connected directly with the observations.

Questions regarding the origin of gaseous envelope or the consequential problem of outflow of matter observed in some systems fall outside the scope of this work and are not discussed. The presence of gas outflow, however, does not exclude the flow produced by the orbital motion. It simply combines with the latter.

## II. Notation

In what follows the subscript $i$ may be 1 or 2 to denote quantities pertaining to one or the other of the stars.

$a_i$ are the radii of the stars which are assumed to be spherical, $A_i$ their distances from the center of mass, and $A = A_1 + A_2$, the distance between the two star centers.

$(x, y, z)$ is a rotating right-handed coordinate system at the center of mass of the binary with the $x$-axis along the line of centers, directed in the positive sense from star 1 to 2 and the $z$-axis normal to the orbital plane of the binary. The stars are assumed to revolve with a constant angular velocity.

$(x_i, y_i, z_i)$ are two right-handed coordinate systems whose origins are the centers of the stars with the $x_i$-axes directed towards the center of mass and the $z_i$-axes parallel to the $z$-axis. The following transformation laws hold among the three coordinate systems:

$$x_1 = A - x_2 = A_1 + x\,,$$
$$y_1 = -y_2 = y\,, \quad (1)$$
$$z_1 = z_2 = z\,.$$

A point may also be designated by its polar coordinates $(r_i, \theta_i, \varphi_i)$ through the following transformations:

$$x_1 = r_1 \sin\theta_1 \cos\varphi_1 = A - r_2 \sin\theta_2 \cos\varphi_2\,,$$
$$y_1 = r_1 \cos\theta_1 = -r_2 \cos\theta_2\,, \quad (2)$$
$$z_1 = r_1 \sin\theta_1 \sin\varphi_1 = r_2 \sin\theta_2 \sin\varphi_2$$
$$r_1 = (A^2 + r_2^2 - 2\,A\,r_2 \sin\theta_2 \cos\varphi_2)^{1/2}\,. \quad (2a)$$

## III. Equations of Motion

The fluid motion investigated below is assumed to be produced by revolution of the binary system. This assumption renders the motion an irrotational flow, derivable from a velocity potential. The fluid is further assumed to be incompressible. The latter assumption is good for subsonic flows and is inadequate for supersonic ones (Landau and Lifshitz, 1963).

Parker's (1963) application of Bernoulli's equation to a gas in the gravitational field of a star shows that a large volume of gas can stay in the vicinity of a star only if it is heated to coronal temperatures. A shell of gas at stellar atmospheric temperatures will collapse in a matter of hours, the period required for free-fall. Should coronal temperatures prevail the flow will be subsonic.

The possibility of transient and/or localized cool streams of gas is not excluded. Such phenomena fall outside the scope of this paper. It is not, however, unlikely that the flow of such streams is strongly affected by the motion suggested above.

Even if one assumes a cool gas, ignoring the consequences of not satisfying Bernoulli's equation the flow established here will at least give the order of magnitude.

The equations of motion and continuity in the rotating coordinates $(x, y, z)$ are

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\,\boldsymbol{u} + 2\,\boldsymbol{\omega}\times\boldsymbol{u} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}) = -\boldsymbol{\nabla}\left(\frac{p}{\varrho} + \Omega\right) \quad (3)$$

and

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \quad (4)$$

respectively. Here $\Omega$ is the gravitational potential of the stars and the other symbols have their conventional meanings. Assume

$$\boldsymbol{u} = -\boldsymbol{\nabla}\Phi - \boldsymbol{\omega}\times\boldsymbol{r}\,. \quad (5)$$

Substituting this velocity in Eqs. (3) and (4) gives

$$\frac{p}{\varrho} = \frac{\partial\Phi}{\partial t} - \frac{1}{2}\,|\boldsymbol{\nabla}\Phi|^2 + \omega\left(y\,\frac{\partial\Phi}{\partial x} - x\,\frac{\partial\Phi}{\partial y}\right) - \Omega \quad (6)$$

and

$$\nabla^2\,\Phi = 0 \quad (7)$$

respectively.

Thus the problem is reduced to that of finding the potential $\Phi$ satisfying Laplace's Eq. (7), vanishing at infinity and satisfying the following boundary condition on both stars

$$u_n = [-\boldsymbol{\nabla}\Phi - \boldsymbol{\omega}\times\boldsymbol{r}]\cdot\boldsymbol{n} = 0 \quad (8)$$

where, $\boldsymbol{n}$ is a unit normal to the surfaces of the stars. Equation (8) ensures the vanishing of the normal component of the velocity on the stellar surfaces.

It is worth noting that the irrotational flow determined by the kinematical Eqs. (4), (7) and (8) is unaffected by the gravitational field of the binary. Dynamical effects appear in the pressure Eq. (6) only.

## IV. The Velocity Potential

A general scheme for obtaining the flow produced by an object immersed in a fluid was first developed by Thomson and Tait (1879, c.f. Lamb, 1932) by applying the lagrangian equations of motion to the fluid and object combined. Let the configuration of the system producing the flow be specified by generalized coordinates $q_1, q_2, \ldots$ The flow being entirely due to the immersed system, will have the following potential

$$\Phi = \dot{q}_1\,\Phi_1 + \dot{q}_2\,\Phi_2 + \cdots \quad (9)$$

where $\Phi_1, \Phi_2, \ldots$ satisfy Laplace's equation. If the velocity normal to the surface or surfaces of the system is given by

$$u_n = \dot{q}_1\,S_1 + \dot{q}_2\,S_2 + \cdots \quad (10)$$

where $S_1, S_2, \ldots$ are some functions of the surface or surfaces bounding the system, then $\Phi_1, \Phi_2, \ldots$ should satisfy the boundary conditions:

$$-\frac{\partial\Phi_1}{\partial n} = S_1\,, \quad -\frac{\partial\Phi_2}{\partial n} = S_2\,, \ldots \quad (11)$$

This method applied to a sphere in uniform linear motion through a fluid gives the familiar dipolar motion. The flow produced by two spheres in uniform and parallel motions along the line of centers or perpendicular to it has also been treated (e.g.

Herman, 1887; Basset, 1887) by means of this same formalism.

In our binary problem, let

$$\Phi = \omega\,(\Phi_1 + \Phi_2)\,. \tag{12}$$

Substituting this in Eq. (7) gives

$$\nabla^2 \Phi_i = 0\,, \qquad i = 1, 2\,. \tag{13}$$

According to Eqs. (8), (10) and (11) the normal gradient of $\Phi_1$ should vanish on star 2 and be equal to $-A_1 \cos\theta_1$ on star 1. Except for a factor $\omega$ the latter is the normal velocity $-(\boldsymbol{\omega} \times \boldsymbol{r}) \cdot \boldsymbol{n}$. To ensure a generalization of our results, however, we write these boundary conditions as follows:

$$-\left[\frac{\partial \Phi_1(r_1, \theta_1, \varphi_1)}{\partial r_1}\right]_{r_1 = a_1} = (Q_1^{(1)} + q_1^{(1)} \sin\theta_1 \cos\varphi_1) \cos\theta_1 \tag{14}$$

and

$$-\left[\frac{\partial \Phi(r_1, \theta_1, \varphi_1)}{\partial r_2}\right]_{r_2 = a_2} = 0 \tag{15}$$

where, $Q_1^{(1)} = A_1$ and $q_1^{(1)} = 0$. Transformation Eq. (1) and (2) are to be used to evaluate Eq. (15). Similar boundary conditions will apply to $\Phi_2$.

To solve Eq. (13) subject to conditions (14) and (15), we employ a method closely resembling the method of images used in electrostatistics. Let

$$\Phi_1 = \Phi_1^{(1)} + \Phi_1^{(2)} + \cdots \tag{16}$$

find $\Phi_1^{(1)}$ so as to satisfy Eq. (14) but not (15). Thus obtained, $\Phi_1^{(1)}$ will be the exact potential due to star 1 in the absence of star 2. Next find $\Phi_1^{(2)}$ such as to cancel the normal velocity $(-\partial \Phi_1^{(1)}/\partial r_2)_{r_2 = a_2}$ on star 2 produced by $\Phi_1^{(1)}$. Again find $\Phi_1^{(3)}$ to cancel the normal velocity $(-\partial \Phi_1^{(2)}/\partial r_1)_{r_1 = a_1}$ on star 1 produced by $\Phi_1^{(2)}$, and so on.

We now proceed to the actual calculation of the potentials $\Phi_1^{(j)}$. Equation (14) is a linear combination of the surface harmonics $Y_1(\theta_1)$ and $Y_2^1(\theta_1, \varphi_1)$, suggesting a combination of the corresponding solid harmonics for $\Phi_1^{(1)}$:

$$\Phi_1^{(1)}(r_1, \theta_1, \varphi_1) = \frac{1}{2} Q_1^{(1)} \frac{a_1^3}{r_1^2} \cos\theta_1 + \frac{1}{3} q_1^{(1)} \frac{a_1^4}{r_1^3} \cdot \cos\theta_1 \sin\theta_1 \cos\varphi_1\,. \tag{17}$$

Thus chosen, $\Phi_1^{(1)}$ satisfies both Eqs. (13) and (14). We transform Eq. (17) by means of Eqs. (2) and then evaluate the normal velocity on the surface of star 2, $(-\partial \Phi_1^{(1)}/\partial r_2)_{r_2 = a_2}$.

$$\Phi_1^{(1)} = -\left(\frac{1}{2} Q_1^{(1)} \frac{a_1^3}{r_1^3} + \frac{1}{3} q_1^{(1)} \frac{A a_1^4}{r_1^5}\right) r_2 \cos\theta_2 + \frac{1}{3} q_1^{(1)} \frac{a_1^4}{r_1^5} r_2^2 \cos\theta_2 \sin\theta_2 \cos\varphi_2\,, \tag{18}$$

$$\begin{aligned} -\left(\frac{\partial \Phi_1^{(1)}}{\partial r_2}\right)_{r_2 = a_2} &= \frac{a_1^3}{r_{12}^3} \cdot \left[\frac{1}{2} Q_1^{(1)} - \left(\frac{3}{2} Q_1^{(1)} a_2^2 - \frac{1}{3} q_1^{(1)} A a_1\right)\right. \\ &\cdot \left.\frac{1}{r_{12}^2} - \frac{5}{3} q_1^{(1)} \frac{A a_1 a_2^2}{r_{12}^4}\right] \cos\theta_2 + \frac{a_1^3 a_2}{r_{12}^5} \\ &\cdot \left[\left(\frac{3}{2} Q_1^{(1)} A - \frac{2}{3} q_1^{(1)} a_2\right) + \frac{5}{3} q_1^{(1)} a_1 (A^2 + a_2^2) \frac{1}{r_{12}^2}\right] \\ &\cdot \cos\theta_2 \sin\theta_2 \cos\varphi_2 - \frac{5}{3} q_1^{(1)} A a_1^4 a_2^2 \frac{1}{r_{12}^2} \\ &\cdot \cos\theta_2 \sin^2\theta_2 \cos^2\varphi_2\,. \end{aligned} \tag{19}$$

We have employed Eq. (2a) to express $r_1$ and $\partial r_1/\partial r_2$ in terms of $r_2$ and have further introduced the notation

$$r_{12} = (r_1)_{r_2 = a_2} = (A^2 + a_2^2 - 2 A a_2 \sin\theta_2 \cos\varphi_2)^{1/2}\,. \tag{20}$$

Let

$$R_{12} = (A^2 + a_2^2)^{1/2} \tag{21}$$

and expand all powers of $r_{12}$ in Eq. (19) about $R_{12}$. Keeping only the first and the second order harmonics we obtain

$$-\left(\frac{\partial \Phi_1^{(1)}}{\partial r_2}\right)_{a_2} = -(Q_1^{(2)} + q_1^{(2)} \sin\theta_2 \cos\varphi_2) \cos\theta_2\,, \tag{22}$$

where

$$Q_1^{(2)} = -\left[\frac{1}{2} Q_1^{(1)} \frac{a_1^3}{R_{12}^3} - \left(\frac{3}{2} Q_1^{(1)} a_2^2 - \frac{1}{3} q_1^{(1)} A a\right) \cdot \frac{a_1^3}{R_{12}^5} - \frac{5}{3} q_1^{(1)} \frac{A a_1^4 a_2^2}{R_{12}^7}\right] \tag{23}$$

and

$$q_1^{(2)} = -\left[(3 Q_1^{(1)} A + q_1^{(1)} a_1) \frac{a_1^3 a_2}{R_{12}^5} - 5\left(\frac{3}{2} Q_1^{(1)} a_2^2 - \frac{1}{3} q_1^{(1)} A a_1\right) \frac{A a_1^3 a_2}{R_{12}^7} - \frac{35}{3} q_1^{(1)} \frac{A^2 a_1^4 a_2^3}{R_{12}^9}\right]. \tag{24}$$

Except for the negative sign, the normal velocity of Eq. (22) is of the same form as Eq. (14). Therefore, the potential cancelling this velocity on the surface of star 2, as in Eq. (17), will be

$$\Phi_1^{(2)} = \frac{1}{2} Q_1^{(2)} \frac{a_2^3}{r_2^2} \cos\theta_2 + \frac{1}{3} q_1^{(2)} \frac{a_2^4}{r_2^3} \cos\theta_2 \sin\theta_2 \cos\varphi_2 \,. \tag{25}$$

The term $\Phi_1^{(2)}$, however, will entail a further correction $\Phi_1^{(3)}$ obtained from Eq. (17) by substituting some coefficients $Q_1^{(3)}$ and $q_1^{(3)}$ for $Q_1^{(1)}$ and $q_1^{(1)}$. A scheme for deriving these coefficients is given below following Eq. (28).

Thus collecting the series $\Phi_1^{(1)}, \Phi_1^{(2)}, \ldots$ and the similar terms $\Phi_2^{(1)}, \Phi_2^{(2)}, \ldots$ for the flow produced by star 2 we arrive at the following total potential:

$$\Phi = \omega \left[\frac{1}{2} \frac{Q_1 a_1^3}{r_1^2} \cos\theta_1 + \frac{1}{3} \frac{q_1 a_1^4}{r_1^3} \cos\theta_1 \sin\theta_1 \cos\varphi_1 + \frac{1}{2} \frac{Q_2 a_2^3}{r_2^2} \cos\theta_2 + \frac{1}{3} \frac{q_2 a_2^4}{r_2^3} \cos\theta_2 \sin\theta_2 \cos\varphi_2\right] \tag{26}$$

where

$$Q_1 = Q_1^{(1)} + Q_1^{(3)} + \cdots + Q_2^{(2)} + Q_2^{(4)} + \cdots \tag{27}$$

and

$$q_1 = q_1^{(1)} + q_1^{(3)} + \cdots + q_2^{(2)} + q_2^{(4)} + \cdots \tag{28}$$

Similar relations, with subscripts 1 and 2 interchanged, hold for $Q_2$ and $q_2$. The coefficients $Q_i^{(j)}$ and $q_i^{(j)}$ for even values of $j$ are given in terms of $Q_i^{(j-1)}$ and $q_i^{(j-1)}$ by Eqs. (13) and (24). If $j$ ($\geqq 3$) is odd subscripts 1 and 2 should be interchanged in these equations. We also recall from the convention following Eq. (15) that $Q_i^{(1)} = A_i$ and $q_i^{(1)} = 0$.

Inspection of Eqs. (17), (25) and (23) reveals that

$$\left|\frac{\Phi_1^{(j)}}{\Phi_1^{(j-1)}}\right| \approx \left|\frac{Q_1^{(j)}}{Q_1^{(j-1)}}\right| \approx \frac{a_1^3}{(A^2 + a_2^2)^{3/2}} \,. \tag{29}$$

This ratio is approximately that of the volume of a star to the volume of a sphere containing the whole binary. In the most unfavorable case of a hypothetical binary with $a_1 = a_2 = 1/2\,A$, this ratio is about 1/10 indicating the rapid convergence of the series (16), (27), and (28). The values of some $Q_i^{(j)}$ and $q_i^{(j)}$ given in Table 1 confirm this statement.

In the development following Eq. (22), in which we omitted the third – and higher – order harmonics, it is, however, fruitless to look for accuracies better than $1/4\,(a_i/r_i)^4$, the magnitude of the contribution of the third-order harmonics. The inclusion of higher harmonics, however, can easily be arranged within the framework of the expansion developed here. This will introduce no conceptual difficulty but does render the calculations tedious.

Table

| $j$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $A_1 = A_2 = .50,\quad a_1 = a_2 = .50$ | | | | |
| $Q_1^{(j)} = Q_2^{(j)}$ | .500000 | −.008944 | +.000160 | −.000003 |
| $q_1^{(j)} = q_2^{(j)}$ | .0 | −.026833 | +.000704 | −.000014 |
| $A_1 = A_2 = .50,\quad a_1 = a_2 = .25$ | | | | |
| $Q_1^{(j)} = Q_2^{(j)}$ | .500000 | −.002937 | +.000021 | |
| $q_1^{(j)} = q_2^{(j)}$ | .0 | −.004294 | +.000032 | |
| $A_1 = .40, A_2 = .60,\quad a_1 = a_2 = .25$ | | | | |
| $Q_1^{(j)}$ | .400000 | −.002350 | +.000025 | |
| $Q_2^{(j)}$ | .600000 | −.003525 | +.000017 | |
| $q_1^{(j)}$ | .0 | −.003436 | +.000039 | |
| $q_2^{(j)}$ | .0 | −.005154 | +.000026 | |

## V. The Velocity and the Streamlines

With respect to a non-rotating frame the instantaneous velocity is $-\nabla\Phi$ whose components are

$$-\Phi_x = \omega y \left[\frac{3}{2}\left\{Q_1 a_1^3 \frac{(A_1 + x)}{r_1^5} + Q_2 a_2^3 \frac{(A_2 - x)}{r_2^5}\right\} - \frac{1}{3}\left\{\frac{q_1 a_1^4}{r_1^5}\left[1 - 5\frac{(A_1 + x)^2}{r_1^2}\right] + \frac{q_2 a_2^4}{r_2^5}\left[1 - 5\frac{(A_2 - x)}{r_2^2}\right]\right\}\right], \tag{30}$$

$$-\Phi_y = -\,\omega\left[\frac{1}{2}\left\{\frac{Q_1 a_1^3}{r_1^3}\left(1 - 3\frac{y^2}{r_1^2}\right) - \frac{Q_2 a_2^3}{r_2^3}\left(1 - 3\frac{y^2}{r_2^2}\right)\right\} - \frac{1}{3}\left\{\frac{q_1 a_1^4}{r_2^5}(A_1 + x)\left(1 - 5\frac{y^2}{r_1^2}\right) - \frac{q_2 a_2^4}{r_2^5}(A_2 - x)\left(1 - 5\frac{y^2}{r_2^2}\right)\right\}\right], \tag{31}$$

$$-\Phi_z = \omega\, y\, z\left[\frac{3}{2}\left(\frac{Q_1 a_1^3}{r_1^5} - \frac{Q_2 a_2^3}{r_2^5}\right) + \frac{5}{3}\left\{q_1 a_1^4 \frac{(A_1 + x)}{r_1^7} - q_2 a_2^4 \frac{(A_2 - x)}{r_2^7}\right\}\right]. \tag{32}$$

Streamlines are obtained by solving the following differential eqs.

$$\frac{dx}{\Phi_x} = \frac{dy}{\Phi_y} = \frac{dz}{\Phi_z} \,. \tag{33}$$

In the orbital plane $\Phi_z = 0$. Consequently streamlines passing through this plane lie on it and satisfy the first of Eq. (33). An approximate integral for the

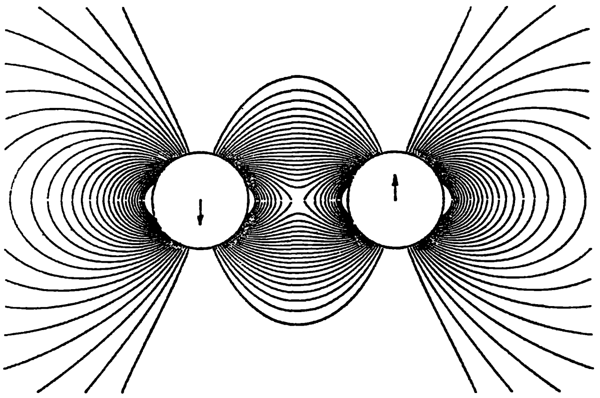

Fig. 1. Streamlines in the orbital plane. The total mass, the separation between the two stars and the angular velocity of rotation are chosen to be unity. The arrows indicate the directions and magnitudes of the orbital velocities in a center of mass system of reference. Crowding of streamlines in a region indicate a larger velocity of flow. Masses and radii of the stars are denoted by $m_1$, $m_2$ and $a_1$, $a_2$ respectively. $m_1 = m_2 = 0.50$, $a_1 = a_2 = 0.25$

latter Equation is[1])

$$\Psi(x, y) = \int (\Phi_y\,dx - \Phi_x\,dy) = \text{const.} \tag{34}$$

Substituting for $\Phi_x$ and $\Phi_y$ in Eq. (34) and integrating give

$$\begin{aligned}\Psi(x, y) = -\,\omega \Bigg[\frac{1}{2y^2}\Bigg\{&Q_1 a_1^3 \frac{(r_1^2 + 2y^2)(A_1 + x)}{r_1^3} \\ &+ Q_2 a_2^3 \frac{(r_2^2 + 2y^2)(A_2 - x)}{r_2^3}\Bigg\} \\ &+ \frac{1}{3}\Bigg\{q_1 a_1^4 \frac{(A_1 + x) - y^2}{r_1^5} \\ &+ q_2 a_2^4 \frac{(A_2 - x) - y^2}{r_2^5}\Bigg\}\Bigg]_{z=0}.\end{aligned} \tag{35}$$

We have, however, integrated the first of Eqs. (33) numerically and plotted the solution in Figs. 1–4. The approximate integral (35) was employed only to generate streamlines corresponding to roughly equidistant values of $\Psi(x, y)$ in the orbital plane.

[1]) By a direct differentation of $\Psi(x, y)$ and employing the fact that $\Phi$ satisfies Laplace's equation one may show that Eq. (34) is an exact integral of the following equation:

$$\frac{dx}{2\,\phi_x - \int \phi_{zz}\,dx} = \frac{dy}{2\,\phi_y - \int \phi_{zz}\,dy}\,.$$

In the orbital plane, excepting the $x$-axis $\Phi_{zz}$ is small. The latter equation does not differ greatly from the first of Eq. (33). On the $x$-axis, however, both terms in the denominator of the left hand side become of the same order and tend to zero. Equation (34) therefore cannot be a good representation of the streamlines in this neighborhood.

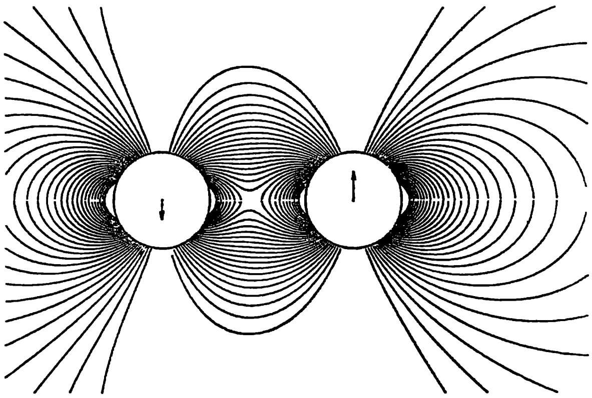

Fig. 2. $m_1 = 0.6$, $m_2 = 0.4$, $a_1 = a_2 = 0.2$. See also legend for Fig. 1

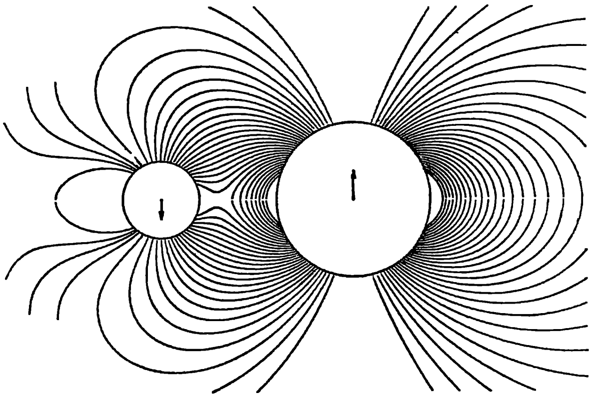

Fig. 3. $m_1 = 0.6$, $m_2 = 0.4$, $a_1 = 0.4$, $a_2 = 0.2$. See also legend for Fig. 1

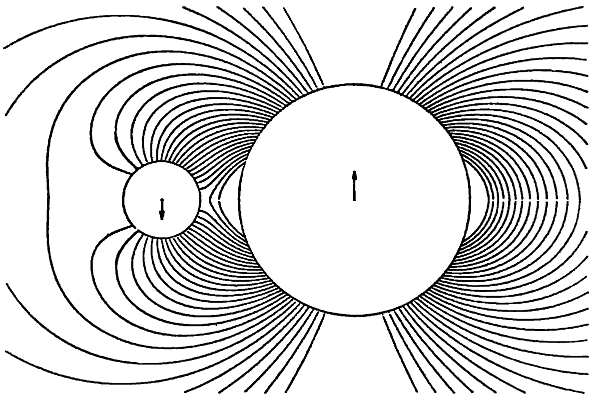

Fig. 4. $m_1 = .6$, $m_2 = .4$, $a_1 = 0.6$, $a_2 = 0.2$. See also legend for Fig. 1

The motion in the envelope may qualitatively be described as follows. To an observer outside the binary system it will seem that stars tend to carry the surrounding gas with them. The gas is pushed out by the preceding hemispheres and is drawn into the vacuum developed behind the following hemispheres. The order of magnitude of the gas velocity in the neighborhood of the stellar surfaces is that of the orbital velocity. It never exceeds the latter and drops off inversely as the cube of the distance. The gas around the less massive secondary has a larger velocity than that around the primary.

Struve (1946) found that for a group of binaries with A-type primary stars and G-type secondary stars the velocity, $V$, of some emission features and the period, $P$, of the system followed the relation $V^3 P \approx$ const. With the picture of gas flow suggested above this empirical relation may be simply explained. In the immediate vicinity of the stars, presumably the most likely place for emission lines, gas velocities are proportional to the orbital velocities of stars. The latter velocities in turn follow Kepler's law $V_1^3 P \propto m_2^3/(m_1 + m_2)^2$ where $V_1$ is the orbital velocity of a star. For Struve's sample of stars the left-hand side of this relation is approximately the same for all systems, as the binary components are roughly of the same stellar type.

Without constructive criticisms of Professor Robert H. Koch this work would not have been accomplished. During the preparation of this paper the author was a visiting Professor at the Department of Astronomy, University of Pennsylvania and enjoyed a sabbatical leave from Pahlavi University, Iran.

## References

Basset, A. B. 1887, *Proc. Lond. Math. Soc.* **43**, 369.
Gould, N. L. 1959, *A. J.* **64**, 136.
Herman, R. A. 1887, *Quart. J. Math.* **22**, 370.
Kopal, Z. 1956, *Ann. Astr.* **19**, 298.
Kuiper, G. P. 1941, *Ap. J.* **93**, 133.
Lamb, H. 1932, *Hydrodynamics*, Cambridge University Press.
Landau, L. D., Lifshitz, E. M. 1963, *Fluid Mechanics*, Addison-Wesley.
Parker, E. N. 1963, *Interplanetary Dynamical Processes*, John Wiley.
Prendergast, K. H. 1960, *Ap. J.* **132**, 162.
Struve, O. 1946, *Ap. J.* **103**, 76.

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