ON A BERNOULLI'S INTEGRAL PERTAINING TO GAS FLOW IN CLOSE BINARY SYSTEMS

(Research Note)

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Abstract. A Bernoulli's integral supplemented with the equation of continuity provides a solution for the motion of gas surrounding a binary system.

There exist two velocity modes whose streamlines are confined within appropriate equipotential surfaces.

1. Introduction

In his theory of corona, Parker (1963) combines Bernoulli's theorem and the equation of continuity to develop structure and flow patterns for coronal motions and the solar wind. This note advances the notion that Parker's method, and especially parts of his mathematical developments, can be profitably applied to gas motions in close binary systems. This approach, while making full use of the concepts of hydrodynamics, incorporates the conditions prevailing in a binary, such as compressibility, gravitational and centrifugal forces, into the theory.

Prendergast (1960) pointed out that the motion of matter surrounding a binary system should be sought within the domain of hydrodynamics. While his remark has found acceptance by most authors, because of mathematical difficulties it does not appear to have been expounded enough to result in any concrete solution (Sobouti's [1970] potential flow, although a solution of hydrodynamical equations, is severely limited by the assumption of incompressibility). Kopal's (1969, 1970) introduction of Roche coordinates and Kitamura's (1970) subsequent investigation of the geometry of this coordinates may prove useful in simplifying the mathematics considerably. In the meantime, however, we wish to emphasize that, regardless of the extent of its relevance to binary systems, Bernoulli's integral provides a solution for the fluid motion in the field of two gravitating bodies. It will also be noteworthy that one of Kopal's Roche coordinates: namely, the total potential, appears explicitly in this solution.

2. Bernoulli's Equation

Euler's equation of steady motion in a rotating frame of reference admits the following Bernoulli's integral

$$\tfrac{1}{2}U^2 - \tfrac{1}{2}[\omega^2 r^2 - (\boldsymbol{\omega}\cdot\mathbf{r})^2] + \Omega + \int \frac{\mathrm{d}P}{\varrho} = C, \tag{1}$$

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where $\mathbf{U}$, $\boldsymbol{\omega}$, $\mathbf{r}$, ϱ, p and Ω are the velocity vector, the rotation vector of the reference frame, the position vector, the density, the pressure and the gravitational potential, respectively. The constant C defines a streamline.

The equation of continuity in terms of cross-section, A, of a streamline is

$$U\varrho A = U_0 \varrho_0 A_0 , \tag{2}$$

where the subscript zero refers to a reference level along a streamline. Let us assume a polytropic equation of state

$$\frac{P}{P_0} = \left(\frac{\varrho}{\varrho_0}\right)^{\alpha}, \quad 1 < \alpha < \gamma \tag{3}$$

where γ is the ratio of specific heats of the gas. A substitution of Equations (2) and (3) in Equation (1) gives

$$\tfrac{1}{2}U^2 + \frac{\alpha}{\alpha - 1}\frac{P_0}{\varrho_0}\left(\frac{U_0 A_0}{UA}\right)^{\alpha - 1} - \tfrac{1}{2}\left[\omega^2 r^2 - (\boldsymbol{\omega}\cdot\mathbf{r})^2\right] + \Omega = C . \tag{4}$$

It is convenient to use the following dimensionless quantities

$$v = \frac{U}{U_0}\chi^{1/(\alpha+1)}, \tag{5a}$$

$$F = \frac{A}{A_0}, \tag{5b}$$

$$\phi = -\left[\Omega - \tfrac{1}{2}\{\omega^2 r^2 - (\boldsymbol{\omega}\cdot\mathbf{r})^2\}\right]\frac{2}{U_0^2}\chi^{2/(\alpha+1)}, \tag{5c}$$

and

$$c = 2\frac{C}{U_0}\chi^{2/(\alpha+1)}, \tag{5d}$$

where

$$\chi = \frac{\alpha - 1}{2\alpha}\frac{\varrho_0 U_0}{P_0}. \tag{5e}$$

It should be noted that the dimensionless variable ϕ, the negative of the total gravitational and centrifugal potential, is always positive – becoming infinite at the center of stars and at the infinity and having minima at the libration points. The dimensionless form of Equation (4) reduces to

$$v^2 + (vF)^{-(\alpha-1)} = \phi + c . \tag{6}$$

Equation (9) has the following asymptotic solutions (Parker, 1963)

$$v = \frac{1}{F(\phi + c)^{1/(\alpha+1)}}\left[1 + \frac{1}{(\alpha - 1)F^2(\phi + c)^{(\alpha+1)/(\alpha-1)}} + \cdots\right], \tag{7a}$$

and

$$v = (\phi + c)^{1/2}\left[1 + \frac{1}{2F^{(\alpha-1)}(\varphi + c)^{(\alpha+1)/2}} + \cdots\right]. \tag{7b}$$

Equation (7a) is obtained by treating the thermal energy term, $(vF)\exp(1-\alpha)$, in Equation (6) as the dominant term. This should be the case if the thermal energy supplied to the gas is the principal cause of the motion. Equation (7b) results on assuming that the kinetic energy term, v^2 is the more important one. This will be the case if the effects of gas pressure are unimportant. Equations (7a) and (7b) will be referred to as the velocity modes (a) and (b) respectively.

Some preliminary remarks may be made at this stage. For a stream constant $C<0$, let us define a 'limiting' equipotential surface by $\phi+C=0$. Outside this surface, i.e. in regions of $\phi+C<0$, the velocities of both modes (a) and (b) are imaginary. There is no motion of the gas across such an equipotential.

The gas density, ϱ is proportional to $1/(vF)$ (Equations [2] and [5a]). We, therefore, conclude that in the case of mode (a), the density vanishes as the limiting equipotential is approached. Since the variations of F is not known, the behavior of v in the neighbourhood of the equipotential cannot be determined readily. If, however, v remain finite as the equipotential is approached, the streamline has to become parallel to the equipotential in order to give vanishing densities.

In the case of mode (b) the velocity vanishes on the equipotential (Equation 7b). Because of lack of information on F, however, the behavior of density cannot be decided readily.

Should one accept that the streamlines are confined within appropriate equipotentials a study of the geometry of such surfaces may then prove instructive in getting possible flow patterns. An extensive geometrical illustration of these surfaces is given by Kitamura (1970). We only point out that flow patterns confined to the immediate neighbourhood of one or both stars, flows that circulate inside a common envelope of both stars, and flows that escape the system from the openings in the equipotentials, near the Lagrangian points and form rings or other features around the whole system are all possibilities furnished by different configurations of the equipotential surfaces.

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