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Excitation of stellar oscillations by tidal processes*

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Abstract. Normal modes of a binary member may be excited by the tidal effect of its companion, causing energy transfer from the orbital motion to the star (Fabian et al.). We decompose the displacement vector field of the oscillating star into irrotational and solenoidal components, and show that only the irrotational motions are responsible for the energy transfer. The tidal capture cross sections are calculated for a wide range of polytropic indices.

Key words: oscillations of stars – close binaries – globular clusters

1. Introduction

Among the possibilities for close binary formation is the suggestion of Fabian et al. (1975). They invoke a tidal process in which the energy from the relative orbital motion of two unbound stars is transferred into the normal modes of oscillations of one or the other member. Press and Teukolsky (1977) analysed this tidal process in some detail and gave mathematical expressions for the energy transfer and the capture cross-section. The expressions involve orbital specifications as well as the eigenvalues and eigenfunctions of the non-radial normal modes. They also present numerical calculations for $n=3$ polytrope. Giersz (1986) used the same formalism and gave capture cross sections for a wide range of stellar masses and radii. Lee and Ostriker (1986) reconsidered the question of tidal capture and among other problems extended the numerical analysis to $n=1.5$, 2, and 3 polytropes. McMillan et al. (1987), and Ray et al. (1987) followed the same procedure as the latter authors for polytropes 1.5 and 3, and added further details to various aspects of tidaly interacting binaries.

In this communication we decompose the eigendisplacements of a normal mode into an irrotational and a 'weighted' solenoidal component. We show that only the irrotational component is responsible for the interaction with the companion star. This analytical features stems from the fact that the tidal gravitational field itself is an irrotational one. And the whole affair is a special aspect of a much wider group theoretic property that requires the symmetries of interacting fields (the displacement and the tidal fields in the present problem) to be the same. In the light of this finding, one not only arrives at a better understanding of the problem, but also can devise efficient methods of numerical calculations. Numerical results are presented for polytropes 1.5, 2, 2.5, 3, 3.25, 3.5, and 4.

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Our formalism for mode analysis is given in Sect. 2. Press-and-Teukolsky's treatment of tidal interactions is summarized in Sect. 3. Incorporation of the two aspects to obtain the overlap integrals is given in Sect. 4. Capture cross sections are discussed in Sect. 5. Concluding remarks, and comparisons with other's calculations are presented in Sect. 6.

2. Normal modes in terms of scalar and vector potentials

Let ρ be the density and $\boldsymbol{\xi}(\boldsymbol{r})$ a displacement field in a star. Let H be a Hilbert space the elements of which are $\boldsymbol{\xi}(\boldsymbol{r})$ and the inner product in it defined as

$$(\xi, \xi')=\int \rho \xi^* \cdot \xi' d^3x = \text{finite}, \ \xi, \xi' \in H, \tag{1}$$

where the integration is over the volume of the star.

The normal modes of the star (ω_n, ξ_n), in which ω_n is the eigenfrequency and ξ_n is the corresponding displacement field belong to H and satisfy the eigenvalue equation

$$\mathscr{W}\xi_n = \omega_n^2 \rho \xi_n, \tag{2}$$

where

$$\mathscr{W}\xi = \nabla(\delta p) + \delta\rho\nabla\Omega + \rho\nabla(\delta\Omega), \tag{3}$$

$$\delta\rho = -\nabla\cdot(\rho\xi), \tag{3a}$$

$$\delta p = \frac{dp}{d\rho}\delta\rho - \left[\left(\frac{\partial p}{\partial \rho}\right)_{\text{ad}} - \frac{dp}{d\rho}\right]\rho\nabla\cdot\xi, \tag{3b}$$

$$\nabla^2(\delta\Omega) = 4\pi G\delta\rho. \tag{3c}$$

The $\mathscr{W}$ operator is self-adjoint. It follows that the eigenvalues ω_n^2 are real and the set $\{\xi_n\}$ is orthogonal and may be normalized to unity, thus

$$(\xi_n, \xi_m) = \delta_{nm}. \tag{4}$$

Dixit, Sarath, and Sobouti (1980) give a basis set for H and expand ξ_n's in terms of this basis. Inverting their expansion it is possible to express their basis in terms of $\{\xi_n\}$ in a unique fashion. It then follows that $\{\xi_n\}$ is also complete and may serve as an orthogonal basis for H.

Using a gauged version of Helmholtz' theorem, Sobouti (1981, 1986) decomposes a general vector field $\xi(\boldsymbol{r})$ in H into an irrotational component and a "weighted" solenoidal component. Thus

$$\xi = \xi_p + \xi_g, \tag{5}$$

where

$$\xi_p = -\nabla\chi_p, \tag{5a}$$

$$\xi_g = \frac{1}{\rho} \nabla \times A_g = \frac{1}{\rho} \nabla \times \nabla \times (\hat{r}\chi_g). \tag{5b}$$

Here $\chi_p(\boldsymbol{r})$ and $\chi_g(\boldsymbol{r})$ are two scalar potentials, $A_g = \nabla \times (\hat{r}\chi_g)$ is a vector potential, and $\hat{r}$ is the unit vector in the radial direction. The nomenclatures 'irrotational' and 'weighted solenoidal' is to indicate that $\nabla \times \xi_p = 0$, and $\nabla \cdot (\rho \xi_g) = 0$. Sobouti also shows that in the vicinity of convective neutrality, i.e., for small values of $(\partial p/\partial \rho)_{\rm ad} - dp/d\rho$, the eigendisplacement vector of the p-modes are exactly of ξ_p-type and those of the g-modes are exactly of ξ_g-type. For larger deviations from convective neutrality this analytical separation breaks down. However, it still turns out that the p-modes have a dominant ξ_p-component and the g-modes a dominant ξ_g-component. The indices p and g in Eqs. (5) are to remind this feature. Of attractive and simplifying aspects of this procedure is that (a) each normal mode ξ_n is expressed in terms of two scalars χ_p and χ_g and (b) the two components are orthogonal

$$(\xi_p, \xi_g) = 0. \tag{6}$$

We emphasize that ξ_p and ξ_g are not the conventional p and g modes of the star. They are defined by Eqs. (5) without being required to satisfy Eqs. (2)–(3). Any normal mode ξ_n, however, will have its ξ_p and ξ_g components.

Sobouti's ansatz for χ_p and χ_g are almost polynomials in r and spherical harmonics in (θ, ϕ). Clement (1986), not satisfied with the convergence of g modes, has proposed his own combinations. He has been able to produce better g eigenfrequencies. But poor features in eigenfunctions still persist. We shall comment on Clement's remark in Sect. 6.

3. Tidal interaction with normal modes

A unit mass of a primary star at position $\boldsymbol{r}$ is acted upon by the differential gravitational field of a companion at position $\boldsymbol{R}(t)$, as follows

$$\boldsymbol{F}_t(\boldsymbol{r}, t) = -\nabla U_t(\boldsymbol{r}, t) = GM_2 \left[\frac{\boldsymbol{R} - \boldsymbol{r}}{|\boldsymbol{R} - \boldsymbol{r}|^3} - \frac{\boldsymbol{R}}{R^3} \right], \tag{7}$$

where M_2 is the mass of the companion, and the common origin of $\boldsymbol{r}$ and $\boldsymbol{R}$ is the center of the primary. The tidal potential corresponding to $\boldsymbol{F}_t$ is easily obtained

$$\begin{aligned} U_t(\boldsymbol{r}, t) &= +GM_2 \left[\frac{1}{|\boldsymbol{R} - \boldsymbol{r}|} - \frac{\boldsymbol{R} \cdot \boldsymbol{r}}{R^3} - \frac{1}{R} \right] \\ &= + \frac{GM_2}{R} \sum_{l=2,3,\ldots} \left(\frac{r}{R} \right)^l P_l(\cos\theta), \end{aligned} \tag{8}$$

where the potential is chosen zero at the center of the primary, and θ is the angle between $\boldsymbol{R}$ and $\boldsymbol{r}$. Note that the summation in Eq. (8) starts from $l = 2$. This tidal potential generates a linear motion within the primary with the velocity field $\boldsymbol{v}(\boldsymbol{r}, t) = \dot{\xi}(\boldsymbol{r}, t)$. The displacement ξ satisfies the inhomogeneous wave equation

$$\rho \ddot{\xi} + \mathscr{W} \xi = -\rho \nabla U_t, \tag{9}$$

where $\mathscr{W}$ is the same as in Eq. (3). The time rate of the energy transfer from the gravitational field to the displacement field ξ is

$$\frac{dE}{dt} = -\int \rho \dot{\xi}^* \cdot \nabla U_t d^3x = -(\dot{\xi}, \nabla U_t). \tag{10}$$

Press and Teukolsky analysed $\xi(\boldsymbol{r}, t)$ and $\nabla U_t(\boldsymbol{r}, t)$ in terms of the complete set of the normal modes, $\{\xi_n\}$, and arrived at the following expression for the total energy transfer

$$\Delta E = 2\pi^2 \sum_n |A_n(\omega_n)|^2, \tag{11}$$

where the summation is over the normal modes, and A_n is defined as

$$A_n(\omega_n) = (\xi_n, \nabla V(\boldsymbol{r}, \omega_n)), \tag{12}$$

and $V(\boldsymbol{r}, \omega_n)$ is the time Fourier transform of $U_t(\boldsymbol{r}, t)$:

$$V(\boldsymbol{r}, \omega_n) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega_n t} U_t(\boldsymbol{r}, t)\, dt. \tag{13}$$

4. The overlap integral in our formalism

Substitution of the decomposition of Eqs. (5) in Eq. (12) gives

$$\begin{aligned} A_n(\omega_n) &= \int \rho \left(-\nabla \chi_p + \frac{1}{\rho} \nabla \times A_g \right) \cdot \nabla V d^3x \\ &= \int \chi_p \nabla \cdot (\rho \nabla V) d^3x - \int \nabla \cdot (\nabla \times A_g) V d^3x, \end{aligned} \tag{14}$$

where each term is integrated by parts and the integrated terms have been put equal to zero. The second integral in Eq. (14) is obviously zero. The first integral simplifies further by noting that ρ is spherically symmetric and $\nabla^2 V = 0$. For V is the potential of the secondary at points within the primary and satisfies Laplace's equation. Thus

$$A_n(\omega_n) = \int \chi_p(\boldsymbol{r}) \frac{d\rho}{dr} \frac{\partial V(\boldsymbol{r}, \omega_n)}{\partial r} d^3x. \tag{15}$$

We conclude that the gravitational field of the secondary excites the linear motion within the primary through their p-components. The accompaning g-motions are of course excited but through the intermediary of the p-motions. Thus, it should not be surprising to conclude at this premature stage that the contribution of the p-modes to the energy deposition, ΔE of Eq. (11), is by far larger than those of the g-modes, a prediction well born out by numerical calculations. In fact, in the vicinity of convective neutrality, i.e., small values of $\varepsilon = (\rho/p)[(\partial p/\partial \rho)_{\rm ad} - dp/d\rho]$ contributions from the g-modes are of the order of ε^2.

Another noteworthy point: That the interaction is between ξ_p and $\boldsymbol{F}_t$ is due to the fact that both fields are derived from scalar potentials. Had the perturbing force been derived from a vector potential (e.g., in magnetic interactions) then ξ_g motions would have entered the play at the expense of the exclusion of ξ_p.

Further reduction of $A_n(\omega_n)$ requires insertion of the orbital motion of $\boldsymbol{R}(t)$ in Eqs. (8) and (13), expansion of $V(\boldsymbol{r}, \omega_n)$ and $\chi_p(\boldsymbol{r})$ in terms of spherical harmonics, and integrations over the angles in Eq. (15). For a parabolic orbit one arrives at the following expression

$$A_{nlm}(\omega_n) = \left(\frac{GM_2^2}{R_1} \right)^{1/2} \left(\frac{R_1}{R_{\rm min}} \right)^{l+1} Q_{nl} K_{nlm}. \tag{16}$$

The overlap integral Q_{nl}, in the present formalism is

$$Q_{nl} = l \int_0^1 r^{l+1} \frac{d\rho}{dr} \chi_{p,nl}(r)\, dr, \tag{17}$$

where $\chi_{p,nl}(r)$ is the r-dependent part of $\chi_p(\boldsymbol{r})$ for the mode in question. The other quantities as given by Press and Teukolsky are

$$K_{nlm}=\frac{W_{lm}}{2\pi}2^{3/2}\eta I_{lm}(\eta\omega_n), \tag{18}$$

$$W_{lm}=(-1)^{(l+m)/2}\left[\frac{4\pi}{2l+1}(l+m)!\,(l-m)!\right]^{1/2}\Bigg/$$

$$\left[2^l\left(\frac{l-m}{2}\right)!\left(\frac{l+m}{2}\right)!\right] \text{ if } l+m \text{ is even,}$$

$$=0 \quad \text{if } l+m \text{ is odd,} \tag{19}$$

$$\eta=\left(\frac{M_1}{M_1+M_2}\right)^{1/2}\left(\frac{R_{\min}}{R_1}\right)^{3/2}, \tag{20}$$

$$I_{lm}(y)=\int_0^\infty dx(1+x^2)^{-l}\cos\left[2^{1/2}y\left(x+\frac{x^3}{3}\right)+2m\tan^{-1}x\right], \tag{21}$$

where M_1, M_2 = masses of primary and secondary, R_1 = radius of primary, $R_{\min}$ = periastron distance. The expression for the energy now becomes

$$\Delta E=\frac{GM_1^2}{R_1}\left(\frac{M_2}{M_1}\right)^2\sum_{l=2,3,\ldots}\left(\frac{R_1}{R_{\min}}\right)^{2l+2}T_l(\eta), \tag{22}$$

where

$$T_l(\eta)=2\pi^2\sum_{n=1}^{\infty}|Q_{nl}|^2\sum_{m=-l}^{l}|K_{nlm}|^2. \tag{23}$$

For computations the following steps were taken. 1) A Rayleigh-Ritz variational method was employed to obtain the eigenfrequencies and the eigenfunctions for various g and p modes (Sobouti, 1977a, b; Sobouti and Silverman, 1978). Computations were carried out for polytropes $n=1.5$, 2, 2.5, 3, 3.25, 3.5, and 4. 2) The information thus obtained was used to extract χ_p for each mode and to calculate Q_{nl} of Eq. (17). These overlap integrals are given in Tables 1 and 2. 3) Equations (18)–(21) were integrated to obtain K_{nlm}. 4) Next $T_1(\eta)$ of Eq. (23) and finally ΔE of Eq. (22) were computed. A plot of $T_2(\eta)$ and $T_3(\eta)$ are given in Figs. 1–3. Five p and nine g modes were found sufficient to make $T_2(\eta)$ and $T_3(\eta)$ converge satisfactorily at large η. McMillan et al. have also noticed that a larger number of g modes is necessary for a good convergence.

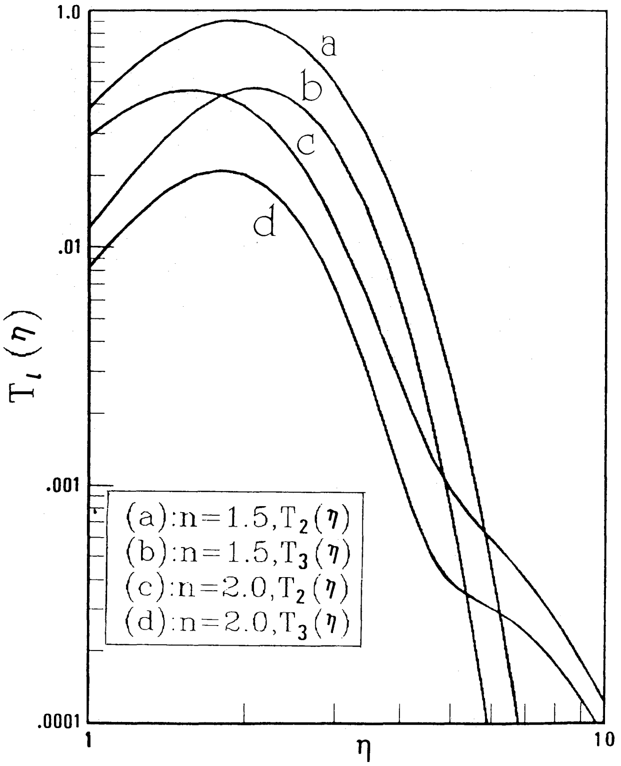

Fig. 1. A dimensionless measure of energy absorption by normal modes versus η, a measure of proximity of the tidal encounter, Eq. (20). T_2 and T_3 for $l=2$ and 3, respectively

Table 1. Overlap integrals ($|Q_{nl}|$) for n = 1.5, 2, 2.5, and 3 polytropes

Mode	$n=1.5$		$n=2$		$n=2.5$		$n=3$	
	$l=2$	$l=3$	$l=2$	$l=3$	$l=2$	$l=3$	$l=2$	$l=3$
p_5 ...	1.618(−4)	2.694(−4)	1.222(−3)	1.657(−3)	4.902(−3)	5.353(−3)	1.337(−2)	1.145(−2)
p_4 ...	6.174(−4)	9.802(−4)	3.085(−3)	4.065(−3)	9.339(−3)	1.011(−2)	2.110(−2)	1.780(−2)
p_3 ...	2.472(−3)	3.753(−3)	8.071(−3)	1.043(−2)	1.866(−2)	1.985(−2)	3.496(−2)	2.880(−2)
p_2 ...	1.062(−2)	1.554(−2)	2.287(−2)	2.861(−2)	3.988(−2)	4.107(−2)	6.161(−2)	4.853(−2)
p_1 ...	5.576(−2)	7.391(−2)	7.574(−2)	8.763(−2)	9.642(−2)	9.256(−2)	1.226(−1)	8.735(−2)
f ...	4.909(−1)	4.677(−1)	4.219(−1)	3.599(−1)	3.446(−1)	2.584(−1)	2.372(−1)	1.518(−1)
g_1 ...	...	...	2.105(−2)	1.435(−2)	5.099(−2)	2.980(−2)	9.940(−2)	6.423(−2)
g_2 ...	...	...	8.595(−3)	7.434(−3)	2.307(−2)	1.733(−2)	4.450(−2)	3.119(−2)
g_3 ...	...	...	3.843(−3)	3.810(−3)	1.191(−2)	1.057(−2)	2.402(−2)	2.139(−2)
g_4 ...	...	...	1.901(−3)	1.995(−3)	6.923(−3)	6.809(−3)	1.446(−2)	1.276(−2)
g_5 ...	...	...	8.287(−4)	1.079(−3)	3.515(−3)	3.543(−3)	9.115(−3)	8.600(−3)
g_6 ...	...	...	4.036(−4)	5.686(−4)	1.284(−3)	1.322(−3)	5.094(−3)	4.946(−3)
g_7 ...	...	...	1.212(−4)	1.382(−4)	3.221(−4)	3.377(−4)	4.661(−4)	4.541(−4)
g_8 ...	...	...	2.253(−5)	2.518(−5)	5.045(−5)	5.356(−5)	6.601(−5)	6.683(−5)
g_9 ...	...	...	2.118(−6)	2.357(−6)	4.048(−6)	4.360(−6)	4.987(−6)	5.228(−6)

Table 2. Overlap integrals ($|Q_{nl}|$) for $n=3.25$, 3.5, and 4 polytropes

Mode	$n=3.25$		$n=3.5$		$n=4$	
	$l=2$	$l=3$	$l=2$	$l=3$	$l=2$	$l=3$
p_5 ...	2.010(−2)	1.475(−2)	2.895(−2)	1.752(−2)	3.299(−3)	1.524(−3)
p_4 ...	2.940(−2)	2.111(−2)	3.990(−2)	2.326(−2)	1.856(−2)	7.603(−3)
p_3 ...	4.558(−2)	3.162(−2)	5.941(−2)	3.264(−2)	5.133(−2)	1.682(−2)
p_2 ...	7.569(−2)	4.917(−2)	9.572(−2)	4.723(−2)	6.326(−2)	2.109(−2)
p_1 ...	1.364(−1)	7.997(−2)	4.346(−2)	5.405(−2)	6.591(−2)	2.413(−2)
f ...	1.408(−1)	1.055(−1)	1.186(−1)	7.678(−2)	7.093(−2)	3.732(−2)
g_1 ...	1.329(−1)	4.212(−2)	1.080(−1)	3.569(−2)	5.740(−2)	3.485(−2)
g_2 ...	6.167(−2)	4.858(−2)	8.855(−2)	3.404(−2)	4.970(−2)	2.695(−2)
g_3 ...	3.764(−2)	2.786(−2)	4.802(−2)	3.342(−2)	2.185(−2)	1.369(−2)
g_4 ...	1.925(−2)	1.475(−2)	2.182(−2)	1.558(−2)	2.176(−2)	1.065(−2)
g_5 ...	7.621(−3)	6.297(−3)	8.047(−3)	6.262(−3)	7.083(−3)	4.884(−3)
g_6 ...	2.287(−3)	2.026(−3)	2.294(−3)	1.944(−3)	1.847(−3)	1.426(−3)
g_7 ...	4.888(−4)	4.578(−4)	4.714(−4)	4.279(−4)	3.575(−4)	3.045(−4)
g_8 ...	6.729(−5)	6.586(−5)	6.311(−5)	6.039(−5)	4.593(−5)	4.238(−5)
g_9 ...	5.025(−6)	5.104(−6)	4.652(−6)	4.635(−6)	3.288(−6)	3.258(−6)

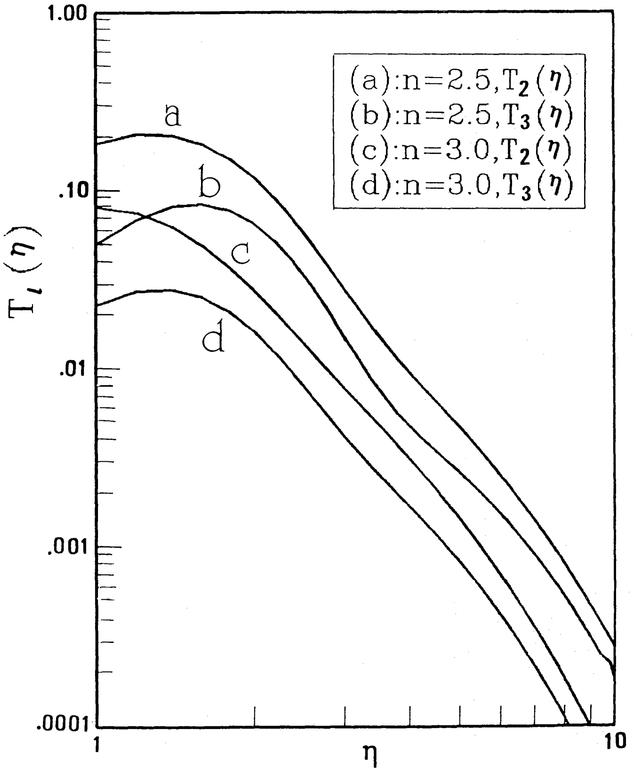

Fig. 2. Same as Fig. 1, for $n=2.5$ and 3

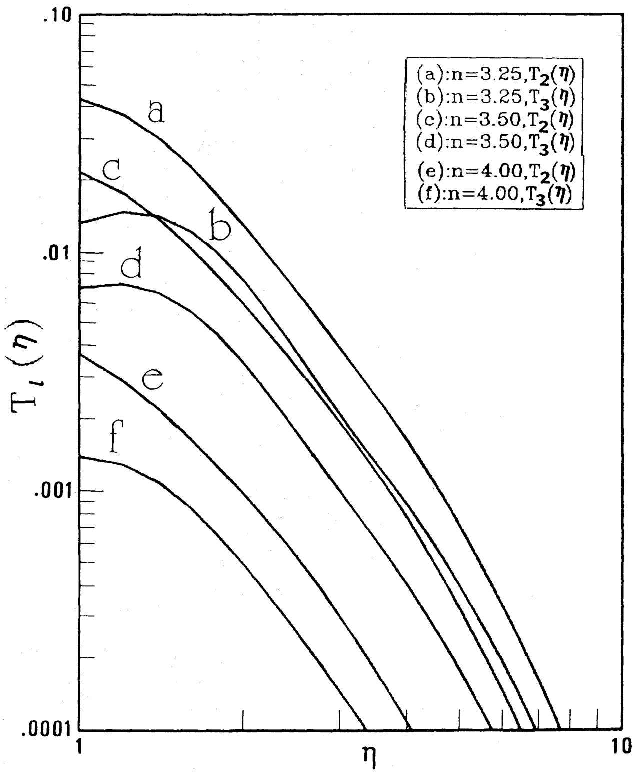

Fig. 3. Same as Fig. 1, for $n=3.25$, 3.5 and 4

5. Capture cross sections and rates

Consider two unbound stars of the total mass M_t, the reduced mass μ and the relative velocity v_∞ at infinity. What should their impact parameter $R_0(v_\infty)$ be to form a binary system? The angular momentum should remain constant in the process, and the energy absorbed by modes should at least be $\Delta E = \Delta E_1 + \Delta E_2 = \frac{1}{2}\mu v_\infty^2$.

These considerations give

$$R_0 = (2GM_t R_{\min}/v_\infty^2)^{1/2}. \tag{24}$$

In Figs. 4 and 5 $\log(VR_0/R_1)$ is plotted against V, where

$$V = \left(\frac{M_1}{M_\odot}\right)^{-1/2} \left(\frac{R_1}{R_\odot}\right)^{1/2} \left(\frac{v_\infty}{10\ \mathrm{km\,s^{-1}}}\right) = 617.44 \left(\frac{v_\infty}{v^*}\right), \tag{25}$$

and $v^* = (2GM_1/R_1)^{1/2}$ is the escape velocity from stellar surfaces. Figures 4 and 5 can be approximated by a single power law. Thus

$$R_0(V)/R_1 = CV^{-\alpha}. \tag{26}$$

For different polytropes the constants C and α are given in Table 3. The data is for the neighborhood of $V = 10\ \mathrm{km\,s^{-1}}$ (appropriate for globular clusters). The exponent α varies from 1.06 to 1.09 for polytropes 1.5 to 3, and remains constant at 1.09 for higher polytropes. The capture cross section $\sigma = \pi R_0^2$ may now be written as follows

$$\sigma = \begin{cases} 12.89\left(\frac{v_\infty}{v^*}\right)^{-2.12} R_1^2, & \text{for } n=1.5 \\ 10.64\left(\frac{v_\infty}{v^*}\right)^{-2.14} R_1^2, & \text{for } n=2 \\ 8.87\left(\frac{v_\infty}{v^*}\right)^{-2.16} R_1^2, & \text{for } n=2.5 \\ 7.24\left(\frac{v_\infty}{v^*}\right)^{-2.18} R_1^2, & \text{for } n=3 \\ 6.83\left(\frac{v_\infty}{v^*}\right)^{-2.18} R_1^2, & \text{for } n=3.25 \\ 6.46\left(\frac{v_\infty}{v^*}\right)^{-2.18} R_1^2, & \text{for } n=3.5 \\ 5.39\left(\frac{v_\infty}{v^*}\right)^{-2.18} R_1^2. & \text{for } n=4 \end{cases} \tag{27}$$

The tidal capture rate per unit volume in a cluster with a velocity distribution $f(v)$ is

$$\Gamma_{\mathrm{cap}} = \frac{N^2}{2}\int \sigma(v) v f(v) d^3v, \tag{28}$$

where N is the number density of stars. For a Maxwellian distribution characterised by a rms velocity v_0, Eqs. (27) and (28) give

$$\Gamma_{\mathrm{cap}} = K\left(\frac{R_1}{R_\odot}\right)^{2-\alpha}\left(\frac{M_1}{M_\odot}\right)^{\alpha}\left(\frac{v_0}{10\ \mathrm{km\,s^{-1}}}\right)^{1-2\alpha} \left(\frac{N}{10\ \mathrm{pc^{-3}}}\right)^2 \mathrm{pc^{-3}s^{-1}}, \tag{29}$$

where

$$K = 1.89\ 10^{-20} C^2 \left(\frac{\Gamma(2-\alpha)}{(0.75)^{2-\alpha}}\right) 10^{-2\alpha}, \tag{30}$$

and $\Gamma(z)$ is the Gamma function. The values of K for various polytropes are also listed in Table 3. In a globular cluster

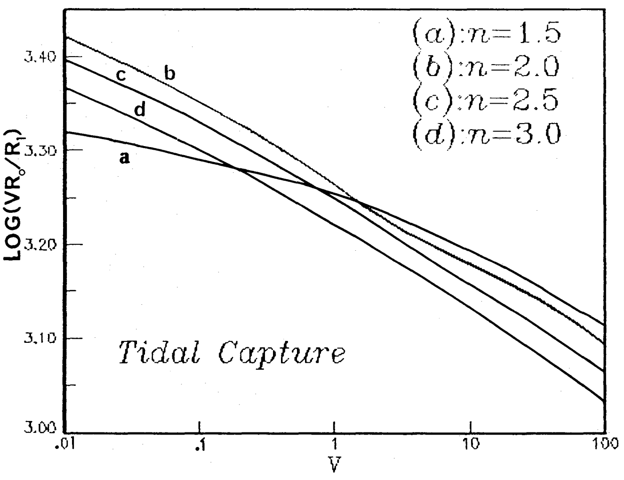

Fig. 4. Tidal capture impact parameter in units of R_1 as a function of relative velocity at infinity. Identical stars of polytropic indices $n = 1.5$, 2, 2.5, 3

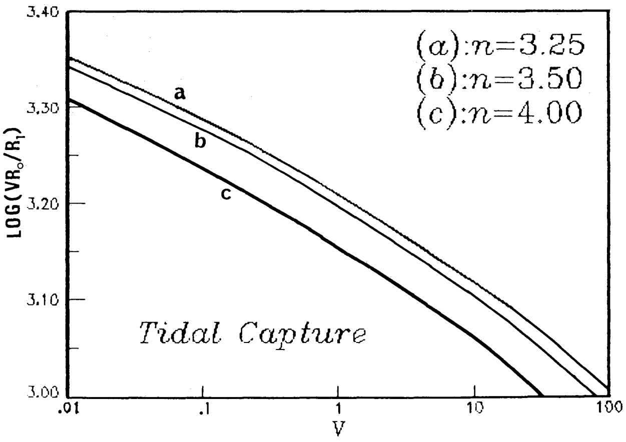

Fig. 5. Same as Fig. 4, $n = 3.25$, 3.5, 4

with parameters $R_1 \sim 5\,10^{10}$ cm, $M_1 \sim 0.5\,M_\odot$, $N \sim 10^4$, $v_0 \sim 10\ \mathrm{km\,s^{-1}}$, $n = 3$, radius 0.5 pc, and of age $\sim 5\,10^{17}$ s, the number of tidally captured binaries is ~ 43. Fabian et al.'s estimate for the same cluster is ~ 50.

6. Concluding remarks

Decomposition of fluid motions into an irrotational and a weighted solenoidal component, Eqs. (5), gives a deeper insight and a simpler computational procedure. One immediately concludes that only the irrotational component of the fluid motions interacts with the tidal gravitational field. Therefore, contributions of

Table 3. Best-fitting constants for tidal capture cross sections and rates for encounters between main-sequence stars determined near $v_\infty = 10\ \mathrm{km\,s^{-1}}$

n	1.5	2.0	2.5	3.0	3.25	3.5	4.0
C	1839	1782	1735	1672	1623	1579	1442
α	1.06	1.07	1.08	1.09	1.09	1.09	1.09
$K/10^{-15}$	0.659	0.594	0.540	0.481	0.453	0.429	0.358

the g modes to the energy transfer from orbital motions to oscillations are far less important than those of the p modes. For the g modes are mainly solenoidal motions. The reason is the common symmetry of the p motions and the tidal fields. Both are derived from scalar potentials. Had the interaction between the binary members been through a vector potential field then the g and toroidal displacements of the fluid would have been the medium of interaction. The authors intend to investigate mode excitation by an external magnetic field in the spirit of this last remark.

Our computations cover the range $n = 1.5$, 2, 2.5, 3, 3.25, 3.5, and 4. In the overlapping region we obtain the same results as the other authors. This is in spite of the fact that we use an entirely different formalism for the overlap integral and a variational technique for mode calculations (Press et al., calculated their modes by Robe's (1968) method; Lee and Ostriker, McMillan et al., and Ray et al., adopted Cox's (1980) and Dziembowski's (1971) procedures). We wish to present this agreement with others and the simplicity of the procedure (namely the expression for the overlap integral in terms of a scalar property of the modes) as an evidence for the validity and strength of expressing the modes in terms of scalar and vector potentials. Clement finds Sobouti's (1981) decomposition of Eqs. (5) 'esthetically pleasing' but expresses a desire for supporting computations. The full agreement of the present with those obtained by other means may also serve as such supporting evidence.

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