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Revised dynamics or dark matter in galactic and extra galactic scales?

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Abstract. Allowing the energy of a gravitational field to serve partially as its own source allows gravitating bodies to exhibit stronger fields, as if they were more massive. Depending on degree of compaction of the body, the field could be one to five times larger than the newtonian field. This is a comfortable range of increase in field strength and may prove to be of convenience in the study of velocity curves of spirals, of velocity dispersions in clusters of galaxies and in interpreting the Tully-Fisher or Faber-Jackson relations in galaxies or systems of galaxies. The revised gravitation admits of superposition principle but only approximately in systems whose components are widely separated. The revised dynamics admits of the equivalence principle in that, the effective force acting on a test particle is derived from a potential, and could be eliminated in a freely falling frame of reference.

Key words: –Dark matter– Missing mass– Alternative dynamics– Spiral galaxies

> The only justification for our concepts and system of concepts is that they serve to represent the complex of our experiences. Beyond this they have no legitimacy. I am convinced that the philosophers have had a harmful effect upon the progress of scientific thinking in removing certain fundamental concepts from the domain of empiricism, where they are under our control, to the intangible heights of the a priori. Albert Einstein, 1922.

1. Introduction

The problem of dark matter surrounding spiral galaxies, wrote Van Albada et al. (1985), *is one of the most enigmatic questions in present day astrophysics. A number of years of intensive research have brought little or no clarification.*

Earlier suggestions to resolve mass discrepancy were on the conservative side. Bahcall and Cassertano (1985) spoke of *missing mass* as synonymous to *missing light*. Tremaine and Lee (1987)with *dark matter* meant the matter whose existence was inferred only through its gravitational effects. Critics were vocal. Outstanding among them Milgrom (1983 a,b,c), the architect of *MOND*, spoke of the *dark side of the dark matter hypothesis* and *of its arbitrariness in that one invokes the dark matter in the correct amount and spatial distribution needed to explain the mass discrepancy in each and every case for itself* (1987).

In the course of the past two decades acurate observations of velocity curves of spiral galaxies have become available.See, e. g., Begeman et al. (1991), Sanders (1996), Sanders and Verheijen (1998), Mc Gauph and de Blok (1998). Credible velocity dispersions of galactic systems have emerged. See, for example, Tullly et al. (1996) for data on Ursa Majoris cluster. On the theoretical side many ingenious and bold candidates for dark matter have been offered. A list compiled by Ostriker and Steinhardt (2003) includes items such as cold collisionless dark matter, strongly self interacting dark matter, warm dark matter, repulsive dark matter, fuzzy dark matter, self annihilating dark matter, decaying dark matter, etc.

In spite of all these developments, however, there is no clue as how to detect the dark matter and get to the physics that it obeys. The issue does not seem less enigmatic than what von Albada and his colleagues described in 1985. Arguments pro and con are the same as a quarter of century ago. Proponents of dark matter assume the validity of Newtonian dynamics at all distance scales and look for one or another form of hypothetical matter to provide the missing gravity. Skeptics, on the other hand, see no logic in resorting to a concept that solves the riddle of dynamics but remains unamenable to further validation by any other known physical means.

A historical reminder might be timely. In the closing decade of the 19th. century, physicists had agreed that light was an electromagnetic wave. And out of the experience with waves in other contexts had required a medium of propagation for it, the ether. Yet ether evaded all attempt of detection no matter how ingeniously the detection devices were designed. On the other hand the notion of medium of propagation seemed so obvious to everyone, that some of the brightest minds of the time invoked

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theories that would have kept the ether, but in a concealed form. The famous transformations of Lorentz were developed primarily to explain the null results of the experiments of Fiseau, Michelson, Michelson & Morely, etc., rather than as a foundation for the special relativity that emerged later. It was up to Einstein to think and later to speak out that *our concepts have no legitimacy beyond what the complex of our experiences bestows upon them.* If experiments do not reveal the ether it could be dispensed with. If experiments show that the speed of light is the same in all reference frames, that could be a fact to build the new physics upon it.

In these early years of the 21st. Century, it would not be unwise to take Einstein's advice seriously. Accept the validity of newtonian dynamics in solar and similar systems, where it has been tested, but look for alternatives in larger galactic and extragalactic scales, where it has not been verified. Many researchers have actually adopted such a point of view before. See Sanders and Mc Gauph (2002) for a review of Milgrom's *MOND* and earlier attempts. Here, we adopt a variational approach to the problem. We amend the classical action by adding a term to it that eventually makes provision for the field energy to serve partially as its own source. The procedure is apt to a general relativistic generalization, in preparation for a future presentation.

2. Variational formulation

The system to be considered is a collection of point masses m_i at positions $\mathbf{x}_i$, $i = 1, \cdots, N$; plus the field $\phi(\mathbf{x})$ at point $\mathbf{x}$. Both newtonian equation of motion and equation of gravitational field are derivable from the following action integral.

$$I = \int L[\mathbf{x}_i, \phi(\mathbf{x})]\ dt, \tag{1}$$

$$L[\mathbf{x}_i, \phi(\mathbf{x})] = \sum m_i \left[\frac{1}{2}\dot{\mathbf{x}}_i^2 + \phi\left(\mathbf{x}_i\right)\right] - \frac{1}{8\pi G}\int \left|\nabla\phi(\mathbf{x}\right|^2 d^3x.$$

At point $\mathbf{x}$ the gravitational energy density is $(8\pi G)^{-1}|\nabla\phi(\mathbf{x})|^2$. By Einstein, this energy has an equivalent mass. We propose to entertain the possibility of using this mass or a fraction α of it as the source of extra gravitation on the particles. The effect on m_i at position $\mathbf{x}_i$ will be $\alpha G m_i (8\pi G c^2)^{-1}|\nabla\phi(\mathbf{x})|^2|\mathbf{x}-\mathbf{x}_i|^{-1}$. The contribution to the action integral from all space and all particles will be

$$I_{int} = \int L_{int}[\mathbf{x}_i, \phi(\mathbf{x})]\ dt,$$

$$L_{int}[\mathbf{x}_i, \phi(\mathbf{x})] = \frac{1}{8\pi}\frac{\alpha}{c^2}\sum_i m_i \int \frac{|\nabla\phi(\mathbf{x}|^2}{|\mathbf{x}-\mathbf{x}_i|} d^3x. \tag{2}$$

The dimensionless constant α is of the order of unity. It is introduced for possible later adjustments. Note that Gm_i/c^2 is of the order of the Schwarschild radius of m_i.

Before proceeding further, let us emphasize that the argument given in composing I_{int} is for mnemonic purposes only. The formal statement of the assumption is the following: The classical action integral from which Newton's laws are inferred is inadequate in galactic and extragalactic scales. We propose to amend it by adding I_{int} to the classical action. The addition is a scalar, quadratic in field gradients, and contains the particle coordinates and masses. It will alter both the field equation and the equations of motion.

The field equation for $\phi(\mathbf{x})$ is obtained by requiring the functional derivative of the total action, $I + I_{int}$, with respect to $\phi(\mathbf{x})$to vanish. Thus,

$$\begin{aligned}\frac{\delta(I+I_{int})}{\delta\phi(\mathbf{x})} \ &= \ \nabla.\left[\nabla\phi(\mathbf{x})\left\{1 - \frac{\alpha G}{c^2}\sum\frac{m_i}{|\mathbf{x}-\mathbf{x}_i|}\right\}\right] \\ &+ \ 4\pi G\sum m_i\delta(\mathbf{x}-\mathbf{x}_i) = 0. \end{aligned} \tag{3}$$

By writing $\delta(\mathbf{x}-\mathbf{x}_i) = \frac{1}{4\pi}\nabla^2|\mathbf{x}-\mathbf{x}_i|^{-1}$ one immediately integrates Eq. (3) into

$$\nabla\phi\left\{1 - \frac{\alpha G}{c^2}\sum_i\frac{m_i}{|\mathbf{x}-\mathbf{x}_i|}\right\} = -G\nabla\sum\frac{m_i}{|\mathbf{x}-\mathbf{x}_i|}. \tag{4}$$

Denoting the newtonian potential by

$$\phi_N(\mathbf{x}) = -G\sum\frac{m_i}{|\mathbf{x}-\mathbf{x}_i|}, \tag{5}$$

Eq.(4) integrates into

$$\begin{aligned}\phi(\mathbf{x}) \ &= \ \frac{c^2}{\alpha}\ln\left[1 + \alpha\frac{\phi_N(\mathbf{x})}{c^2}\right] \\ &= \ \phi_N\left[1 - \frac{1}{2}\alpha\frac{\phi_N}{c^2} + \frac{1}{3}\alpha^2\frac{\phi_N^2}{C^4} + \cdots\right].\end{aligned} \tag{6}$$

Equation of motion for m_i is obtained likewise,

$$\begin{aligned}&\frac{\delta(I+I_{int})}{\delta\mathbf{x}_i} = \\ &m_i\left[\ddot{\mathbf{x}}_i - \nabla_i\phi(\mathbf{x}_i) - \frac{1}{8\pi}\frac{\alpha}{c^2}\nabla_i\int\frac{|\nabla\phi(\mathbf{x})|^2}{|\mathbf{x}-\mathbf{x}_i|}d^3x\right] = 0.\end{aligned} \tag{7}$$

Newtonian limits in all cases are obtained by letting $\alpha = 0$. To elucidate the significance of the proposed amendment to the classical action and the way it relates to mass discrepancy, several simple examples are worked out in section 3.

3. Applications

3.1. Gravitational field of spheres

Consider a sphere of uniform density ρ, radius R, and total mass M: From Eqs. (5) and (6) the outside solution is

$$\phi(r) = \frac{GM}{s}\ln\frac{r-s}{r} = -\frac{GM}{r}\left(1 + \frac{1}{2}\frac{s}{r} + \cdots\right), \quad r \geq R$$

$$\frac{d\phi}{dr} = \frac{GM}{r(r-s)}, \quad r \geq R, \tag{8}$$

where $s = \alpha GM/c^2$ is of the order of Schwarschild radius of M. For the inside, noting that $\phi_N(r) = -\frac{1}{2}\frac{GM}{R^3}\left(3R^2 - r^2\right)$, one obtains

$$\phi(r) = \frac{c^2}{\alpha}\ln\left[1 - \frac{3}{2}\frac{s}{R} + \frac{1}{2}\frac{s}{R}\frac{r^2}{R^2}\right], \quad r \leq R,$$
$$\frac{d\phi}{dr} = \frac{GM}{R^2}\frac{r}{R}\left[1 - \frac{3}{2}\frac{s}{R} + \frac{1}{2}\frac{s}{R}\frac{r^2}{R^2}\right]^{-1}, \quad r \leq R. \qquad (9)$$

3.2. Motion of test satellites

Equation of motion is obtained by substituting Eqs. (8) & (9) in Eq. (7) and reducing it. The result is

$$m\ddot{\mathbf{r}} = -\frac{GmM}{r^2}\left[1 + u\left(\frac{s}{R}\right) + \frac{s}{r-s}\right]\frac{\mathbf{r}}{r}, \qquad (10)$$

where $u(s/R)$ is derived, numerically calculated, and plotted in the Appendix. As the central body contracts and R reduces from an initially large value to its allowed minimum $\frac{3}{2}s; u(s/R)$ increases from zero to 4. The first term on the right hand side of Eq. (10) is recognized as the classical expression. The second term, $u(s/R)$, is the pivotal one and as mentioned it could be four times as large as the classical term. In newtonian parlance this amounts to saying that a compact object may, gravitationally, present itself up to five times more massive than the same object in a diffuse state. This is what we propose as a partial answer to mass discrepancy!
The third term in Eq. (10) arises from deviations of the force law from r^{-2}. It is infinitesimal compared with the previous two terms, though it may observably perturb elliptical orbits in secular time scales.

3.3. N-body problem

Consider a collection of spheres each of mass M_i, radius R_i, and positions $\mathbf{x}_i$. Assume the system members are well separated such that $|\mathbf{x}_i - \mathbf{x}_j| = r_{ij} \gg R_i$ for all $i \neq j$. It is demonstrated in the Appendix, that on account of this assumption, the integral appearing in Eq. (7) splits into a sum given by Eq. (A. 7). Thus, one finds

$$\ddot{\mathbf{x}}_i - G\sum M_j^{\text{eff}}\frac{(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3}\Bigg] = 0. \qquad (11)$$

where $M_j^{\text{eff}} = M_j(1+u_j), u_j = u(s_j/R_j)$. Actually a term of the order $G\sum_j M_j s_j |r_{ij} - s_j|^{-1}$, is neglected in Eq (11), a) because of the extra small factor s_j and b) because of its steeper, r^{-3}, decrease with distance compared with r^{-2} dependence of the retained terms.

Albeit the approximations, Eq. (11) is identical with the newtonian many body equation of motion, except that instead of the classical masses, the effective masses, $M_j^{\text{eff}} = M_j(1+u_j)$, play the role. Depending on the degree of compactness of the constituent bodies, the effective masses could, in the case of uniform densities, be 1 to 5 times larger than the classical ones. Having noted this, it is easy to write the first integrals of Eq. (11):

$$\text{Momentum}: \mathbf{P} = \sum M_i^{\text{eff}}\dot{\mathbf{x}}_i = const. \qquad (12)$$

$$\text{Angular momentum}: \mathbf{L} = \sum M_i^{\text{eff}}\mathbf{x}_i \times \dot{\mathbf{x}}_i = const. \qquad (13)$$

$$\text{Energy}: E = T + V = \text{const.}, \qquad (14)$$
$$T = \frac{1}{2}\sum_i M_i^{\text{eff}}\dot{\mathbf{x}}_i^2, \quad V = -\frac{1}{2}G\sum_{i,j} M_i^{\text{eff}}M_j^{\text{eff}}/r_{ij}$$

$$\text{Virial theorem}: 2\bar{T} + \bar{V} = 0, \qquad (15)$$

where 'bars' on T and V indicate averages over system members or over time periods.

Increased effective mass in Eq. (11) and the virial theorem may prove useful in explaining the missing mass or missing gravity issues, in the velocity curves of galaxies and/or velocity dispersions in clusters of galaxies.

4. Concluding remarks

> *... it is contrary to the mode of thinking in science to conceive of some thing that can act itself, but which cannot be acted upon. Albert Einstein, 1922.*

The quotation is from Einstein's argument to set the stage for the rejection of absolute space-time continuum of special relativity in the presence of matter, and his admiration of Mach's reservation to ascribe an absolute meaning to *inertia* irrespective of matter elsewhere. In an attempt to amend newtonian dynamics, we have followed Einstein's advice and tried to alter the laws of gravity and of motion simultaneously. The interaction lagrangian of Eq. (2) contains both the particle coordinates, $\mathbf{x}_i$, and the field gradient, $\nabla\phi$.

Noteworthy in L_{int} is the field energy that eventually appears as the increased effective mass, $M^{\text{eff}} = M(1+u)$ in Eqs. (10-15). We recall that the main issues in the analysis of the velocity curves of spirals are: a) insufficiency of the observed and/or estimated stellar masses to provide the required dynamical effect, and b) the flatness of the velocity curves beyond what the distribution of observed masses would permit. Similarly, in clusters of galaxies, again the observed masses are much smaller than the virial ones, inferred from the velocity dispersion in the cluster. The increased effective mass discussed above could provide a partial answer in both cases of luminous mass deficiency. The flatness of the rotation curves is a matter of the distribution of effective masses. For example, if there are enough low luminosity collapsed faint objects in the outskirts of spiral arms the velocity curves could be flattened enough.

The field in Eq. (6), unlike the Newtonian one, is not proportional to the mass of the constituent members and does not admit of superposition principle. Even in the case of approximate superposition in systems with widely separated components, the field of each component is a highly nonlinear function of its mass. This nonlinearity could prove useful in discussions of Tully-Fisher relation in spirals, an empirical relation between the brightness of the galaxy and the asymptotic orbital speed at the flat end of the velocity curve. The Tully-Fisher relation, with the assumption of a further empirical mass to light ratio for the galaxy, translates into a power law between the asymptotic speed and the total mass, $v_{asymp} \propto M^{\beta}$, $2.5 < \beta < 3.5$. In the present dynamics, β will sensitivity depend on the compaction factor, the fraction, and the distribution of the compact members of the galaxy. Same considerations and reservations holds for the Faber-Jackson relation, another empirical relation between the velocity dispersion in cluster of galaxies and the luminosity of the cluster.

In section 3.3, conservation laws were derived for the case of approximate superposition. One can, however, do better. The total lagrangian, $L + L_{\rm int}$, is invariant under time translations, coordinate translations, and coordinate rotations. Therefore, seven first integrals of motion, corresponding to energy, momenta, and angular momenta, should exist. This point of view will be presented elsewhere.

Birkhof's theorem states that in newtonian and relativistic regimes the gravitational field of a spherically symmetric system is independent from the internal structure of the system. This is not the case in the present theory as it is highlighted by the presence of $u(s/R)$ in Eq. (10). This may open the possibility of studying the history of contraction or collapse of an object by logging the external gravitational field of the object as it evolves.

From Eq.(7), the effective force on a test particle is derived from a potential. It could be eliminated in a freely falling frame of reference, meaning that the dynamics developed here admits of equivalence principle. This is hardly surprising. For, in the action integrals of Eqs. (1) and (2) one single mass m_i is used to compose the kinetic and the gravitational potential energies of the system.

Finally a criticism: a) The mass equivalent of energy is a relativistic concept. Yet, a gravitational effect was attributed to it through the law of gravity of Newton. b) The effective mass attains its full significance when the source object shrinks into sizes of the order of Schwarzschild radius. Logically, a covariant general relativistic approach should be adopted. Presently, we are looking for a field equation and a geodesic one in which some sort of mutual interactions between distant points, similar to one in $L_{\rm int}$ are taken into account. As in Eq. (2), there will be a free parameter α in the formulation. We will require the emerging dynamics to reduce a) to the general relativistic one in the limit $\alpha \to 0$; b) to the present dynamics in the limit of weak fields but $\alpha \neq 0$, and c) to the newtonian one in the limit of zero α and weak field. This will be presented elsewhere.

Appendix A: Reduction of L_{int}, Equation(2)

We have already seen the formal role of the interaction lagrangian, L_{int} in altering the equations of motion and of the field. Its numerical values are needed in the study of orbits, Eq.(7); virial theorem, Eq. (15), etc. Here, we calculate it a) for a single uniform spherical mass, and b) for a collection of unform spheres.

a)For a spherically symmetric field the non newtonian integral appearing in Eq. (7) is

$$l_{int}(r) = \frac{1}{8\pi}\frac{\alpha}{c^2}\int \frac{|d\phi(r')/dr'|^2}{|\mathbf{r}-\mathbf{r}'|} d^3r'. \qquad \text{(A.1)}$$

In an expansion of $[\mathbf{r}-\mathbf{r}']^{-1}$ in Legendre polynomials, only the $l=0$ term will have non vanishing contribution; for $d\phi/dr'$ has no directional dependence. We carry out angular integrations and split the radial range of the integration into three intervals $0 < r' < R$, $R < r' < r$ and $r < r' < \infty$. Thus,

$$l_{int} = \frac{\alpha}{2c^2}\left[\frac{1}{r}\int_0^R \left|\frac{d\phi(r')}{dr'}\right|^2 r'^2 dr' + \frac{1}{r}\int_R^r \left|\frac{d\phi(r')}{dr'}\right|^2 r'^2 dr' + \int_r^\infty \left|\frac{d\phi(r')}{dr'}\right|^2 r' dr'\right]. \qquad \text{(A.2)}$$

The expression for the field gradient in the first integral is the interior solution of Eq. (9) and in the other two are the outer solution of Eq. (8). The remaining mathematical manipulations are elementary, though lengthy. One eventually arrives at

$$l_{int} = \frac{GM}{r} u\left(\frac{s}{R}\right), \quad s = \frac{\alpha GM}{c^2}, \qquad \text{(A.3)}$$

where

$$u(y) = -\frac{1}{2} + \frac{3}{y}\left[1 - \sqrt{\frac{(2-3y)}{y}}\arctan\sqrt{\frac{y}{(2-3y)}}\right] \qquad \text{(A.4)}$$

The parameter s/R indicates the degree of compaction of the central body. As the body contracts from an initially dispersed state, $(s/R) \approx 0$, to its maximum allowable compact state, $(s/R = 2/3)$, $u(s/R)$ increases from zero to 4. A plot of $u(s/R)$ is given in figure below. In newtonian language, the effect on the motion of an orbiting satellite is to make the central body look $(1+u)$ times more massive than what it does in newtonian regime.

b)Multi-component systems: In Eq. (A.1) The main contribution to the integral comes from the immediate vicinity of the central body. As one approaches the gravitating body from outside, $|d\phi(r')/dr'|^2$ grows roughly as r'^{-4} up to the surface of the body and fades out to zero at its center. Assume a system consisting of two well separated spheres (M_1, R_1) at $\mathbf{x}_1$ and (M_2, R_2) at $\mathbf{x}_2$ with

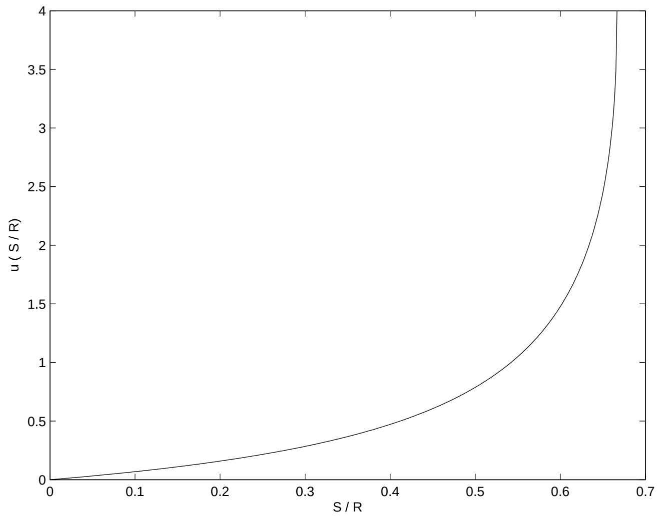

Fig. A.1. $u(s/R)$ versus s/R, the degree of compactness of the central body.

$|\mathbf{x}_1 - \mathbf{x}_2| \gg R_1 \& R_2$. From Eqs. (5) and (6), we read

$$|\nabla\phi(\mathbf{x}')|^2 = \left[1 - \frac{\alpha G}{c^2}\left(\frac{M_1}{|\mathbf{x}'-\mathbf{x}_1|} + \frac{M_2}{|\mathbf{x}'-\mathbf{x}_2|}\right)\right]^{-2} \times G^2\left[\frac{M_1^2}{|\mathbf{x}'-\mathbf{x}_1|^4} + \frac{M_2^2}{|\mathbf{x}'-\mathbf{x}_2|^4} + 2M_1M_2\frac{(\mathbf{x}'-\mathbf{x}_1).(\mathbf{x}'-\mathbf{x}_2)}{|\mathbf{x}'-\mathbf{x}_1|^3|\mathbf{x}'-\mathbf{x}_2|^3}\right]. \quad (A.5)$$

Again, the main contributions come from the vicinities of the two spheres. In the immediate vicinity of sphere 1, however, $G^2M_1^2|\mathbf{x}' - \mathbf{x}_1|^{-4}$ is large. While, with the assumption of large separation of the two spheres, $G^2M_2^2|\mathbf{x}'-\mathbf{x}_2|^{-4}$ is insignificant. Vice versa for the vicinity of sphere 2. The expression containing $(\mathbf{x}'-\mathbf{x}_1).(\mathbf{x}'-\mathbf{x}_2)$ practically adds up to zero upon integration over $\mathbf{x}'$, because of its directional dependencies. We conclude that

$$l_{int}(x) \approx \frac{GM_1u_1}{|\mathbf{x}-\mathbf{x}_1|} + \frac{GM_2u_2}{|\mathbf{x}-\mathbf{x}_2|}, \quad u_i = u\left(\frac{s_i}{R_i}\right). \quad (A.6)$$

Similarly, for a system of many, but well separated, components one obtains

$$l_{int}(x) \approx G\sum_i \frac{M_iu_i}{|\mathbf{x}-\mathbf{x}_i|}, \qquad u_i(s_i/R_i). \quad (A.7)$$

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