Dark companion of baryonic matter. III

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Wherever one talks of dark matter, one does so where there is an observable matter and an associated unsolved dynamical issue to be settled. We promote this observation to the status of an axiom and conjecture that there is a dark companion to every baryonic matter, subject to certain rules as regards its size, distribution. To pursue the proposition in a systematic way we resort to the rotation curves of spiral galaxies. They have non classical features. First, we design a spacetime metric around the galaxy to accommodate these features. Next we calculate the density and pressure of a hypothetical dark matter that could generate such a spacetime. In the weak field regime and for a spherical distribution of mass M, we are able to assign a dark perfect gas companion, whose density is almost proportional to $M^{1/2}$ and fades away almost as r^{-2}. However, in view of this orderly relation between the observable mass and its dark companion, one may choose to interpret the whole scenario as an alternative theory of gravitation.

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I. INTRODUCTION

That the baryonic content of galaxies, cluster of galaxies, or for that matter the universe at large, does not provide sufficient gravitation to explain the observed dynamics of the systems is an established fact. To resolve the dilemma, dark matter (energy) scenarios and/or alternative theories of gravitation have been speculated and debated. The fact, however, remains that the proponent of dark matter (energy) have always looked for it in baryonic environments. No one has, so far, reported a case where there is no ordinary matter, but there is a dynamical puzzle to be resolved. In view of this negative observation, it is not unreasonable to conjecture that:

'Any baryonic matter has an ever attendant dark companion, and there are rule to this companionship as regards the size and the distribution of the matter and its twin companion.'

On the other hand such a point of view, that denies the independent existence of the dark matter, is equivalent to the assumption that the known theories of gravitation, based on baryonic matter alone, do not tell the whole story and there is room for amendments. This conclusion in turn reduces the distinction between the dark matter scenarios and alternative theories to the level of semantics: As long as the dark matter betrays no interaction with the baryonic matter other than the gravitational one, one has the option either to assume a dark component to every baryonic matter subject to certain rules, and account for its gravitational field in the conventional way; or simply adhere to the baryonic matter but come up with an alternative law of gravitation.

This paper, like its two precursors [1], [2], is an inverse approach to understand the idiosyncracies of the rotation curves of spiral galaxies. Based on observations we first design a spacetime metric that is capable of supporting the non-classical features of the rotation curves. This step amounts to actually giving the gravitational potential at the outer reaches of a galaxy. Next we attribute the deviations from the conventional baryon induced gravitation to a dark companion to the galaxy, and give a rule for the size and distribution of its density and pressure.

II. OBSERVED FACTS AND IMPLICATION

There are three main characteristics to the rotation curves of spirals

- They decline, if at all, much less steeply than the Keplerian curves do, see, e. g. [3] - [10].
- Beyond the visible disks of the galaxies, orbital speeds are, more often than not, proportional to the fourth root of the mass of the galaxy, the Tully-Fisher relation [11].
- Deviation from the classical concepts, in this case the gravitation, show up in large scale systems and at large distances, or in the description of Milgrom at small gravitational accelerations [12].

These observed facts are our starting points. The galaxy, though a flattened system, is approximated by a spherically symmetric distribution of baryonic matter. Accordingly the spacetime external to it will be static and spherically symmetric:

$$ds^2 = -B(r)dr^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (1)$$

We adopt a dark matter language and assume that the galaxy processes a static dark perfect gas companion of density $\rho_d(r)$, of pressure $p_d(r) << \rho_d(r)$, and of 4-velocity,

$$U_t = -B^{1/2},\ U_r = U_\theta = U_\varphi = 0.$$

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In the baryonic vacuum, Einstein's field equations become

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\left[p_d \; g_{\mu\nu} + (p_d + \rho_d) U_\mu U_\nu\right], \qquad (2)$$

where we have let $8\pi G$ and c^2 equal to 1. To respect the Bianchi identities, one must require the 4-divergence of the right hand side of Eq. (2) to varnish. This, in turn, leads to the hydrostatic equilibrium of the dark fluid and to a differential equation for p_d. From Eq. (2) the two combinations,

$$R_{tt}/B + R_{rr}/A + 2R_{\theta\theta}/r^2 \text{ and } R_{tt}/B + R_{rr}/A,$$

give

$$\frac{1}{r^2}\left[\frac{d}{dr}\left(\frac{r}{A}\right) - 1\right] = -\rho_d, \qquad (3)$$

$$\frac{1}{rA}\left(\frac{B'}{B} + \frac{A'}{A}\right) = \rho_d + p_d, \qquad (4)$$

respectively. To the first order of smallness, we neglect p_d in comparison with ρ_d, eliminate ρ_d between the two equations, and find

$$\frac{B'}{B} = \frac{1}{r}(A-1). \qquad (5)$$

We now assume $(A-1)$ is analytic and differentiable, and has the following series expansion at large r's:

$$A(r) - 1 = \left(\frac{r_0}{r}\right)^\alpha \sum_{n=0} \frac{s_n}{r^n}, \qquad (6)$$

where the indicial exponent, α, is dimensionless, and r_0 is an arbitrary length scale, presumably of the order of the size of the galaxy. The constant parameters, s_n, are of dimension (length)^n. Their size will be discussed shortly. It should be pointed out that the expansion of Eq (6) is for regions external to the galaxy. In particular, the center $r = 0$ is not included in the indented domain and no question of singularity will arise.

Next we substitute Eq. (6) in Eq. (5) and integrate for $B(r)$. Depending on whether α is zero or not, two different solutions emerge:

$$B = \exp\left[-\left(\frac{r_0}{r}\right)^\alpha \sum_{n=0} \frac{s_n}{(n+\alpha) r^n}\right], \quad \text{for } \alpha \neq 0, \qquad (7)$$

$$= \left(\frac{r}{r_0}\right)^{s_0} \exp\left[-\sum_{n=1} \frac{s_n}{n r^n}\right], \quad \text{for } \alpha = 0. \qquad (8)$$

Both gravitation- and speed-wise, galactic environments are non relativistic. In the weak field regime, we retain the first two terms in the series expansion of the exponential terms, approximate the gravitational potential by $\phi(r) = \frac{1}{2}(B-1)$, calculate the circular speed of a test object orbiting the galaxy from $v^2 = r d\phi/dr$, and find

$$v^2 = \frac{1}{2}\left(\frac{r_0}{r}\right)^\alpha \left[s_0 + \frac{s_1}{r} + \cdots\right], \quad \alpha \neq 0, \qquad (9)$$

$$= \frac{1}{2}\left(\frac{r}{r_0}\right)^{s_0} \left[s_0 + \frac{s_1}{r} + \cdots\right], \quad \alpha = 0. \qquad (10)$$

At large distances from the galaxy, the s_0 terms in Eqs. (9) and (10) are the dominant ones. As one moves closer, s_1 terms gain over s_0. Further inward, s_2 and higher terms may take turn. It should, however, be noted the formalism devised here is to deal with velocity anomalies at the outer reaches of the galaxies, beyond their visible extensions. No chance will arise for s_2 and higher order terms to play roles.

Logarithmic slopes of the rotation curves, $\Delta = d\ln v^2/d\ln r$, and their asymptotic behaviors are

$$\text{a)} \quad \Delta = \left[-\alpha s_0 - (1+\alpha)\frac{s_1}{r}\right] / \left[s_0 + \frac{s_1}{r}\right], \quad \alpha \neq 0$$
$$\to -\alpha \quad \text{at large } r$$
$$\to -(1+\alpha) \quad \text{at small } r. \qquad (11)$$

$$\text{b)} \quad \Delta = \left[s_0^2 - (1-s_0)\frac{s_1}{r}\right] / \left[s_0 + \frac{s_1}{r}\right], \quad \alpha = 0$$
$$\to s_0 \quad \text{at large } r$$
$$\to -(1-s_0) \quad \text{at small } r. \qquad (12)$$

At far distances, one has the falling slope $-\alpha$ in case (a), and the rising slope s_0 in case (b). In either case one, however, knows that the observed asymptotic slopes are much less steep than the Keplerian slope, -1. Therefore, α in case (a) and s_0 in case (b) should be much smaller than 1. At closer distances the slopes are almost Keplerian, except for small abberation by $+\alpha$ or $-s_0$. This, however, should not be taken seriously. It is a consequence of the assumption of spherical symmetry of the model and is not expected to be present in actual flat galaxies.

III. DETERMINATION OF α AND s_n'S

α): A study of the asymptotic slopes of the rotation curves of spirals can, in principle, give α of Eq. (11), if the slope is negative, or s_0 of Eq. (12) if it is positive. Here, however, we are content with an order of magnitude of these parameters. In their list of 1100 rotation curves, primarily used to derive a universal rotation curve, Persic et al [10] find a subset of 27 reliable curves extending out to $2R_{\text{optical}}$ and 200 statistically significant ones farther out than R_{optical}. They define a dark matter indicator, $\delta = [v(2R_{\text{opt}}) - v(R_{\text{opt}})]/v(R_{\text{opt}})$, which is almost one half of the slopes of Eqs. (11) and (12), in the interval $(1-2)R_{\text{opt}}$. Thus

$$\Delta(R_{\text{opt}}) \approx 2\delta = -2[0.05 + 0.16\log(L/10^{10.4} L_\odot)], \qquad (13)$$

where L is the luminosity of the galaxy. They note, the expression is valid in the magnitude range $-23.2 < M_I < -18.5$. Depending on L, negative Δ's are roughly, α of Eq. (11) and the positive ones are s_0 of Eq. (12). In either case they are small and fall in the range of ± 10 percent. Actual *'asymptotic'* slopes, however, should be well below what Eq. (13) indicates. For, one can hardly convince oneself that rotation curves at distances of $(1.5-2)R_{\rm opt}$ have actually reached their asymptotic regime. To summarize, we are inclined, after a qualitative examination of a good number of rotation curves in [9], [10], [8], [13] and others, to infer from Eq. (13) the value '$\alpha \leq$ few percents' for the negative asymptotic slopes of Eq. (11), with a fair confidence. The case of asymptotically positive slopes, if they occur in nature at all, is discussed below.

s_0): The Tully-Fisher relation, initially a power law expression between the circular rotation speeds at the outer reaches of spiral galaxies and their luminosities, can be expressed as a power law relation between the asymptotic speeds and galactic masses. Thus, $v_\infty \propto M^\beta$. A range of values for the exponent, β, can be found in the literature [14], [15], and [13]. We adhere to the commonly quoted value $\beta = 1/4$.

The length scale r_0 in Eqs. (9) and (10) is arbitrary. We choose it roughly the distance to which the rotation curves are extended out to. Because of the smallness of α and s_0 the factors $(r_0/r)^\alpha$ and $(r/r_0)^{s_0}$ approximate to 1 and the asymptotic speed in both equations becomes $v_\infty^2 \approx s_0/2$. This, by Tully-Fisher relation, gives

$$s_0 = \lambda \left(\frac{M}{M_\odot}\right)^{1/2} \tag{14}$$

where M is the baryonic mass of the galaxy, stars+gas, etc. To determine the proportionality constant, λ, one turns to observations. From a list of 31 galaxies in [8], we find $\lambda \approx 3\times 10^{-12}$ [16]. A better estimate is available through MOND. The weak acceleration limit of MOND [12] is

$$v^2/r = \frac{1}{2}\lambda (M/M_\odot)^{1/2} c^2/r = (a_0 GM/r^2)^{1/2},$$

where we have restored the factor c^2 which was suppressed so far, and $a_0 = 1.2\times 10^{-8}$cm sec^{-2} [5] is the universal acceleration of MOND. This yields

$$\lambda = 2(GM_\odot a_0)^{1/2}/c^2 = 2.8\times 10^{-12}. \tag{15}$$

s_1): The term $s_1/2r$ in Eqs. (9) and (10), operative at closer distances, is actually the classical newtonian term. Thus, s_1 should be identified with the Schwarzschild radius of the galactic mass:

$$s_1 = 2GM/c^2. \tag{16}$$

For the remaining s_2 and higher terms we have no suggestion at present. If they exist at all, our idealized spherically symmetric model does not sufficiently closely mimic the actual flattened galaxies to draw meaningful conclusions.

IV. THE DARK COMPANION

From Eqs. (6) and (3) the density of the dark fluid is

$$\rho_d = \left(\frac{r_0}{r}\right)^\alpha \frac{1}{r^2}\left[(1-\alpha)\lambda\left(\frac{M}{M_\odot}\right)^{1/2} - \alpha\frac{s_1}{r}\right], \tag{17}$$

where we have substituted for s_0 from Eq. (14). The expression is valid for $\alpha = 0$ as well. For all practical purposes, the second term in the bracket can be neglected. That the dark density is almost proportional to the square root of the mass of the galaxy is a direct consequence of the Tully-Fisher relation. That it fades away exactly or approximately as r^{-2}, depending on whether $\alpha = 0$ or not, is in accord with ΛCDM simulations of [17] and others.

The dark matter inside a radius r, $M_d(r) = 4\pi\int \rho r^2 dr$, is

$$M_d(r) = 4\pi\lambda\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{r_0}{r}\right)^\alpha r. \tag{18}$$

It is said that the role of the dark matter is more prominent in intrinsically fainter galaxies than in brighter ones, see e. g. [10]. This is inferred from the fact that deviations of the actual rotation curves from the baryonic-matter-based Keplerian ones is larger in fainter galaxies than in the brighter ones. This can be understood by considering the ratio $M_d(r)/M \propto r^{(1-\alpha)} M^{(-1/2)}$. At a given r, normalized to the optical radius of the galaxy, say, this ratio decreases as the galactic mass or equivalently its luminosity increases.

The pressure of the dark fluid is obtained by letting the 4- divergence of the right hand side of Eq. (2) vanish. it leads to the hydrostatic equilibrium of the dark matter:

$$\frac{p_d'}{p_d+\rho_d} \approx \frac{p_d'}{\rho_d} = -\frac{1}{2r}(A-1). \tag{19}$$

Integration is straightforward. The first two terms in the series are

$$p(r) = \frac{1}{4}\left(\frac{r_0}{r}\right)^{2\alpha}\frac{s_0}{r^2}\left[(1-2\alpha)\,s_0 + \frac{2}{3}\left(1-\frac{2}{3}\alpha\right)\frac{s_1}{r} + ...\right], \tag{20}$$

where we have expanded all coefficients involving α and kept only the terms linear in it. The equation of state is barrotropic, $p(\rho)$. It is obtained by eliminating r between Eqs. (17) and (20).

V. INNER SOLUTIONS

The formalism developed here is for regions external to the baryonic matter. Exact interior solutions are involved and are not easily available. The weak field versions, however, can be obtained by redefining M of Eqs. (14) and (16) as the baryonic mass inside the radius r. Thus

$$M(r)=4\pi\int_0^r \rho_b r^2 dr,$$
$$s_0(r)=\lambda\left[M(r)/M_\odot\right]^{1/2},$$
$$s_1(r)=2GM(r)/c^2.$$

The use of $M(r)$ to calculate s_1 in the baryonic interior is known to GR and to the newtonian gravitation. The proof of its use, to infer $s_0(r)$, is involved. It is the subject of a forthcoming paper by [18]. There we also show that the rotation curves calculated on this premise are as good as, if not better than, those obtained by other technics.

VI. CONCLUDING REMARKS

The proposed formalism is a modified GR paradigm or, equivalently, a dark matter scenario to understand the non classical behavior of the rotation curves of spiral galaxies. We approximate the galaxy by a spherical distribution of baryonic matter, attribute a dark perfect gas companion to it , and find its size and distribution by comparing the rotation curve of our hypothetical model with those of the actual galaxies. However, as long as the dark companion displays no physical characteristics other than its gravitation, one has the option to interpret the scenario as an alternative theory of gravitation. For example, one may maintain that the gravitation outside a baryonic sphere is not what Newton or Schwarzschild profess, but rather what one infers from the spacetime metric of Eqs. (6) - (8).

Regions exterior to the baryonic matter are not dark matter vacua. Therefore, the Ricci scalar does not vanish. Its spatial behavior is that of $\rho_d(r)$ as can be inferred from the contraction of Eq. (2). There are also excess lensings and excess periastron precessions caused by the dark matter. These are discussed in [1] and [2].

The formalism is good for spherical distributions of baryonic matters. An axiomatic generalization to non spherical configurations or to many body systems requires further deliberations and more accurate observational data for guidance. One might need further postulates not contemplated so far. The difficulty lies in the fact that there is no superposition principle to resort to. One may not add the fields of the dark companions of two separate baryonic systems; for s_0 of Eq. (14) is not linear in M. As a way out we are planning to expand an extended non spherical distribution into its localized mass-multipole moments and see if it is possible to assign a dark multipole for each baryonic multipole, more or less in the same way done for spherical distributions.

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